MODULE 2

UNIT 1 MOTION IN MORE THAN ONE DIMENSION

CONTENTS

1.0 INTRODUCTION

In treating the topic, motion, we have so far discussed only motion along a straight line or rectilinear motion. In this chapter, we shall consider motion in more than one dimension. This is the same thing as discussing motion in a plane and in three dimensions. You have realised, from your studies of unit 1 to 4 that our physical world is in three dimensional space. So, as a particle moves, its co-ordinates with reference to a specified frame changes in two or three dimensions depending on where the motion is taking place. Having realised from unit 5 that the parameters for describing motion which include displacement/distance, velocity and acceleration are vector quantities, we shall draw on our knowledge of vectors from units 3 and 4 to understand this Unit better.

We shall also study circular motion which will give us an insight into satellite motion, and then conclude the Unit with studies of Relative Motion. Other types of motion and causes of motion will be developed in the subsequent Units.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- determine the displacement, velocity and acceleration of a particle in two or three dimensions in any given frame of reference.
- distinguish between average and instantaneous velocity, and average and instantaneous acceleration in two or three dimensions.
- determine relative velocity and acceleration of one particle with respect to another particle
- solve problems concerning relative motion and uniform circular motion.

3.0 MAIN CONTENT

3.1 Displacement, Velocity and Acceleration

Let us consider the motion of a particle in space (Fig 3.1)

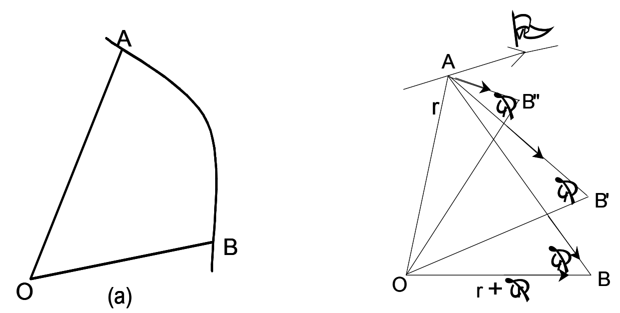


Fig. 3.1

$\Delta\vec{r}$ *and* $\vec{r} + \Delta\vec{r}$ If the particle is at position A at some instant of time t and at position B at another instant of time t + Δt. Recall that the position of a particle in a particular frame of reference is given by a position vector drawn from the origin of the coordinate system in that frame to the position of the particle. In our diagram (Fig. 3.l), let the position vectors of A and B with respect to 0 be respectively. The displacement of the particle in the time interval is equal to in the direction AB. Thus, the average velocity of the particle during the time t is given by

$$\vec{v}_{av} = \frac{\Delta \vec{r}}{\Delta t} \qquad 3.1$$

$\vec{\Delta r}_{av}$ The direction is the same as that of since Δt is a scalar quantity. $\vec{v}_{av}$ We note that is the velocity at which the particle would have travelled the distance AB in uniform and rectilinear motion during the time interval Δt.

SELF-ASSESSMENT EXERCISE 1

If the displacement versus time equation of a particle falling freely from rest is given by

$$x = (4.9m\ s^{-2})t^2$$

Where x is in metres, t is in seconds. Calculate the average velocity of the particle between time, t_1 = 1s and t_2 = 2s and also between t_3 = 3s and t_4 = 4s.
When you solved exercises 3.1, you noticed that the values of average velocities during the two time intervals are not the same.

Such a motion is described as non-uniform motion. A practical example of non uniform motion is the motion of a bus leaving one bus stop and travelling up to the next bus stop. The velocity of the bus at a given instant of time can be found.

$\Delta \vec{r}$ We remark that the velocity of a particle may change as a result of change in magnitude, direction or both. In Figure 3.1b above, the average velocity during the time interval Δt is directed along the chord AB but the motion has taken place along the arc (AB). The average velocities during the intervals Δt^1(i.e A to B^1) and Δt^{11} (i.e. A to B^{11}) are different both in magnitude and direction. The time interval Δt^{11} is smaller than Δt^1, which is in turn smaller than Δt. Note that as we decrease the interval of time, the point B approaches A, i.e. the chord approximates the actual motion of the particle better. These points finally merge and the direction of coincides with the tangent to the curve at the point of merger.

$$\frac{\Delta \vec{r}}{\Delta t} as \Delta t \to 0,$$

$\overline{v} \frac{\vec{r}}{\Delta t}$ As Δt decreases, the ratio approaches a limit. The vector, having the magnitude equal to the limit of the ratios called the instantaneous velocity of the particle at time t.

The instantaneous velocity is the direction of the tangent to the curve at the given moment of motion.
Thus,

$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{d \vec{r}}{dt} \qquad 3.1$$

$\vec{r}$ In other words, the instantaneous velocity is the derivative of with respect to time.

$$\vec{v} = \frac{d \vec{r}}{dt} \qquad 3.2a$$

It follows from equation 3.2a that if has components x, y, z then

differentiating the RHS we get

since i, j, k are independent of time

$$\vec{\vee} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(xi + yj + zk)$$

$$\vec{\vee} = x\frac{di}{dt} + i\frac{dx}{dt} + y\frac{dj}{dt} + j\frac{dy}{dt} + z\frac{dk}{dt} + k\frac{dz}{dt}$$

$$= \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k$$

$$= \vee_x\, i + \vee_y\, j + \vee_z\, k \text{ where}$$

$$\vec{\vee}_x = \frac{dx}{dt}, \vec{\vee}_y = \frac{dy}{dt}, \vec{\vee}_z = \frac{dz}{dt} \qquad 3.2b$$

Note that if we were using coordinates alone to write the equations for velocity, we would have to write three equations as in Equation 3.2b. The advantage of the use of vectors is that it enables us to write a single equation as in equation 3.2a.

Representing the instantaneous velocities of the particle when it passes through points A and B of its path as shown in Figure 3.2,

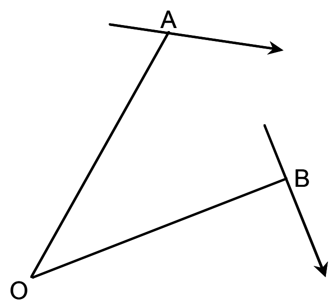


Fig. 3.2

We see that the velocity at B is different from that at A. This means that the velocity is changing in magnitude and direction. Thus the particle experiences an acceleration. Definition of average acceleration is given thus:

$\vec{a}_{av} = \frac{\vec{v}+\Delta\vec{v}}{\Delta t}$If the velocity of the particle changes from within the time interval from t to t + t, then the average acceleration during this interval of time is given by

$$a_{av} = \frac{\Delta \vee}{\Delta t} \qquad 3.3$$

$\Delta v / \Delta t$ The direction of is along . Remember that Δt is a scalar quantity. Now, as the interval of time Δt decreases, the ratio approaches a limit. Hence we define the instantaneous acceleration of a particle at any particular instant of motion as

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{\vee}}{\Delta t} = \frac{d\vec{\vee}}{dt} \qquad 3.4a$$

$$\vec{a}_x = \frac{d\vec{\vee}_x}{dt}, \vec{a}_y = \frac{d\vec{\vee}_y}{dt}, a_z = \frac{d\vec{\vee}_z}{dt} \qquad 3.4b$$

So, from our knowledge of calculus, acceleration is the derivative of velocity with respect to time, i.e. and in component form, we have

SELF-ASSESSMENT EXERCISE 2

Given a wire helix of radius R oriented vertically along the z-axis. If a frictionless bead slides down along the wire (Fig.3.3), and its position vector varies with time as

$$\vec{r}(t) = (R\cos bt^2)i + (R\sin bt^2)j - \tfrac{1}{2}ct^2 k$$

$\vec{a}(t)$ where b and c are constants, find and , where and are the velocity and acceleration expressed as functions of t.

Solution:

$\vec{r}(t)$ From the expression for given. We know that for the three axes, x, y, z.

$x = R\cos bt^2$, $y = R\sin bt^2$, $z = ½ ct^2$

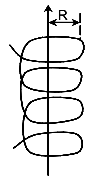


Fig. 3.3

Recall that

$$\vec{v}(t) = \frac{d\vec{r}}{dt} = \frac{dx}{dt}i + \frac{dy}{dt}j + \frac{dz}{dt}k$$

$$\therefore \quad \vec{v}(t) = (-2tb\ R\sin bt^2)i + (2t\,bR\cos bt^2)j - (ct)k$$

$\vec{a}(t)$ The acceleration is given by

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = (4t^2b^2R\cos bt^2 - 2Rb\sin bt^2)i$$
$$+(-4t^2Rb^2 Sin bt^2 + 2Rb\cos bt^2)j - ck$$

SELF-ASSESSMENT EXERCISE 3

A particle moves along the curve y = Ax^2 such that x = Bt, A and B are constants. (a) Express the position vector of the particle in the form $\vec{r}(t)$ = xi, + yj,

(b) calculate the speed of the particle along this path at any instant t. $\left[v = \left|\frac{dr}{dt}\right|\right]$

Solution:

$$(a) \qquad \vec{r}(t) = Bti + AB^2t^2j$$

$$(b) \qquad \vec{v} = \frac{d}{dt}\left\{\vec{r}(t)\right\} = Bi + 2AB^2tj$$

$$\therefore\ Speed = \left|\vec{v}\right| = v = \sqrt{B^2 + 4A^2t^2B^4}$$
$$= B\sqrt{1 + 4A^2t^2B^2}$$

3.2 Uniform Circular Motion

We shall now use the concepts we have developed so far to study uniform circular motion and you will see how simple it will all become. Uniform circular motion plays an important role in physics. Uniform circular motion approximates many diverse phenomena, such as rotation of artificial satellites in circular orbits, designing of roads, motion of electrons in a magnetic field etc.

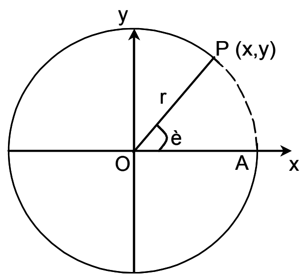


Fig.3.4 Uniform circular motion

In Figure 3.4, let us assume that a particle P is performing a circular motion along the circle, part of which has been represented by the curve with broken lines. This particle, therefore, maintains a constant distance r from the centre of the circle, O. Let us also assume it turns through a constant angle 2 in a fixed time. Let A be the position of the particle along x axis at time t = 0. Now, t seconds later, it is at point P after describing an angle 2 (= < AOP). Through 0 we draw y-axis perpendicular to x-axis. Let the coordinates of P with respect to the mutually perpendicular axes x and y be (x, y). From our knowledge of trigonometry and our course on resolution of vectors in unit, 2 and 3 we have that:

x = r Cos Ω
y = r Sin Ω 3.5a

Now, if the angle described per second by the particle be a constant equal to Ω (pronounced 'omega') radians, then Ω = Δt and eqn. 3.5 can be written as

x = r Cos Ωt
y = r Sin Ωt 3.5b

$$\overrightarrow{r} = xi + yj \qquad 3.6$$

or

$$\overrightarrow{r} = r\cos \omega t i + r \sin wtj,$$

T is also known as the angular speed of the particle. We note that the position vector of the particle at P is given by

= -r2 sin Tti + r 2cos 2tj 3.7

= $V_x I + V_y j$

where V_x = - rT sin Tt, V_y = rT cos Tt 3.8

The magnitude of velocity is therefore,

$$\vec{v} = \frac{d\vec{r}}{dt}$$

$$\left|\vec{v}\right| = v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

$$= \sqrt{r^2 w^2} = rw \qquad 3.9$$

$\vec{v} . \vec{r}$ What is the direction of this velocity? To find out, let us calculate

$\vec{v} . \vec{r}$ = (- rT Sin Tti + rT Cos Ttj).(Cos Tti + r Sin Ttj)

= - r^2T Sin Tt cos Tt + r^2 Cos Tt Sin Tt = 0

$\vec{v} . \vec{r} = 0$ we see that

$\vec{a}_R$ Since V. r = 0 is always perpendicular to. This implies that V is always along the tangent to the circular path. Eqn 3.9 reveals that has a constant magnitude.

We have found that for circular motion, the particles velocity constantly changes direction because it (the velocity) is always along the tangent at any point. So, we conclude that the velocity vector is not constant, i.e., the particle has an acceleration. Let us denote the particle acceleration by and then find the appropriate expression for it .

Recall that acceleration

$$= d\frac{\vec{v}}{dt}$$

We, therefore,

$$\vec{a}_R = rw^2 \cos wti - rw^2 \sin wtj \qquad 3.10$$

$$= -w^2(\cos wti + r \sin wtj)$$

$$\therefore \vec{a}_r = -\omega^2 \vec{r} \qquad 3.11$$

have from Eqn (3.8) that
Since v = Tr from Eqn 3.9, we get

$$\left|\vec{a}_R\right| - \omega r = \frac{v^2 r}{r^2} = \frac{v^2}{r} \qquad 3.12$$

$\vec{r}$ The negative sign in the expression for the acceleration Eqn (3.11) indicates that the acceleration is opposite to i.e. towards the centre of the circle. I would then like you to remember that a particle moving with uniform angular speed in a circle, experiences an acceleration directed towards the centre. This is known as centripetal acceleration.

Example

Let us calculate the period of revolution of a satellite moving around the earth in a circular equatorial orbit, (Fig. 3.5).
Let the velocity of the satellite in the

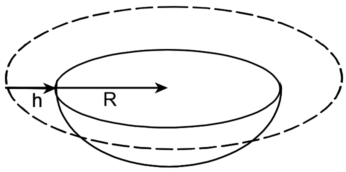


Fig.3.5

$\vec{v}$ orbit be, and the radius of the orbit be r. Like any free object near the earth's surface, the satellite has an acceleration towards the centre of the earth (= g, say), which is the centripetal acceleration. It is this acceleration that causes it to follow the circular path. Hence from Eqn (3.12), we have

$$g^1 = \frac{v^2}{r}$$

$$or\, v^2 = g^1 r$$

If the angular speed of the satellite is T, we get from Eqn (3.9) that

$$\omega^2 r^2 = g^1 r$$

or

$$\omega^2 = \frac{g^1}{r}$$

Again the time period T is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g^1}}$$

or

$$T = 2\pi\sqrt{\frac{R+h}{g^1}}$$

Where R = the radius of the earth and h = the height of the satellite above the surface of the earth.

The orbit of the first artificial satellite Sputnik, was almost circular at a mean height of 1.7 x 10^5m above the surface of the earth, where the value of acceleration due to gravity is 9.26m s^{-2}.

Thus the time taken for the satellite to complete one revolution round the earth was

$$T = 2\pi\sqrt{\frac{(6.37x\,10^6 + 0.17\,x\,10^6)m}{(9.26)m\ s^{-2}}}$$

$$= 5.28\,x\,10^3\,s = 1hr.28\min$$

SELF ASSESSMENT EXERCISE 3.3

A flat horizontal road is being designed for 60kmh^{-1} speed limit. If the maximum acceleration of a car travelling on the road is to be 1.5 m s^2 at the above speed limit, what must be the minimum radius of curvature for curves in the road?

Solution

$$\frac{v^2}{r} > a \ \ or \frac{v^2}{a} > r$$

or $$r < \frac{v^2}{a}, i.e\ r_{\min} = \frac{v^2}{a}$$

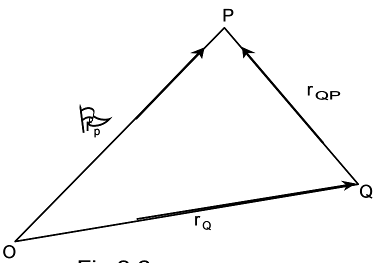


Fig.3.6

Since $v = 60km\ h^{-1}$, $a = 1.5ms^{-1}$

$$\therefore r_{\min} = 1.8\,x\,10^2\,m$$

Let us recapitulate what you have learnt so far. You now know that the language for describing motions is displacement, velocity and acceleration. You have also learnt about these quantities using vectors.

We have also pointed out that the position, velocity and acceleration of a particle can only be defined with respect to some reference frame. The friends travelling in the same car are at rest with respect to each other, while they are in relative motion with respect to a person standing on the roadside. The velocity of their car as measured by the person standing on the roadside will be different from that measured by an "Okada" cyclist moving along the same road. Hence, saying that a car moves at say, 60kmh^{-1} means that it moves at 60kmh^{-1} relative to the earth. But the earth itself is moving at 30km h^{-1} relative to the sun. Thus the speed of the car relative to the sun is much, greater than 60km h^{-1}. By these examples we are only trying to show that all motion is relative. This is interesting, isn't it? Often in practical situation, we need to determine the relative position, velocity and acceleration of a particle or an object with respect to another one. In the next section we shall find out how this is done.

3.5 Relative Motion

In this section, we shall discuss relative motion. Your knowledge of units 1, 3 and 4 will be applied here.

$\vec{r}_p$ *and* $\vec{r}_Q$ Let be the position vectors of particles P and Q, respectively, at any instant of time, with respect to a fixed origin 0. This has been drawn in Figure 3.6 above.

$$\vec{r}_Q + \vec{r}_{Qp} = \vec{r}_p$$

$$\text{or } \vec{r}_{Qp} = \vec{r}_p - \vec{r}_Q \qquad 3.16$$

$\vec{r}_{Qp}$, Thus, the relative velocity of P with respect to Q is got by differentiating with respect to time.

Thus,

$$\vee_{QP} = \frac{d}{dt}\vec{r}_{QP} = \frac{d\,\vec{r}_p}{dt} = \frac{d\,\vec{r}_Q}{dt}$$

or

$$\vec{v}_{QP} = \vee_P - \vec{v}_Q \qquad 3.17$$

$\vec{a}_{Qp}$ Relative acceleration of P with respect to Q us given by,

$$\vec{a}_{Qp} = \frac{d}{dt}(\vec{v}_{QP}) = \frac{d\vec{v}_p}{dt} - \frac{d\vec{v}_Q}{dt}$$

or

$$\vec{a}_{QP} = \vec{a}_P - \vec{a}_Q \qquad 3.18$$

$\vec{a}_{..} = 0$ If is constant then and we conclude that

$$\vec{a}_{Qp} = \vec{a}_P$$

This means that the relative acceleration of P with respect to Q is the same as the acceleration of P with respect to O, provided Q has a constant velocity with respect to 0.

Let us consider the practical problem of Navigation and avoiding collisions at sea. Imagine that two ships S_1 and S_2 moving with constant velocities are at the positions A and B shown in Figure 3.7 at some instant of time. The vectors V_1 and V_2 represent their velocities with respect to the sea. The paths of the ships extended along their directions of motion from the initial points A and B intersect at point P.

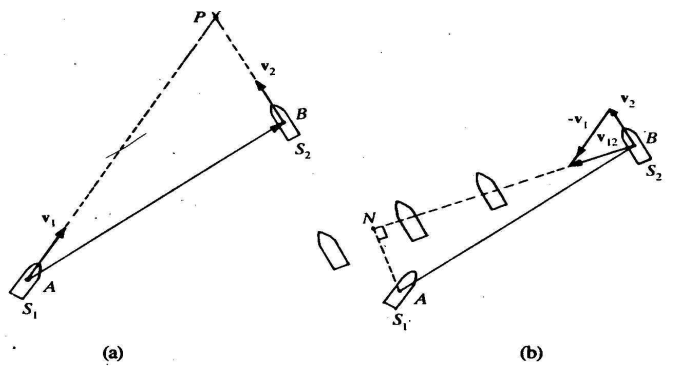


Fig 3.7 (a) Path of two ships moving at constant velocity along courses that intersect;
(b) Path of S_2 relative to S_1 showing that they do not collide even though their paths cross.

$\vec{v}_{12} = \vec{v}_2 - \vec{v}_1$ Will the ships collide, or will they pass one another at a distance?

The relative velocity of ship S_2 with respect to ship S_1 is given from Eqn 3.17 as

$\vec{V}_{12}$ is shown in Figure 3.7b.

$\vec{v}_{12}$ Now, with respect to ship S_1, ship S_2 follows the straight line .
It will miss S_1 by the distance AN. If you have travelled in a ship and experienced an event of this sort, on an open sea with land marks in sight, you will know that it is a curious experience. The observed motion of the other ship seem to be unrelated to the direction in which it is going.

$\vec{v}$ We can now generalize our observations using equation 3.17 and 3.18 concerning relative motion. Let an object move with velocity relative to a frame of reference S, if another frame of reference S^1 moves with velocity V relative to S (Fig. 3.8), then the velocity V^1 of the object with respect to the frame S^1 is given by

$$\vec{v}^{\,1} = \vec{v} - \vec{V} \qquad 3.19$$

$\vec{v}$ If a is constant, then

$$\vec{a}^{\,1} = \vec{a} \qquad 3.20$$

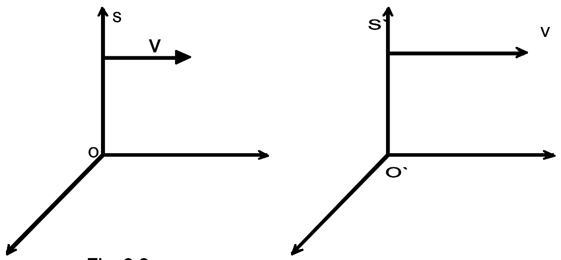


Fig. 3.8

Thus the acceleration of an object is the same in all frames of reference moving at constant velocity with respect to one another. The discussion has but tested our earlier conclusion in Unit 1 that absolute motion is trivial (i.e. unrealistic). We need always to study the motion of one object with respect to another.

4.0 CONCLUSION

In this unit you learnt that:

(i) Motion involves change in the position of an object with time.
(ii) The language used to describe motion are displacement, velocity and acceleration.
(iii) You have also learnt how to determine velocity and acceleration both along a straight line or on a circular motion.
(iv) You have also learnt about relative motion and how it is determined.

5.0 SUMMARY

What you have learnt in this unit are:

- A body is said to be in motion if it changes position with time
- A frame of reference is required to determine any kind of variation of position with time.
- That the instantaneous velocity and instantaneous acceleration of the particle are:

$$\vec{v} = \frac{d\vec{r}}{dt} = v_x i + v_y i + v_z k$$

$$where\, v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

$$and\ \vec{a} = \frac{d\vec{v}}{dt} - \frac{d^2\vec{r}}{dt^2} = a_x i + a_y j + a_z k$$

$$where\ \vec{a} = \frac{d\vec{v}_x}{dt} = \frac{d^2\vec{x}}{dt^2},\ \vec{a}_y = \frac{d\vec{v}_y}{dt} = \frac{d^2 y}{dt^2}$$

$$a_z = \frac{d\vec{v}_z}{dt} = \frac{d^2 z}{dt^2}$$

(iv) For a particle performing uniform circular motion, the instantaneous velocity is always directed along the tangent and. Has magnitude v = T where r is the radius of the circle and is the angular speed of the particle.

(v) That the instantaneous acceleration is directed towards the centre, and has magnitude

$$\left|\vec{a}_R\right| = \frac{v^2}{r} = \omega^2 r$$

(vi) That motion is relative

(vii) That the relative position and velocity of a particle P with respect to a particle, Q are given as

$$\vec{r}_{QP} = \vec{r}_p - \vec{r}_Q \qquad and$$

$$\vec{v}_{QP} = \vec{v}_p - \vec{v}_Q$$

where r_P and r_Q are the position vectors of P and Q in a given frame of reference. V_P and V_Q are the velocities of P and Q in this frame.

6.0 TUTOR-MARKED ASSIGNMENT

1. Why is the statement "I am moving " meaningless?
2. An automobile A, traveling relative to the earth at 65km h^{-1} on a straight level road, is ahead of an Okada cyclist (motor cyclist) B traveling in the same direction at 80km h^{-1} What is the velocity of B relative to A?
3. A small body of mass 0.2 kg moves uniformly in a circle on a horizontal frictionless surface, attached by a cord 0.2m long to a pin set in the surface. If the body makes two complete revolutions per second, find the force P exerted on it by the cord.

7.0 REFERENCES/FURTHER READING

Spiegel, M. R. (1959). *Vector Analysis-Schaum's Series*. New York: McGraw Hill Book Company.

Stroud, K. A. (1995). *Engineering Mathematics*. (4th ed.). London: Macmillan Press Ltd.

Das Sarma, J. M. (1978). *An Introduction to Higher Secondary Physics.* India: Modern Book Agency Private Ltd.

Fishbane, P. M., Gasiorowicz, S & Thronton S. T. (1996). *Physics for Scientist and Engineer.* (2nd ed.). Vol. 1 New Jersey: Prentice Hall.

Indira Gandi National Open University School of Sciences. PHE 01, Elementary Mechanics.

UNIT 2 FORCE

CONTENTS

1.0 INTRODUCTION

In the last two Units we explored the parameters that describe motion such as velocity and acceleration. Such a description is called Kinematics. Kinematics alone cannot predict the possible motion of an object. In this Unit, we shall look at other things that cause changes in motion of an object. The studies of the causes of motion are called Dynamics. A scientist called Sir Isaac Newton described the laws that govern motion in 1687. These are based on careful and extensive observations of motion and its changes. It may interest you to know that these laws actually provide an accurate description of motion of all objects, whether they are small or big, whether they are simple or complicated, though with minute exceptions. These exceptions include motions within the atoms and motions near the speed of light (300,000km s^{-1}). I would like you to note that Newton's laws represent tremendous achievement in their simplicity and breadth of what they cover. We use Newton's law to calculate the motion of a body given the force acting on it.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define a force
- State the conditions for equilibrium of a rigid body acted upon by a system of forces.
- State the three Newton's laws of motion for a particle in linear motion
- solve problems using conditions for equilibrium of forces and Newton's laws of motion.

3.0 MAIN CONTENT

3.1 Definition of Force

What makes things move? I invite you to keep this question at the back of your mind as you study this Unit. In this Unit we shall, in answer to the question above, look a bit to the history of physics. Way back in the fourth Century B.C., Aristotle proffered an answer to the question above. And for nearly 2000 years following his work most scientists believed in his answer that a force-which may be a push or a pull-on something was needed to keep the thing moving. The motion ceased when the force was removed. This stands to reason because from our experience we know that when we pull or push a wheel it moves. But when we stop pushing or pulling the wheelbarrow, it remains at relative rest. Therefore, when we push or pull on a body, we are said to exert a force on the body. Non- living things can also exert force on other things. For example, a relaxed spring exerts force on the body to which its ends are attached when compressed and released.

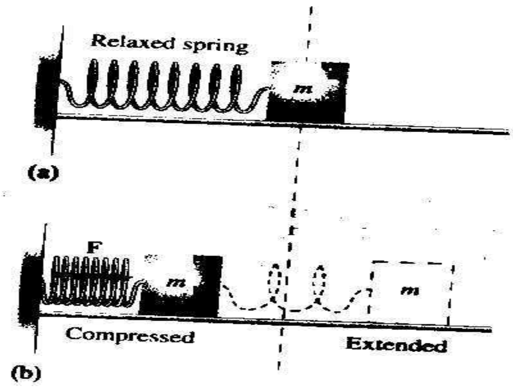


Fig 3.1

In Figure 3.1 we show a mass, m attached to the end of a released spring. The end of the spring is then pushed to the left and released. It is seen to exert a force on the mass and pushes it to the right. Also, in our daily life, we experience what we call gravitational force. For example, stop reading and throw any object around you vertically upwards. What do you observe? You see that the object got to a certain height and started coming down. What happens in effect is that the earth exerts a force of gravity on it to attract it to itself (the earth). This type of force we call weight. The earth exerts this pull on every physical body. Gravitational, electrical and magnetic forces can act through empty space without contact. Other forces can be termed contact forces.

Contact forces are forces resulting from direct contact of two or more objects. Contact forces are said to be mainly as a result of attraction and repulsion of the electrons and nuclei making up the atom of materials.
To describe a force, we need to describe the direction in which it acts and also the magnitude of the force. This shows us that force is a vector quantity.

3.1.1 Graphical Representation of Force

Since forces are vectors, forces are represented exactly like vectors. So, everything we studied in Units 3 and 4 about vectors apply to forces including vector representation, addition, subtraction etc. So, I would like you, at this juncture to go back and read Units 3 and 4 again to refresh your memory. But for the sake of concretising what you have learnt, let us give one example of how forces are represented. If you slide a box along the floor by pulling it with a string or by pushing it with a stick, the box moves (Fig 3.2)

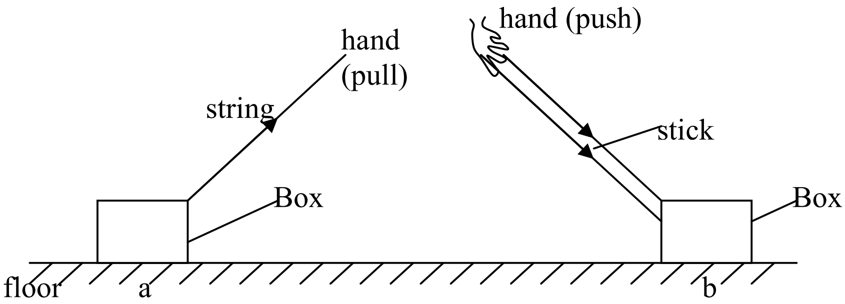


Fig. 3.2

Note that it is not the objects (hand, stick, or string) that make the box to move but the force exerted by these objects. If we imagine the magnitude of the pull or push to be "10N". Then, writing just 10N on the diagram would not completely describe the force because it does not indicate the direction in which the force acts. One might write "10N, 30^0 above horizontal to the right" or "10N, 45^0 below the horizontal to the right". But all the above could be more briefly conveyed by representing the force by a line with an arrow head. The length of the arrow to some chosen scale gives the magnitude of the force and the direction of the arrow indicates the direction of the force. An example is given below in Fig. 3.3

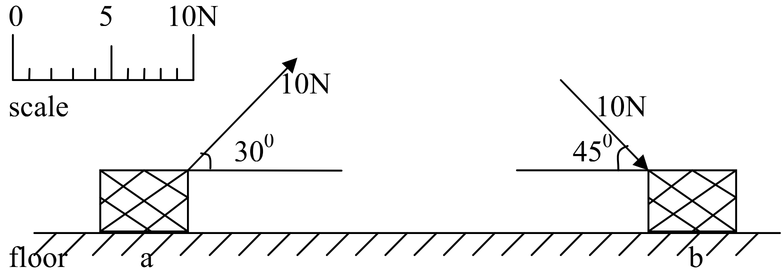


Fig. 3.3

This is the force diagram corresponding to Figure 3.2. We neglect other forces acting on the box.

3.1.2 Equilibrium

We have seen that one effect of a force is to change the motion of the object on which it acts. Force also can alter the dimensions of an object. The motion of an object is made up of both translational motion and rotational motion of the object where applicable. In some cases a single force can produce a change in both translational and rotational motion of a body at once. But when several forces act on a body simultaneously, the effect can cancel each other resulting in no change either in translational or rotational motion. When this happens, the body is said to be in equilibrium. This means that

(i) the body as a whole either remains at rest or moves in a straight line with constant speed and
(ii) the body is not rotating at all or is rotating at a constant rate.

Now, let us look at an example to explain what we mean.

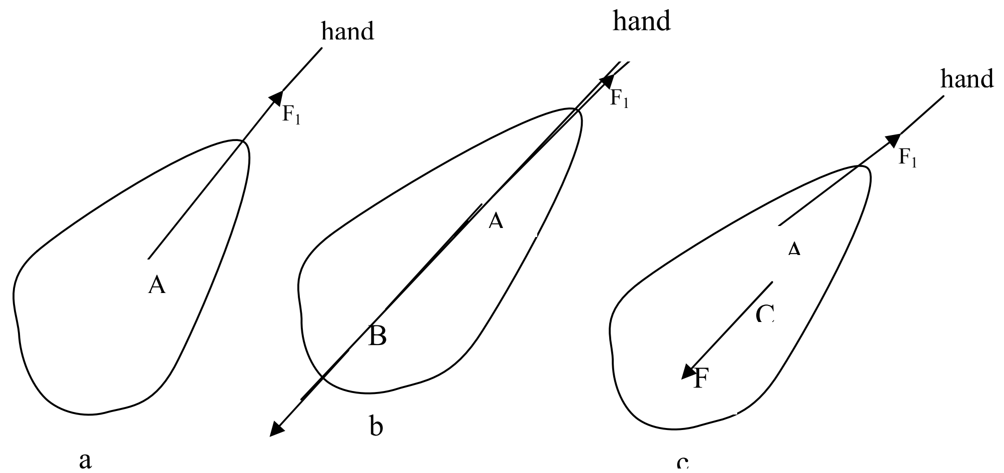


Fig 3.4

The forces acting on a body under different conditions are as indicated in Figure 3.4. If force $\vec{F}_1$ only is applied as is in Figure 3.4a, the body originally at rest will move and also rotate clockwise. So it no longer remains in equilibrium. But if an equal force is applied to it in the opposite direction (Fig. 3.4b) and it has the same line of action, then the resultant force is zero and equilibrium will be maintained. Otherwise translational but not rotational motion will set in (Fig. 3.4c). The force , in this case, will form what we call a couple. This will be discussed later.

Mathematically if

$$\vec{F}_2 = -\vec{F}_1 \qquad 3.1$$

then the Resultant, $\vec{R}$ is

$$\vec{R} = \vec{F}_1 + \vec{F}_2 = \vec{F}_1 - \vec{F}_1 = 0 \qquad 3.2$$

Let us adopt the convention that when we say that two forces are "equal and opposite" we mean that their magnitudes are equal and that one is the negative of the other. This meaning is what is conveyed throughout this course when three nonparallel coplanar forces

$\vec{F}_1, \vec{F}_2, \vec{F}_3$ act on a rigid body, for equilibrium to be maintained, the resultant of the forces must be zero. Let us look at Figure 3.5

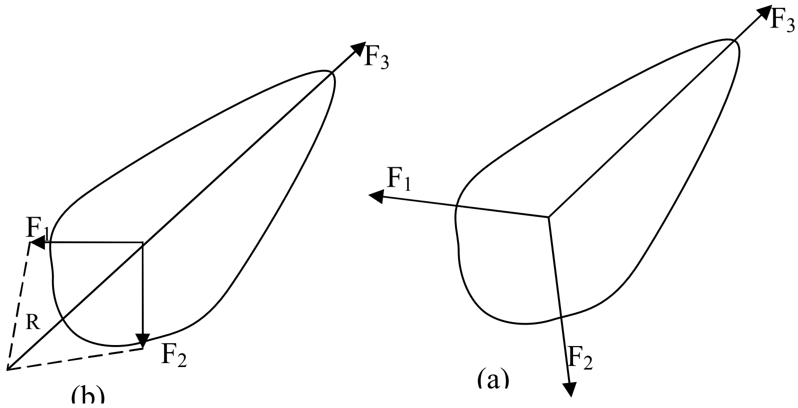


Fig 3.5

A force applied to a rigid body is taken to be acting anywhere along its line of action. Therefore, we can transfer the two forces F_1 and F_2 Figure 3.5a to the point of intersection of their lines of action. We then obtain their resultant, R as indicated in Figure 3.5b. By so doing, we have reduced the force to just two i.e. $\vec{R}\, and\, \vec{F}_3$. For equilibrium to be maintained, these two forces $\vec{R}\, and\, \vec{F}_3$ must.

(i) be equal in magnitude
(ii) be opposite in direction
(iii) have the same line of action.

It then follows from the first two conditions that the resultant of the three forces $\vec{F}_1, \vec{F}_2\, and\, F_3$ is zero. Note that the third condition can only be fulfilled if the line of action of $\vec{F}_3$ passes through the intersection of the lines of forces of $\vec{F}_1\, and\, \vec{F}_2$ as shown in Figure 3.5b. Another important point to note is that when the lines of action of several forces pass through a point, the forces are said to be concurrent. The body in Figure 3.5b can be in equilibrium only when the three forces are concurrent.

Stable, Unstable and Neutral Equilibrium

On displacing a body in equilibrium slightly, the magnitudes, directions and lines of action of the forces acting on it may all change.

Stable equilibrium

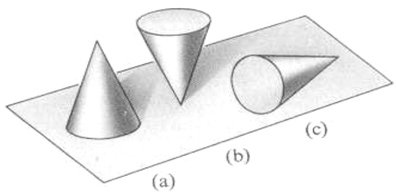
This happens when the forces in the displaced position act such that they return the body in its original position Fig. 3.6a.

Unstable equilibrium

If the forces act to increase the displacement still further, the equilibrium is unstable. Fig. (3.6b).

Neutral equilibrium

If the body after being displaced is still in equilibrium, the equilibrium is neutral. (Fig.3.6c)



(a) Stable, (b) unstable, and (c) neutral equilibrium.

Moments

When the door of a room is opened, the applied force is said to exert a moment, or turning effect about the hinges attached to the back edge of the door and the wall. The magnitude of the moment of a force P about a point 0 is defined as the product of the force P and the perpendicular distance OA from 0 to the line of action of P. See the Figure 3.7a below:

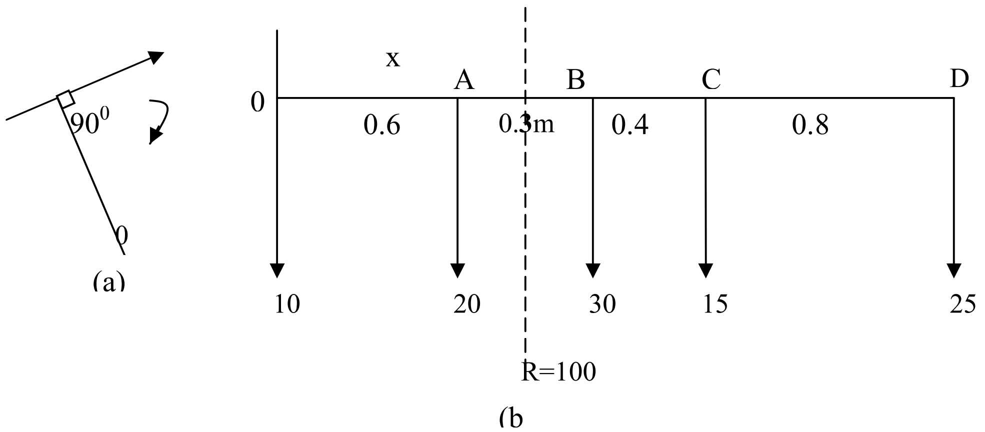


Fig 3.7

Thus, moment about point 0 = P x A0. The magnitude of the moment is expressed in Newton metre (Nm) when P is in Newtons and A0 is in metres. By convention, we shall take an anticlockwise moment as positive in sign and a clockwise moment as negative in sign.

Parallel Forces

If a rod carries loads of 10, 20, 30, 15 and 25N at point 0, A, B, C, D respectively, the resultant, R of the weights which are parallel forces for all the forces in Figure (3.7b) is

resultant, R = (10 + 20+ 30 + 15 + 25) N
= 100N

From experimental results and theory it was seen that the moment of the resultant of a number of forces about any point is equal to the algebraic sum of the moments of the individual forces about the same point. This result helps us to locate where the resultant of R acts.

Taking moments about 0 for all forces in Figure (3.7b) we have (20 x 0.6) + (30 x 0.9)+ (15x 1.3) + (25 x 2.1) because the distances between the forces are 0.6m, 0.3m, 0.4m, 0.8m, as shown. If xm is the distance of the line of action of R from 0, then, the moment of R about 0 = R x X = 100 x X
100x = (20 x 0.6) + (30 x 0.9) + (15 x 1.3) + (25 x 2.1)

i.e.
X = 1.1m

Equilibrium of Parallel Forces

The resultant of a number of forces in equilibrium is zero. Recall that we saw this in Unit 7. It therefore follows that the algebraic sum of the moments of all the forces about any point is zero provided the forces are in equilibrium. What does this mean? It means that the total clockwise moment of the forces about any point = the total anticlockwise moment of the remaining force about the same point.

SELF-ASSESSMENT EXERCISE 1

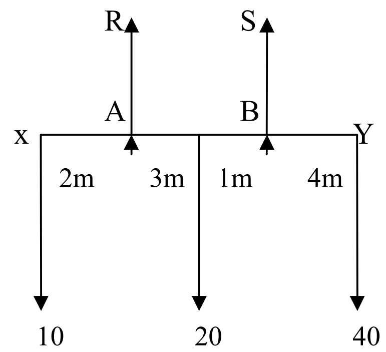


Fig 3.8

Suppose a light beam XY rests on two points A and B and has loads of 10, 20, and 4N at points, X_1 0, Y respectively, then for equilibrium in the vertical direction to hold

R + S = (10 + 20 + 4)N

= 34N

Then, to find R, we take moments about a suitable point such as B. Note that at point B, the moment of S in zero.
Then for the other forces we have
10 x 6 + 20 x 1 - R x 4-4 x4 = 0
hence, we see that

R = 16N

So, from the value for S + R above, it follows that S = 34 - 16 = 18N

SELF ASSESSMENT EXERCISE 3.2

Suppose that a 12m ladder of 20kg is placed at an angle of 60^0 to the horizontal, with one end B leaning against a smooth wall and the other end A on the ground.

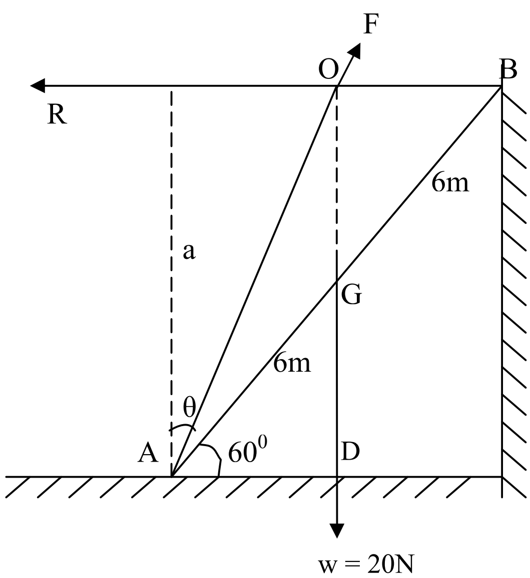


Fig 3.9

The force $\vec{R}$ at B on the ladder is called the reaction of the wall, and if the latter is smooth,
$\vec{R}$ acts perpendicularly to the wall. Let us assume that the weight of the ladder, w acts from the midpoint of the ladder $\vec{G}$, the forces $\vec{R}$ and $\vec{G}$ meet at 0 as shown above.

Consequently, the frictional force $\vec{F}$ at A passes through 0. Use the triangle of forces to find the unknown forces $\vec{R}$, $\vec{F}$

Solution

Since DA is parallel to R, AO is parallel to F, and OD is parallel to W, the triangle of forces is represented by AOD. By means of a scale drawing R and F can be found, since

$$\frac{w(20)}{OD} = \frac{F}{AO} = \frac{R}{DA}$$

A quicker method is to take moments about A for all the forces. The algebraic sum of the moments is zero about any point since the object is in equilibrium and hence,

R x a - w x AD = 0

where a is the perpendicular distance from A to R. (F has zero moment about A)

But a = 12 sin 60^0 , and AD = 6 cos 60^0

R x 12 sin 60^0 - 20 x 6 cos 60^0 = 0

$$R = 10 \frac{\cos 60^0}{\text{Sin } 60^0} = 5.8\text{N}$$

Suppose θ is the angle F makes with the vertical, resolving forces vertically, F cos = w = 20N. Resolving horizontally, F sin = R = 5.8N

$$\therefore\ F^2 \cos^2\theta + F^2 \sin^2\theta = F = 20^2 + 5.8^2$$

$$\therefore\ F = \sqrt{20^2 + 5.8^2}$$

$$= 20.8N$$

We have used graphical method to provide satisfactory solution of problems in equilibrium. But it is much easer to use rectangular components of the forces to sum up forces acting on a body. We refer to this as analytical method. Recall from your knowledge of resolution of vectors into its Cartesian coordinates, that the resultant, R or a set of coplanar forces (i.e. forces acting in one plane) are $R_x = \sum f_x$ i.e. sum of all x - components of the forces

$R_y = \sum f_y$ i.e. sum of all y - components of the forces. Hence, when a body is in equilibrium, the resultant of all the forces acting on it is zero. This means that all the Cartesian components of the vectors must sum up to be zero.

$$R = 0 \, or \sum f_x = 0, \sum f_y = 0 \qquad 3.3$$

These set of equations are called the first condition of equilibrium. The second condition is that the forces must have no tendency to rotate the body.

Note that the first condition of equilibrium ensures that a body be in translational equilibrium while the second condition ensures that it be in rotational equilibrium. These two conditions are the basis for Newton's first law.

Consider the body in Figure (3.8) below part (a) hanging at rest from the ceiling by a vertical cord.

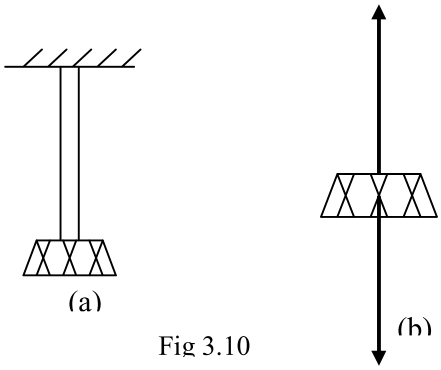


Fig 3.10

Part (b) of the Figure is the free-body diagram for the body. The forces acting on it are its weight w_1 and the upward forceT_1 exerted on it by the cord. Resolve the forces along the x and y-axes and find the conditions of equilibrium.

Solution
Let the x axis be along the horizontal and the y axis be along the vertical axis. There are no x components of the forces

$$\therefore \sum f_x = 0$$

The y -component of the forces are W_1 and T_1. For equilibrium to hold, $\sum f_y = 0$. This means that

$$T_1 - w_1 = 0 = \sum f_y$$
$$\therefore T_1 - w_1\ (\mathit{first\ law})$$

Now for their line of actions to be the same, the centre of gravity must lie vertically below the point where the cord is attached.

SELF-ASSESSMENT EXERCISE 2

In the Figure below a block of weight w hangs from a cord which is knotted at 0 to two other cords fastened to the ceiling. Find the tensions in these three cords. The weight of the cords are taken to be negligible.

Solution
If we have to apply the conditions of equilibrium to find an unknown force, then we must consider a body in equilibrium. In our problem, the hanging box is in equilibrium as shown in the diagram (Fig 3.11)

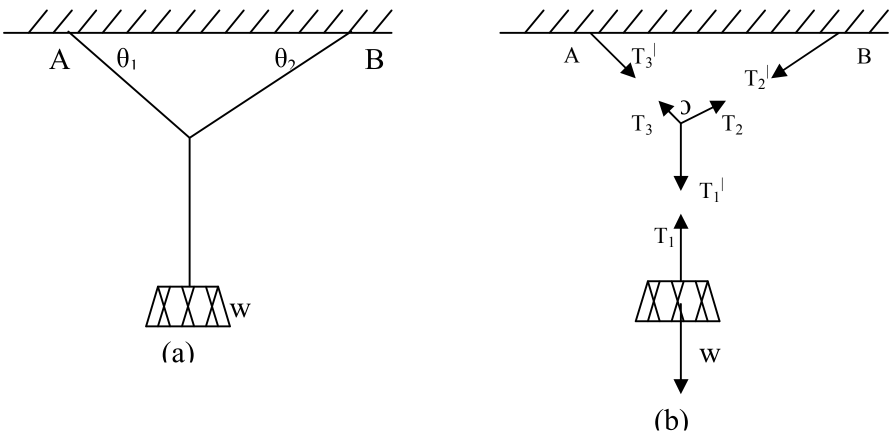


Fig 3.11

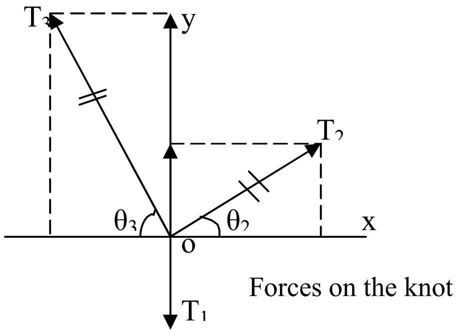


Fig 3.12

The tension in the vertical cord supporting the block is equal to the weight of the block. Note that the inclined cords do not exert forces on the block, but they do act on the knot at 0. So, we consider the knot as a particle in equilibrium with negligible weight.

In the free-body diagrams for the knot and the block shown above, T_1, T_2 and T_3 represent the forces extended on the knot by the three cords. T_1, T_2 and T_3 are the reactions to these forces.

Now, because the hanging block is in equilibrium

$$T_1^1 = w \text{ (first law)}$$

Since T_1 and T_1^1 form an action-reaction pair,

$T_1^1 = T_1$ (Third law)

Hence $T_1 = w$

Now, to find the forces T_2 and T_3, resolve them into their Cartesian components (see the Figure (3.12) above)

$$\therefore \quad \sum f_x = T_2 \cos\theta_2 - T_3 \cos\theta_3 = 0$$
$$\sum f_y = T_2 \sin\theta_2 + T_3 \sin\theta_3 - T_1 = 0$$

Since T_1 and w are known, then these two equations can be solved simultaneously to find T_2 and T_3. Putting in numerical values, we have if w = 50N, $T_2 = 30^0$, $T_3 = 60^0$

Then,

T_1 = 50 N and the two preceding equations become

$$T_2 \left(\frac{\sqrt{3}}{2}\right) - T_3 \left(\frac{1}{2}\right) = 0$$

and

$$T_2 \left(\frac{1}{2}\right) + T_3 \left(\frac{\sqrt{3}}{2}\right) = 50N$$

Solving simultaneously the results are

$$T_2 = 25N, \; T_3 = 43.3N$$

3.2 Newton's Laws of Motion

Newton's ***first*** law of motion describes what happens to atoms, oranges, and any other objects moving or at rest when they are left alone. It is natural to think that a moving object will eventually come to rest when left alone. The ancient Greeks believed so, but scientific observations have proved them wrong. From Galileo's experiments on the motion of objects on smooth planes, continuity of motion was established. This happened in the first part of the seventeenth century. Later, Isaac Newton extended Galileo's work and with great insight and power of abstraction (Fishbane et al.), correctly and simply stated what happens:

> when an object is left alone,
> it maintains a constant velocity.

This law is Newton's first law or the law of inertia. Notice that an object is at rest is a special, case of an object with constant velocity. This first law of motion stated in Newton's words is as follows:

> "Every body continues in its state or rest or of uniform motion in straight line unless it is compelled to change that state by forces impressed on it"

With the help of this law, we can define force as an external cause which changes or tends to change the state of rest or of uniform motion of a body.

Have you noticed that the first law does not tell you anything about the observer? But we know from our discussions on relative motion in Unit 1 and 6 , that the description of motion depends very much on the observer. So, it would be worthwhile to know: for what kind of observer does Newton's first law of motion hold? In answer to this, let us look at this scenario.

P

(a) O

←

(b) O $\xrightarrow{\quad}_{O^{1}}\rightarrow$

Fig 3.13

Suppose that an observer P is at rest with respect to an observer O who is also at rest Fig (3.13a). Let another observer O^1 be accelerating with respect to O. P will appear to O^1 to be accelerating in a direction opposite to the acceleration of O^1 Fig (3.13). According to Newton's first law, the cause of the acceleration is some force. So, O^1 will infer that P is being acted upon by a force. But O knows that no force is acting on P. It only appears to be accelerated to $O^{1.}$ Hence, the first law does not hold good for O^1. It rather holds good for O.

An observer like O is at rest or is moving with a constant velocity is called an inertial observer and the one like O^1 is called a non-inertial observer.

But, how do we know whether an observer is inertial or not? For this, we need to measure the observer's velocity with respect to some standard. It is a common practice to consider the earth as a standard. This we also saw in our Unit 1 of this course. Now, the place where one is performing one's experiment have an acceleration towards the polar axis due to the daily rotation of the earth. Again the centre of the earth has an acceleration towards the sun owing to its yearly motion around the sun. The sun also has acceleration towards the centre of the Galaxy, and so on. Hence the search for an absolute inertial frame is unending.

So, we modify the definition of the inertial observer. We say that:

> two observers are inertial with each either if they are either at rest or in uniform motion with respect to one another. If an observer has an

acceleration with respect to another, then, they are non-inertial with respect to one another.

Thus a car moving with a constant velocity and a man standing on a road are inertial with respect to one another while a car in the process of gathering speed, and the man standing are non- inertial with respect to each other.

The first law tells you how to detect the presence or absence of force on a body. In a sense, it tells you what a force does-it produces acceleration (either positive or negative) in a body. But the first law does not give quantitative, measurable definition of force. This is what the second law does. It gives quantitative, measurable definition of force.

Newton's Second Law of Motion

If you are struck by a very fast moving hockey ball you get injured, but if you are hit by a flower moving with the same velocity as that of the ball, you do not feel perturbed at all. However, if you are struck by a slower ball, the injury is less serious. This indicates that any kind of impact made by an object depends on two things viz.

(i) its mass and
(ii) its velocity

Hence, Newton felt the necessity of defining the product of mass and velocity which later came to be known as linear momentum. Mathematically speaking, linear momentum is given by

$$\vec{P} = m\vec{v} \qquad 3.4$$

Thus P is a vector quantity in the direction of velocity. The introduction of the above quantity paved the way for stating the second law, which in Newton's words are as follows:

> "The change of motion of an object is proportional to the force impressed; and is made in the direction of the straight line in which the force is impressed "

By "change of motion," Newton meant the rate of change of momentum with time. So mathematically we have

$$\vec{F} \propto \frac{d(\vec{P})}{dt}$$

or

$$\vec{F} = k\frac{d(\vec{P})}{dt} \qquad 3.5$$

where $\vec{F}$ is the impressed force ark k is a constant of proportionality.
The differential operator

$\frac{d}{dt}$ indicates the rate of change with time. Now, if the mass of the body remains constant (i.e. neither the body is gaining in mass like a conveyor belt nor it is disintergrating like a rocket), then

$$\frac{d\vec{p}}{dt} = \frac{d(mv)}{dt} = m\frac{d\vec{v}}{dt} = m\vec{a}$$

where

$\vec{a} = \frac{d\vec{v}}{dt} =$ the acceleration of the body.

Thus from Eqn. 3.5 we get

$$\vec{F} = km\vec{a} \text{ and} \qquad 3.6a$$

$$\left|\vec{F}\right| = kma \qquad 3.6b$$

We saw earlier that the need for a second law was felt in other to provide a quantitative definition of force. Something must be done with the constant k. We have realised that the task of a force $\vec{F}$ acting on a body of mass m is to produce in it an accelerations $\vec{a}$. Hence, anything appearing in the expression for force other than m and $\vec{a}$ must be a pure number, i.e. k is a pure number. So we can afford to make a choice for its numerical value.

We define unit of force as one which produces unit acceleration in its direction when it acts on a unit mass. So we obtain from Eqn. (3.6b) that 1 = k . 1.1 or k = 1. Thus, Eqn. (3.5), and (3.6) take the form

$$\vec{F} = \frac{d\vec{p}}{dt} \quad and \qquad 3.7a$$

for constant mass $\vec{F} = m\vec{a}$ 3.7b

Now, we know from Unit 6 that if the position vector of a particle is $\vec{r}$ at a time t then its velocity $\vec{v}$ and acceleration $\vec{a}$ are given by equations.

$$\vec{v} = \frac{d\vec{r}}{dt} \, and \, \vec{a} = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) = \frac{d\vec{v}}{dt}$$

Substituting for

$\vec{a} \, and \, \vec{v}$ in Eqn. 3.7 we get

$$\vec{F} = m\frac{dv}{dt} = m\frac{d}{dt}\left(\frac{dr}{dt}\right)$$

or

$$\vec{F} = \frac{d^2\vec{r}}{dt^2} \qquad 3.8$$

Eqn. 3.8 is a second order differential equation in $\vec{r}$. If we know the force $\vec{F}$ acting on a body of mass m, we can integrate Eqn. (3.8) to determine r as a function of t. The function $\vec{r}(t)$ would give us the path of the particle. Since Eqn. (3.8) is of second order, we shall come across two constants of integration. So we require two initial conditions to work out a solution of this equation. Conversely, if we know the path or trajectory of an accelerating particle, we can use Eqn. (3.8) to determine the force acting on the body. Eqn. (3.8) also enables us to determine unknown masses from measured forces and accelerations. Don't you see that calculations in this area have been made so easy by the second law of Newton?

So far, we have considered only one force acting on the body. But often several forces act on the same body. For example, the force of gravity, the force of air on the wings and body of the plane and the force associated with engine thrust act on a flying jet. (Fig 3.13)

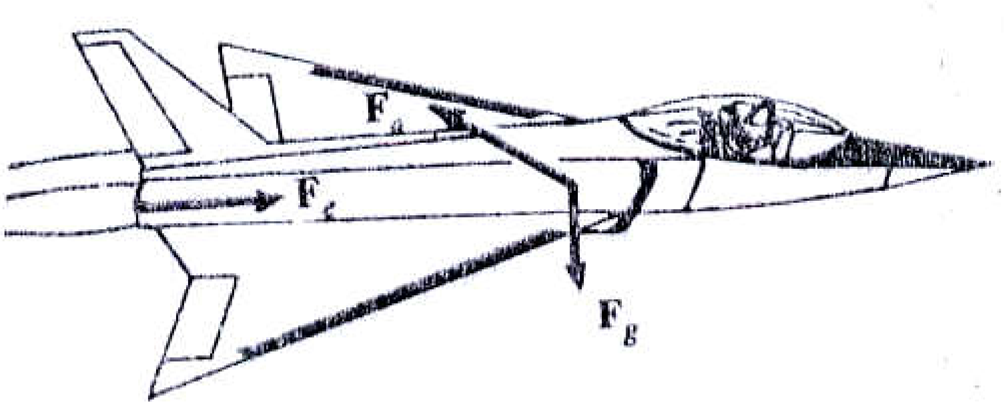


Fig. 3.13 Forces on a Jet: F_g the thrust of the engine, F_a ,
the force of the air provides both lift and drag, F_g the force of gravity.

In such cases, we add the individual forces vectorially, to find the *net force* acting on the object. The object's mass and acceleration are related to this net force by Newton's second law. You may now like to apply Newton's second law to a simple situation.

Units of Force

In Unit 2 we discussed the dimensions and units of mass, length and time. Because acceleration has dimensions of $[LT^{-2}]$ and units ms^{-2} in S.I units, force has dimensions of $[M.LT^{-2}]$ and in S. I, units of kg ms^{-2} or Newtons (N):

$$1N \equiv 1kg.ms^{-2} \qquad 3.9$$

In other words, a 1 N force exerted upon an object with a mass of 1kg will produce an acceleration of 1 ms^{-2}.

SELF-ASSESSMENT EXERCISE 1

A force of 200N pulls a box of mass 50kg and overcomes a constant frictional force of 40N. What is the acceleration of the sledge?

Solution 3.1
The Resultant force, F = 200N - 40N = 160N
from

$$F = ma = 50\,x\,a$$

$$\therefore \quad a = \frac{160N}{50kg}$$

$$= 3.2ms^{-2}$$

$$or \quad a = 3.2N$$

SELF-ASSESSMENT EXERCISE 2

An object of mass 2.0kg is attached to the hook of a spring balance, and the latter is suspended vertically from the roof of a lift. What is the reading on the spring balance when the lift is (i) ascending with an acceleration of 20cm s^{-2} (ii) descending with an acceleration of 10cms^{-2}

Solution 3.2

(i) The object is acted upon by two forces
  (a) The tension TN in the spring-balance, which acts upwards
  (b) Its weight 20N which acts downwards.

Since the object moves upwards, T is greater than 20N. Hence the resultant or net force, F acting on the object is

(T - 20) N approximately

Now F = ma
where a is the acceleration

(i) T (T - 20) N = 2kg x 0.2 ms^{-2}
  T = 20. 4 N   Answer

(ii) When the lift descends with an acceleration of 10 cm s^{-2} or 0.1ms^{-2}, the weight, 20 N is now greater than T_1 N the tension in the spring balance.

(iii)
 2 Resultant force = (20 - T_1)N = 20 -T_1
 2 F = (20 - T)N = ma = 2kg x 0.1ms^{-2}
 2 T = 20 - 0.2
  = 19.8 N   Answer

Newton's Third Law of Motion

So far we have been trying to understand how and why a single body moves. We have identified force as the cause of change in the motion of a body. But how does one exert a force on his body? Inevitably, there is an agent that makes this possible. Very often, your hands or feet are the agents. In football, your feet bring the ball into motion. Thus, forces arise from interactions between systems. This fact is made clear in Newton's third law of motion. To put it in his own words:

" To every action there is an equal and opposite reaction."

Here the words 'action' and 'reaction' means forces as defined by the first and second laws. If a body A exerts a force, F_{AB} on a body B, then the body B in turn exerts a force F_{BA} on A, such that

$$F_{AB} = -F_{BA}$$

So, we have $F_{AB} + F_{BA} = 0$

Notice that Newton's third law deals with two forces, each acting on a different body. You may now like to work out an exercise based on the third law.

SELF-ASSESSMENT EXERCISE 3

(a) When a footballer kicks the ball, the ball and the man experience forces of the same magnitude but in opposite directions according to the third law. The ball moves but the man does not move. Why?

(b) The earth attracts an apple with a force of magnitude F. What is the magnitude of the force with which the apple attracts the earth? The apple moves towards the earth. Why does not the reverse happen?

Solution

(3a) The reaction force acts on the man. Due to the large mass(inertia) of the man the force is not able to make him move.

(b) Apple also attracts the earth with a force of magnitude F. The acceleration of the apple and the earth are, respectively, F/m_a and F/m , where m_a and m are the masses of the apple and the earth, respectively. Since $m >> m_a$ $F/m << F/m_a$. Hence the earth does not move appreciably.

Newton's laws of motion provide a means of understanding most aspects of motion. In the next Unit, we shall study impulse and momentum.

4.0 CONCLUSION

In this unit, you have learnt that

- Objects are kept in motion as a result of externally implied forces on them
- Even inanimate objects can exert forces
- A body can only be in static or dynamic equilibrium if all the forces acting on it cancel each other.
- The three Newton's laws of motion are applied in solving problems relating to motion and forces that keep objects in motion

5.0 SUMMARY

What you have learnt in this unit are:

- that the study of the parametres that describe linear motion is called kinematics
- that the studies of the causes of motion is called dynamics
- that force is a push or a pull exerted on a body by another body.
- that there are gravitational forces and contact forces
- that forces can be represented graphically just like vectors
- that the conditions for equilibrium when a system of forces are acting on a rigid body are:
  (i) the resultant of all the forces sum up to zero

i.e.

$$\vec{R} = F_1 + F_2 + F_3 \ldots\ldots\ldots = 0$$

or

$$\vec{R} = \sum F_x + \sum F_y + \sum F_2 = 0$$

(iii) The forces must have no tendency to rotate the body.
The three Newton's laws express the dynamics of motion how forces acting between objects determine the subsequent motion of those objects. The first law states what happens to an object-moving or at rest when it is left alone. The second law is

$$F = ma$$

The third law is

$$F_{BA} = - F_{AB}$$

i.e. As regards forces between objects that if A and B interact and forces are acting between them, then by this third law the force an object A due to object B is F_{AB} and is equal and opposite to the force an object B due to object A which is F_{BA} .

- that in S. I, force is measured in Newtons abbreviated N where N = 1kg ms^{-2}
- Newton's laws help us to determine the motion of an object if we know the nature of the forces that act on it.
- Conversely, the laws enable us to measure forces acting on an object by measuring the objects motion.
- that observers in reference frames moving with respect to one another observe the motion of a given object differently.

6.0 TUTOR-MARKED ASSIGNMENT

(1) Astronauts on the Skylab mission of the 1970s found their masses by using a chair on which a known force was exerted by a spring. With an astronaut strapped in the chair, the 15kg chair underwent an acceleration of $2.04 \times 10^{-2} ms^{-2}$ when the spring force was 2.07N. What was the astronaut's mass?

(2) Three children each tug at the same plank. All the forces are in the horizontal plane. The three forces on the plank have the vectorial decomposition $F_1 = -5k$ units, $F_2 = 5i$ units and $F_3 = (- 5_i + 5k)$ units in terms of their unit vectors. What is the force on the box? What can you say about its consequent motion? Ignore the force of gravity.

(3) A particle of mass m is hung by two lights strings. The ends A and B are held by hands. The strings OA and OB make angles 2 with the vertical.

Find the values of T and T^1 in terms of m and T. T is tension in hand A and T^1 is tension in hand B.

7.0 REFERENCES/FURTHER READING

Indira Gandi National Open University School of Sciences. PHE -01, Elementary Mechanics-Concepts in Mechanics 1.

Sears, F. W, Zemansky M, W; Young H:D (1975). *College Physic.* (4th ed.). Reading, U. K: Addison-Wesley Publ. Co.

Spiegel M. R. (1959) *Vector Anaylsis-Schaum's Series*. New York: McGraw Hill Book Company.

Stroud K.A. (1995). *Engineering Mathematics*. (4th ed.). London: Macmillian Press Ltd.

Das Sarma, J. M. (1978). *An Introduction to Higher Secondary Physics*. India: Modern Book Agency Private Ltd.

Fishbane, P. M. Gasiorowicz, S & Thronton, S. T (1996). *Physics for Scientists and Engineers.* (2nd ed.). Vol. 1 New Jersey: Prentice Hall.

UNIT 3 THE PROJECTILE MOTION

CONTENT

1.0 INTRODUCTION

In the preceding unit we discussed the concepts of force and acceleration. We have applied Newton`s first law in solving problems in equilibrium. In this unit we shall apply Newton's second law to study projectile motion which is a type of motion in a plain under the influence of the earth`s gravitational field .This science explorers how a body behaves with the resultant force on it is not zero. The chief parameters we shall learn to celebrate here and the range, the maximum height and the time of flight of a particle undergoing projectile motion.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define a projectile motion and a projectile
- state the condition in which a projectile motion is possible
- represent projectile motion graphically
- compute the time of flight, highest point reached, maximum range attained by a projectile given initial conditions
- find the angle of projection of a projectile given the necessary parametres.

3.0 MAIN CONTENT

3.1 Definition of Projectile Motion

To appreciate what you will learn in this unit, find an open space in your neighborhood where you can conveniently throw up a small stone at an angle to the horizontal. Then throw the stone as indicated above. Return to your room and try to sketch the path traced by the stone.
Is your sketch similar to Figure (3.1) below?

The stone or object thrown into space is called a projectile. The shape of the path traced by the projectile is called a parabola. The maximum; horizontal distance traveled is the range, R.

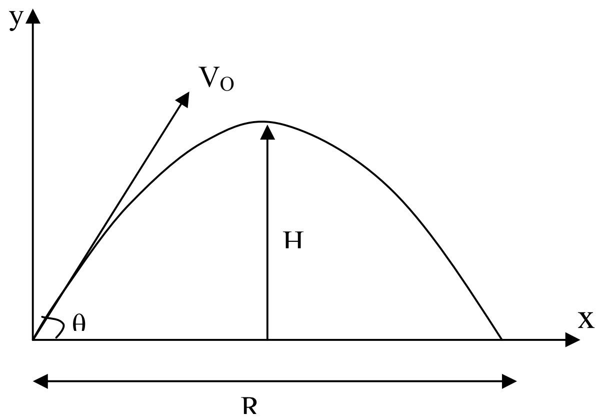


Fig 3.1

SELF-ASSESSMENT EXERCISE 1

Give more examples of projectile motion.
Projectile motion is a good example of motion in 2 dimensions. The initial velocity of projection at an angle as shown in Figure 3.1 is always resolved into two components there is the vertical component by which it attains some height at any instant of time in the y-axis and some horizontal component by which it covers some range, R in the x-axis. Hence, the motion can be described as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.

SELF-ASSESSMENT EXERCISE 2

Close your book and with a sheet of paper draw again the path traced by the stone you threw outside. Is your representation any better now?

The Trajectory

We can find the trajectory of a ball undergoing projectiles motion by plotting its height y versus its x-position. We know both x and y as functions of time, and we can eliminate the time dependence by using appropriate equations of motion.
Therefore from

$$x = 0 + (v_0 \cos\theta_0)\,t + \frac{1}{2}(0)t^2 \qquad 1$$

$$x = v_0 \cos\theta_0 t \qquad 2$$

$$\therefore t = \frac{x}{v_0 \cos\theta_0} \qquad 3$$

Now using the equation for y

$$y = 0 + (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \qquad 4$$

$$= (v_0 \sin\theta_0)t - \frac{1}{2}gt^2 \qquad 5$$

y becomes after substituting for t

$$y = 0 + (v_0 \sin\theta_0)\frac{x}{v_0 \cos\theta_0} - \frac{1}{2}g\left(\frac{x}{v_0 \cos\theta_0}\right)^2 \qquad 6$$

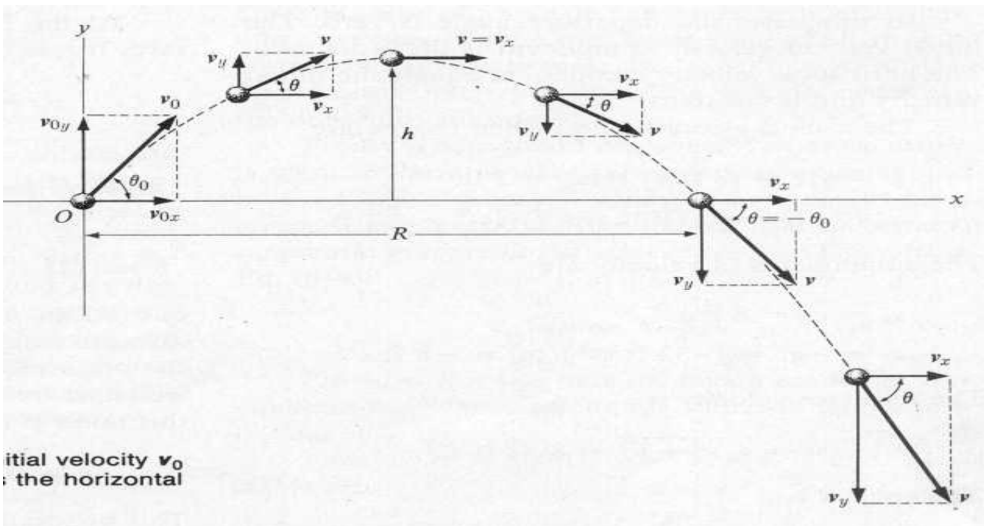
i.e.

$$y = (\tan\theta_0)\,x\left(-\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2 \qquad 7$$

We see that the coefficients of x and x^2 are both constants, so this equation have the form

$$y = c_1x - c_2x^2 \qquad 8$$

This is the general equation of a parabola. Hence we conclude that the trajectory of all objects moving with a constant acceleration is parabolic.
So, plotting different values of x, with their corresponding values of y will trace the trajectory of a projectile. Fig.(3.2)



Is there any other thing you think could affect the motion of a projectile besides gravity?

Yes there is. You know that our atmosphere is not a vacuum. The air in the atmosphere do resist the motion of the projectile. But because the effect is so small we generally neglect its resistive force on the projectile. Note that this could be a source of error in our experiments.

SELF-ASSESSMENT EXERCISE

A ball is projected horizontally with velocity v_o of magnitude $8ms^{-1}$. Find its position and velocity after ¼ s

Solution

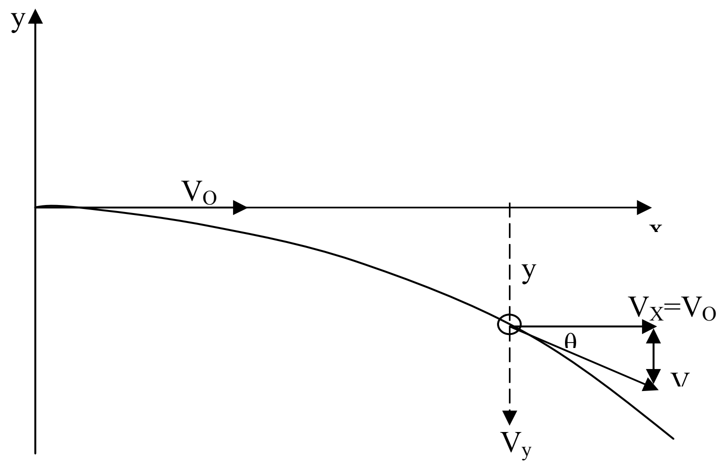


Fig 3.3 Trajectory of a body projected horizontally

The trajectory of the ball is represented in the diagram above Fig (3.3). We notice that the angle of projection is zero. This means that the initial vertical component of velocity is zero. Thus, the horizontal component of velocity is equal to the initial velocity and we recall that it is constant.

The x and y coordinates when $t = \frac{1}{4}s$ and g = 10m s^{-2} are

$$x = v_x t = (8ms^{-1})(\frac{1}{4}s) = 2m$$

and

$$y = -\frac{1}{2}gt^2 = -\frac{1}{2}(10ms^{-2})(\frac{1}{4}s)^2 = 0.32m$$

The components of velocity are

$$v_x = v_0 = 8ms^{-1}$$

$$v_y = -gt = (-10ms^{-2})(\frac{1}{4}s)$$

$$= 2.5ms^{-1}$$

The Flight Time

Let T be the total time of flight of a ball. The ball reaches its maximum height, H exactly half way through its motion, Fig. 3.4

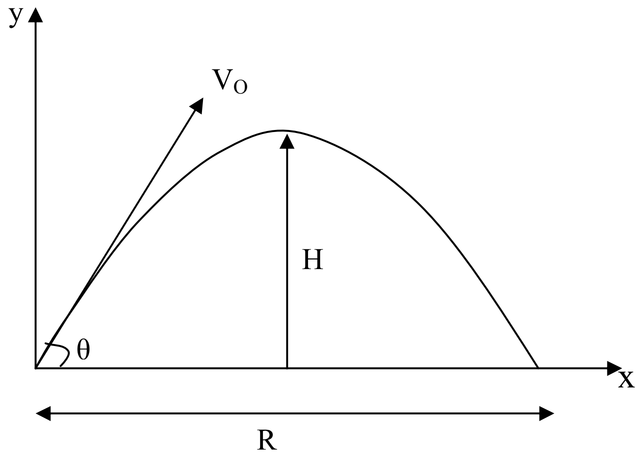


Fig 3.4

At this point its motion is horizontal; i.e. the vertical component of velocity is zero. This occurs at time t = T/2. Now, we can find T/2 by putting $V_y = 0$ in the following Eqn.

$$v_y = v_0 \sin\theta_0 - gt$$

becomes,

$0 = V_0 \sin 2_0 - gt$

so far t = T/2 we have

$$0 = v_0 \sin\theta_0 - \frac{gT}{2}$$

$$\therefore T = \frac{2v_0}{g}\sin\theta_0$$

Range

We defined the range R of a projectile launched from the ground y = 0, to be the horizontal distance that the projectile travels over level ground. Fig (3.2). The quantity R is the value of x when the projectile has returned to the ground. That is, when y again equals zero. Therefore from equations we have

$$0 = R\,(c_1 - c_2 R) \qquad 9$$

The value R = 0 satisfies the condition y = 0 in this equation. Note that this is the starting point of the projectile motion. Since it is launched from the ground, its x position is zero at launch time.

Also if the factor (c_1 - c_2 R) = 0 in equation 8 :. R = c_1/c_2 . This case corresponds to the projectile having landed back on the ground after its flight.

Substituting the values of c_1 and c_2 from equation 7 we get

$$R = \frac{c_1}{c_2} = \frac{\tan\theta_0 (2 \vee_0^2 \cos^2\theta_0)}{g} \qquad 10$$

$$= \frac{2 \vee_0^2}{g}(\frac{\sin\theta_0}{\cos\theta_0})\cos^2\theta_0 \qquad 11$$

Simplifying, we get

$$R = \frac{c_1}{c_2} = \frac{v_0}{g} 2\sin\theta\cos\theta \qquad 12$$

from trignometry, $\sin(2\theta_0) = 2\sin\theta_0\cos\theta_0$

Then using it, we find that

$$R = \frac{v_0 \sin 2\theta_0}{g} \qquad 13$$

The range varies with the initial angle, 2 of the projectile as seen in equation 13. We see that for 2 = 0, then R = 0. If 2 = 90^0 again R = 0 ie when a projectile is launched straight up, it comes back straight down. As 2 increases from 0 to 45^0 and then to 90^0, sin(2θ) first increases from 0 to 1 then decreases back to 0 respectively. This means that there are two initial angles to launch the projectile in order to get the same range for a given initial speed.

Note that the range reaches a maximum value when (sin 22_0) reaches a maximum value of 1 with reference to Eqn. 13. This occurs for 22 = 90^0, or 2 = 45^0 in which case

$$R_{\max} = \frac{\vee_0^2}{g} \qquad 14$$

If the projectile is shot at an angle higher or lower than 45^0, the range is shorter.

Maximum Height

The maximum height, y_{max} = h is reached at time T/2
θ from Eqn. 4 we find that the height at this time is

$$h = (v_0 \sin\theta_0)\frac{2v_0}{2g}\sin\theta_0 - \frac{1}{2}g(\frac{2v_0}{2g}\sin\theta)^2 \qquad 15$$

$$= v_0^2(\frac{\sin^2\theta_0}{g}) - gv_0^2\frac{\sin^2\theta_0}{2g^2}$$

$$= \frac{v_0^2\sin^2\theta_0}{2g} \qquad 16$$

SELF-ASSESSMENT EXERCISE

A group of engineering students constructs a nozzle device that lobs water balloons at a target. The device is constructed so that the launching speed is $12ms^{-1}$. The target is 14m away at the same elevation on the other side of the fence. How can they accomplish this mission?(Hint use $g = 9.8ms^{-2}$)

Solution

Analysing the problem, we see that the range equation for ground level is relevant. The range varies with the initial angle, so the students need to find a value of T that will give a range of 14m. We apply Eqn. 13 which is

$$R = \frac{v_0^2\sin 2\theta_0}{g}$$

to get

$$R = 14m = \frac{(12ms^{-1})^2\sin(2\theta_0)}{9.8ms^{-2}}$$

$$\therefore \sin 2\theta = 0.95$$

This equation has two solutions-that is, $2\theta_0 = 72^0$ and $2\theta_0 = 108^0$
Hence $\theta_0 = 36$ and 54^0

These are the two possible initial angles that the students will use that result in the given range. For a given velocity of projection there are in general two angles of inclination that will achieve the same range for a projectile. If one of these is θ, the other is what?

SELF-ASSESSMENT EXERCISE

A mass is projected horizontally from the top of a cliff with velocity V. Three seconds later, the direction of the velocity of the mass is 45^0 to the horizonntal.

Take the acceleration of free fall g to be $10ms^{-2}$, find the value of the projection velocity, V.

Solution

If the mass falls at 45^0 to the vertical, then, the horizontal and vertical components of velocity must be equal. The vertical component can be calculated using the equation of motion,

$$V = \mu + at$$

for a $=10ms^{-2}$, u = initial velocity, t = 3s

$$\mu = 0$$

$V = 0 + 10 \times 3$

$= 30ms^{-1}$

4.0 CONCLUSION

In this unit, you have learnt:

- that projectile motion is a type of motion with constant acceleration.
- that projectile motion is an example of motion in two dimensions in the Earth's gravitational field.
- that we apply the laws of motion in solving problems that describe projectile motion.
- how to represent projectile motion graphically.
- that the concept of projectile motion can be employed in warfare.

5.0 SUMMARY

What you have learnt in this unit are:

- that an object given an initial velocity and which subsequently follows a path determined by the gravitational force acting on it and by the frictional resistance of the atmosphere is called projectile.
- that the path followed by the projectile is called a trajectory
- that projectile motion is an application of Newton's second law of motion from which we have that a = F/m.
- that the forward component of velocity does not come into play in the projectile flight.
- that projectile motion can be described as a combination of horizontal motion with constant velocity and vertical motion with constant acceleration.
- that projectile motion is a form of parabolic motion.
- that the parameters are

$$\vee_x = \vee_{0x} = \vee_0 \cos\theta_0$$

$$\vee_y = \vee_{0y} - gt = \vee_0 \sin\theta_0 - gt$$

$$\tan\theta = \frac{\vee_y}{\vee_x}$$

The x - coordinate is

$$x = v_{0_x} t = (v_0 \cos\theta_0)t$$

the y-coordinate is

$$y = v_{0_y} t - \frac{1}{2}gt^2 = (v_0 \sin\theta_0)t - \frac{1}{2}gt^2$$

The resultant velocity

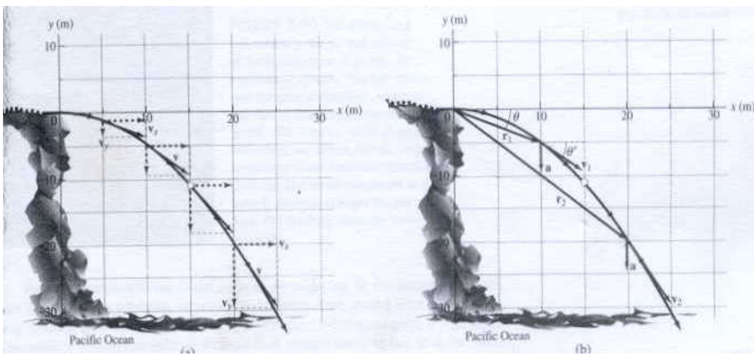
$$v = \sqrt{v_x^{\,2} + v_y^{\,2}}$$

The range, R is

$$R = v_x t = v_0 \cos\theta_0 \, x \frac{2 v_0 \sin\theta_0}{g}$$

6.0 TUTOR-MARKED ASSIGNMENT

1. A wayward ball rolls off the edge of a vertical cliff over-looking the Niger River. The ball has a horizontal component of velocity of $10ms^{-1}$ and no vertical component when it leaves the cliff. Describe the subsequent motion.
2. A boy would rather shoot mangoes down from a tree than climb the tree or wait for the mango to drop on its own. The boy aims his catapult at a mango on the tree, but just when his stone leaves the catapult, the mango falls from the tree. Show that the rock will hit the mango.
3.



For constant acceleration we apply equations

$$x = x_0 + v_{0x} t + \frac{1}{2}a_x t^2 \; and$$

$$v_y = v_{0y} + a_y t$$

We must specify the balls initial position and velocity by using the given information in the question.
We then determine the velocity component

4. What was the maximum height attained by a ball projected off the cliff with an elevation angle 36^0 to the horizontal and how long was the ball in flight?
   The other relevant information is
   height of cliff = 52m
   initial velocity = $48ms^{-1}$
   Total horizontal distance travelled = 281m

5. A projectile is shot at an angle of 34^0 to the horizontal with an initial speed of $225ms^{-1}$. What is the speed at the maximum height of the trajectory?

7.0 REFERENCES/FURTHER READING

Sears, F.W., Zemansky, M.W. & Young, H.D. (1974). Reading, U.K: Addison-Wesley Pub. Co. Inc.

Das Sarma, J.M. (1978). *An Introduction to Higher Secondary Physics.* Modern Book Agency Private Ltd.

Fishbane, P. M; Gasiorowicz, S. & Thranton, S.T., (1996). *Physics for Scientists and Engineers.* (2nd ed.). Vol.1 New Jersey: Prentice Hall.

UNIT 4 IMPULSE AND LINEAR MOMENTUM

CONTENTS

1.0 INTRODUCTION

In Unit 7 we dealt with Force and Newton's laws of motion. In this Unit we shall treat impulse and momentum as a consequence of the action of force. Pulse is a force acting for a very small or short duration of time as in a sudden impact of an object on another like in the impact of batting a tennis ball or an upsurge of current etc. Momentum of an object plays an important role in Newton's second law. A force produces a change in momentum. When a system of particles is isolated, the total momentum is constant. This principle, known as the principle of conservation of momentum is particularly useful for understanding the behaviour of colliding objects. We shall learn about this principle in this unit of the course. But first of all, we shall introduce the concept of impulse and momentum and how they are applied in motion of rockets.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define impulse and linear momentum.
- write the mathematical definition of impulse and linear momentum
- solve problems in linear momentum
- describe the motion of rockets using the linear momentum principle
- state the conditions for the conservation of linear momentum
- apply the principles of conservation of linear momentum.

3.0 MAIN CONTENT

3.1 Definition of Impulse and Momentum

Imagine that a particle of mass m is moving along a straight line, Let us assume that the force acting on the particle is constant and directed along the line of motion of the particle. If the particle's velocity at some initial time t = 0 is V_0, then its velocity at a later time, t, is given by $V = V_0 + at$

I know you recognise this expression as one of the equations of motion we treated in units 5 and 6. Here, the constant of acceleration, a, is given by F/m from Newton's second law. Making the substitution for a we get.

$$Ft = mv - mv_0 \qquad 3.1$$

you will notice that the left hand side of equation 3.1 is the product of the force and the time during which the force acts. This expression, (Ft) is called the impulse of the force. Generally, if a constant force, F acts for a short interval t_1 to time t_2 , the impulse of the force is defined mathematically as

Impulse = F (t_2- t_1) = f□t

where $\Delta t = t_2 - t_1$ is very small interval. We notice that in Eqn.(3.1) the right hand side of it contains the product of mass and velocity of the particle at two different times. The product, mv has a special name called momentum. This is very easy to remember. The experience you get when someone suddenly bumps into you unsuspectingly at a bend in the street is an impact of momentum. Momentum during a linear motion is also called linear momentum. We often use the symbol P to represent momentum.

Momentum = P = mv

So, for the time intervals t_1 and t_2 with corresponding particle velocities of V_1 and V_2, the impulse is given by,

$$F(t_2 - t_1) = mv_2 - mv_1 \qquad 3.2$$

We note that this relation between impulse and force is the same as that between work and kinetic energy change which we shall discuss later.

The differences between them I would like you to also note are that:

(i) impulse is a product of force and time but work is a product of force and distance and depends on the angle between force and the displacement.

(ii) force and velocity are vector quantities and , impulse and momentum are vector quantities but work and energy are scalars. In linear motion the force and velocity may be resolved, as we found earlier in this course, into their components along the x-axis and could have either positive or negative values.

SELF-ASSESSMENT EXERCISE 1

A particle of mass 2kg moves along the x-axis with an initial velocity of 3 ms^{-1}. A force F = - 6N (i.e. the force is moving in the negative x-direction) is applied for a period of 3s. Find the initial velocity.

Solution

We apply the following eqn,

$$F(t_2 - t_1) = m\vee_2 - m\vee_1$$

thus

$$(-6N)(3s) = (2kg)\vee_2 - (2kg)(3ms^{-1})$$

or $\vee_2 = -\ 6ms^{-1}$

The final velocity of the particle is in the negative x - direction that is why we have a negative sign in the value for velocity.

The unit of impulse is the same as the unit of the product of force and time in whatever system the calculation is made. Thus in the S.I system, the unit is one Newton second (1 Ns) in cgs system it is one dyne second (1 dyne s) and in the engineering system it is one pound second (1 lb s).
The unit of momentum in the S.I system is 1 kilogram metre per second (1 kg ms^{-1}).
Since

$$1kgm\,s^{-1} = (1kgms^{-2})s = 1Ns,$$

this implies that momentum and impulse have the same units in a particular system.

Generally, impulse are forces that vary with time. For sufficiently small time intervals, Δt, the force acting could be taken to be constant. So, the impulse during a time Δt is FΔt. This is shown graphically in Figure 3.1.

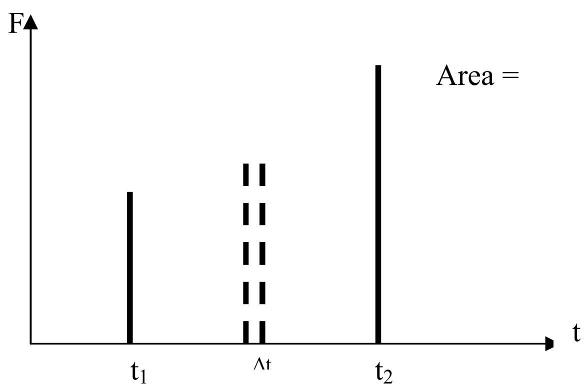


Fig 3.1

Graphically, $F\Delta t$ is represented by the area of the strip of width □t as shown under the curse of F versus t. The total impulse is given by the areas under the curve between the initial time, t_1 and final time of action of the force t_2. Momentum increases algebraically with increase in positive impulse but decreases with negative impulse.

Note that if the impulse is zero, there is no change in momentum.
The total impulse could also be found by integrating F□t as □t tends to zero or as t_2 approaches t_1 thus,

$$\lim_{t_2 - t_1 \to 0} F\Delta t = \int_{t_1}^{t_2} F(t)dt \qquad 3.3$$

This value also gives the change in linear momentum of an object in which such a force acts.

$$\int_{t_1}^{t_2} \frac{dp}{dt} dt = p(t_2) - p(t_1) = \Delta p \qquad 3.4$$

We have now seen that impulse of a force is change in linear momentum. If a force acts during a time interval Δt but is variable, then to calculate impulse we would need to know the function F (t) explicitly. However, this is usually not known. A way out is to define the average for $\overline{F}$ by the equation

$$\overline{F}_{ave} = \frac{1}{\Delta t} \int_{t_1}^{t_2} F(t)dt \qquad 3.5$$

where $\Delta t = t_2 - t_1$
so from Eqns. 3.4 and 3.5 we get

$$\text{Total Impulse} = F_{ave}\, \Delta t = \Delta p \qquad 3.6$$

There are many examples which illustrate the relationship between the average force, its duration and change of linear momentum. A tennis player hits the ball while serving with a great force to impart linear momentum to the ball. To impart maximum possible momentum, the player follows through with the serve. This action prolongs the time of contact between the ball and the racket. Therefore to bring about the maximum possible change in the linear momentum, we should apply a large force as possible over a long time interval as possible. You may now like to apply these ideas to solve a problem

SELF-ASSESSMENT EXERCISE 2

(1) A ball of mass 0.25kg moving horizontally with a velocity $20ms^{-1}$ is struck by a bat. The duration of contact is 10^{-2} s. After leaving the bat, the speed of the ball is 40 ms^{-1} in a direction opposite to its original direction of motion. Calculate the average force exerted by the bat.

(2) Give an example in which a weak force acts for a long time to generate a substantial impulse

Solution:
Let J = impulse
(i) Impulse , J = p

$$= (0.25kg) \times \{40-(-20)\} ms^{-1}$$
$$= 15kgms^{-1}$$
$$\Delta t = 10^{-2}s$$
$$\Delta F_{average} = \frac{J}{\Delta t} = 1500N$$

(2) The gravitational force of attraction between sun and earth is very weak but it has been acting since their formation and so it can generate a substantial impulse.

Motion with Variable Mass

If the mass of a system varies with time, we can express Newton's second law of motion as

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt} \qquad 3.7$$

Under the special case when v is constant, Eqn. (3.7) becomes

$$F = v\frac{dm}{dt} \qquad 3.8$$

Let us study an example of this special type

Example

Sand falls on to a conveyer belt B (Fig. 3.2) at the constant rate of $0.2kgs^{-1}$. Find the force required to maintain a constant velocity of 10m/s of the belt. Here, we shall apply Eqn. (3.8) as velocity remains constant. Since the mass is increasing *dm/dt* is positive. The direction of F, therefore, is same as that of v, i.e. the direction of motion of the conveyer belt. Thus, using Eq. 3.8 we get

$$F = (10ms^{-1}) \times (0.2 \text{ kg } ms^{-1}) = 2kg\ ms^{-2} = 2N.$$

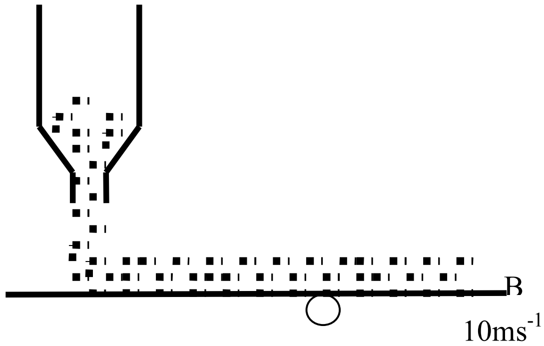


Fig 3.2

Another example of a varying mass system is the rocket. In a rocket (Fig. 3.3) a stream of gas produced at a very high temperature and pressure escapes at a very high velocity through an exhaust nozzle. Thus, the rocket losses mass and

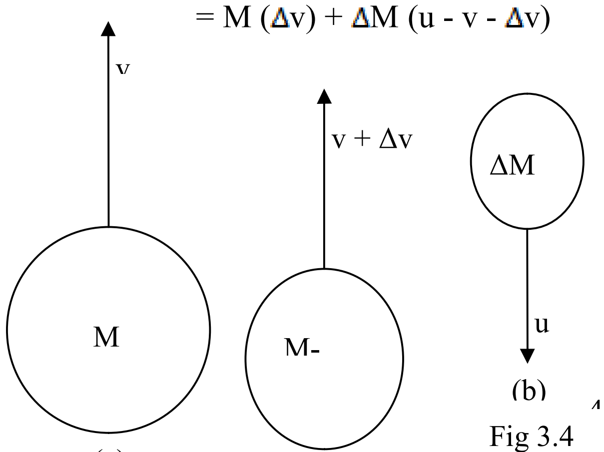
$\frac{dm}{dt}$ is negative. So the Main Content of the rocket experiences a huge force in a direction opposite to that of the exhaust causing it to move. This is a very simplified way of dealing with the motion of a rocket. We shall next analyse the motion of a rocket with a little more rigour using the idea of impulse.

3.2 Motion of a Rocket

Let us assume that the rocket has a total mass M at a time t. It moves with a velocity V and ejects a mass □M during a time interval Δt. The situation is explained schematically in Fig. (3.3 And 3.4a and b)
At time t the total initial momentum of the system = Mv (Fig.3.4a). At time t + Δt the total final momentum of the system = (M - ΔM) (v + Δv) + (ΔM)u (Fig. 3.4b).

Notice that we have used the positive sign for u because the total final momentum of the system in Fig 3.4b is a vector sum and not the difference of the momenta of M and (M - ΔM). Let us now apply Eq.3.6. If we take the vertically upward direction as positive the impulse is - Mg Δt and is equal to the change in linear momentum.

So,
$$-Mg\,\Delta t = (M - \Delta M)(v + \Delta v) + (\Delta M)\,u - Mv$$
$$= M(\Delta v) + \Delta M\,(u - v - \Delta v)$$



(a) (b)

Fig 3.4

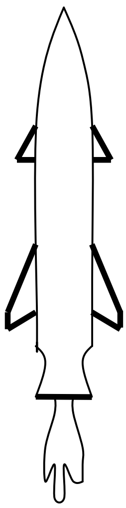


Fig 3.3

to simplify the above relation: recall that $V_Qp = Vp - V_Q$

$\therefore -g = \frac{\Delta v}{\Delta t} + \frac{1}{M}\frac{\Delta M}{\Delta t} u_{rel}$ where $u_{rel} = u - (v + \Delta v)$ is the relative velocity of the exhaust with respect to the rocket.

Now, in the limit

$\Delta t \to 0$, we have

$$-g = \frac{dv}{dt} - \frac{1}{M}\frac{dM}{dt} u_{rel} \qquad 3.9$$

The negative sign on the right-hand side of Eqn. 3.9 appears as

$\lim_{\Delta t \to 0} \frac{\Delta M}{\Delta t} = -\frac{dM}{dt}$, because M decreases with t.

so, when we apply Eq. 3.9 in numerical problems we just replace

$\frac{dM}{dt}$ by its magnitude. On integrating Eq. 3.9 with respect to t, we get

$$\int_0^1 \frac{dv}{dt} dt = -gt + u_{rel} \int_{M_0}^{M} \frac{dM}{M}$$

where M_0 is the initial mass of the rocket and M is its mass at time t. Now, if v_0 is the initial velocity, then we get

$$v - v_0 = u_{rel}\, In \frac{M}{M_0} - gt \qquad 3.10$$

We shall illustrate Eq. 3.10 with the help of an example.

Example

The stages of a two-stage rocket separately have masses 100kg and 10kg and contain 800 kg and 90 kg of fuel, respectively. What is the final velocity that can be achieved with exhaust velocity of 1.5 kms^{-1} relative to the rocket ? (Neglect any effect of gravity). Since we are neglecting gravity Eq. 3.10 reduces to

$$v - v_0 = u_{rel}\, In \frac{M}{M_0} \qquad 3.11$$

Now, let the unit vector along the vertically upward direction be

$\hat{n}$. So, Eq. 3.11 can be written as

$v\hat{n} - v_0\hat{n} = -(u_{rel}\,\hat{n})\, In \frac{M}{M_0}$, *where* $u_{rel} = -u_{rel}\hat{n}$, as the relative velocity of the exhaust points vertically downward.

$$v - v_0 = u_{rel} \, In \frac{M}{M_0} \qquad 3.11a$$

For our problem,

$u_{rel} = 1.5$ kms^{-1}

For the first stage, $v_0 = 0$
$M_0 = (800 + 90 + 100 + 10)$ kg = 1000kg
M = (90 + 10 + 100)kg = 200kg, as the 800 kg fuel gets burnt in the first stage.

Hence, from Eq.3.11 a, we get

$$v = -\,(1.5kms^{-1})\left(1n\frac{200}{1000}\right)$$
$$= (-\,1.5kms^{-1})\,(1n\,2 - 1n\,10)$$
$$= 1.5\,x\,1.6\,kms^{-1}$$
$$= 2.4kms^{-1}$$

Note that the above will be the initial velocity for the second stage. Also note that at the beginning of the second stage there occurs another drop in mass to the extent of the mass
of the first stage (i.e. 100kg). For the second stage,

$$v_0 = 2.4kms^{-1}$$
$$M_0 = (90 + 10)kg = 100\,kg, M = 10kg$$
$$v = (2.4 - 1.5\,1n\frac{10}{100})kms^{-1}$$
$$= (2.4 + 1.5\,x\,2.3)kms^{-1} = 5.85\,kms^{-1} = 5.8kms^{-1}$$

The final result of this Example has to be rounded off to two significant digits. Here we have a special case as the digit to be discarded is 5. By convention , we have rounded off to the nearest even number.
Let us now follow up this example with an exercise

SELF-ASSESSMENT EXERCISE 3

Find the final velocity of the rocket in the Example above taking it to be single-stage, i.e. its mass is 100kg and it carries 890kg of fuel. Hence comment whether the two-stage rocket has an advantage over single stage or not.

Solution
Had it been a single stage rocket, then $v_0 = 0$
$M_0 = (890 + 100)\text{kg} = 990\text{kg}$
$M = 100\text{kg}$

$V = (-\ 1.5\text{km s}^{-1})[\ln \frac{100}{990}]$

$= (-\ 1.5\text{km s}^{-1})(\text{In } 10 - \text{In}99)$
$=3.4\text{kms}^{1}$ which is 41% less than the value of velocity (5.8kms^{-1}) attained in a double-stage rocket. Hence double-stage has an advantage over the single-stage.

3.3 Linear Momentum

Let us first study a system of two interacting particles'1' and '2' having masses m_1 and m_2 (Fig.3.5). Let p_1 and p_2 be their linear momenta. The total linear momentum p of this system is simply the vector sum of the linear momenta of theses two particles.

$$p = p_1 + p_2 \qquad 3.12$$

From Newton's second law, the rate to change of p_1 is the vector sum of all the forces acting on 1, i.e. the total external force F_{el} on it and the internal force f_{21} due to 2:

$$F_{el} + f_{21} = \frac{dp_1}{dt} \qquad 3.13a$$

Similarly, for particle 2:

$$F_{el} + f_{21} = \frac{dp_2}{dt} \qquad 3.13b$$

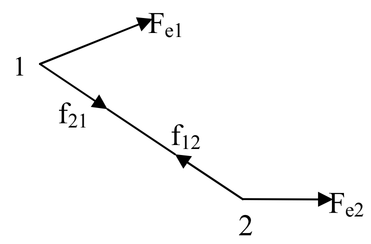


Fig 3.5

From Newton's third law, we know that $f_{12} = -f_{21}$. Therefore, on adding Equation .3.13 a and 3.13b, we get

$$F_{e1} + Fe_2 = \frac{dp_1}{dt} + \frac{dp_2}{dt},$$ which may be written as

$F_e = \frac{d}{dt}(p_1 + p_2)$, where F_e is the net external force on the system.

Therefore, from Eq. 3.12

$$F_e = \frac{dp}{dt}. \qquad 3.14$$

Thus, in a system of interacting particles, it is the net external force which produces acceleration and not the internal forces. Now, we shall see how Equation. 3.14 leads to the principle of conservation of linear momentum.

3.4 Conservation of Linear Momentum

In the special case when the net external force F_e is zero, Equation 3.14 gives

$$\frac{dp}{dt} = 0, \qquad 3.15$$

so that $p = p_1 + p_2$ = a constant vector.
This is the principle of conservation of linear momentum for a two-particle system. It is equally valid for a system of any number of particles. Its formal proof for a many-particle system will be given later. It states that:

"if the net external force acting on a system is zero, then its total linear momentum is conserved".

Let us now apply this principle.

Example

A vessel at rest explodes, breaking into three pieces. Two pieces having equal mass fly off perpendicular to one another with the some speed of 30 ms^{-1}. Show that immediately after the explosion the third piece moves in the plane of the other two pieces. If the third piece has three times the mass of either of the other piece, what is the magnitude of its velocity immediately after the explosion?

The process is explained in the schematic diagram Fig (3.6). The vessel was at rest prior to the explosion. So its linear momentum was zero. Since no net external force acts on the system, its total linear momentum is conserved. Therefore, the final linear momentum is also zero, i.e.

$$p_1 + p_2 + p_3 = 0 \qquad 3.16a$$

or $$p_1 + p_2 = -p_3 \qquad 3.16b$$

($p_1 + p_2$) lies in the plane contained by p_1 and p_2. So in accordance with Eq.3.16b, $-p_3$ must also lie in that plane. Hence, p_3 lies in the same plane as p_1 and p_2. Now, from Eq.3.16

$$(p_1 + p_2).(p_1 + p_2) = (-p_3).(-p_3), \qquad 3.16c$$

or $$p_1^2 + p_2^2 + p_1.p_2 = p^2{}_3$$

But $$p_1.p_2 = 0 (\therefore p_1 \text{ is perpendicular to } p_2). \qquad 3.16d$$

So $$p_3^2 = p_1^2 + p_2^2,$$

or $$(3mv)^2 = (mu)^2 + (mu)^2,$$

or $$9m^2v^2 = 2m^2u^2, or\ v = \frac{\sqrt{2}}{3}u.$$

According to the problem

$$u = 30ms^{-1} .\therefore v = 10\sqrt{2}ms^{-1} \qquad 3.16b$$

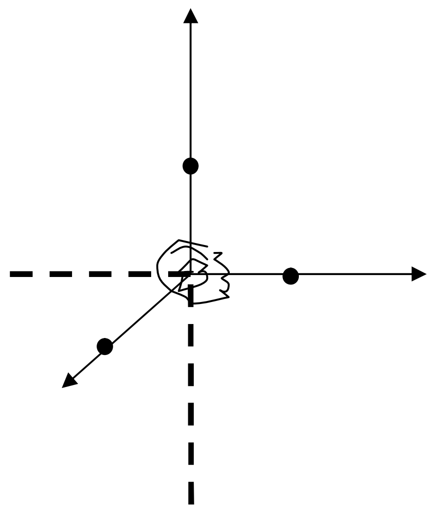


Fig 3.6

There is another method of finding the magnitude of the velocity. We can express Equation 3.16b in terms of the components of p_1,p_2 and p_3 in two mutually perpendicular directions of x and y-axes. Let p_1 be along x-axes, p_2 along y-axis and let p_3 make an angle θ with x-axis. Then Equation 3.16b gives:

$$p_1\hat{i} + p_2\hat{j} = -(p_3\cos\theta\hat{i} + p_3\sin\theta\hat{j}). \qquad 3.17$$

This equation is satisfied iff (see Eq. 1.6)

$$-p_3\cos\theta = p_1, \quad -p_3\sin\theta = p_2 \qquad 3.18$$
$$or\ p_3^2 = p_1{}^2 + p_2{}^2, \text{Which is Eqn. 3.16c}$$

SELF-ASSESSMENT EXERCISE 4

Find the direction of v in the example above.
From the above example and the way we obtained the principle of conservation of momentum, it may appear that the principle is limited in its application. This is because we have assumed that no net external force acts on the system of particles. However, the scope of the principle is much broader.

There are many cases in which an external force, such as gravity, is very weak compared to the internal forces. The explosion of a rocket in mid air is an example. Since the explosion lasts for a very brief time, the external force can be neglected in this case. In examples of this type, linear momentum is conserved to a very good approximation.

Again, if a force is applied to a system by an external agent, then the system exerts an equal and opposite force on the agent. Now if we consider the agent and the system to be a part of a new, larger system, then the momentum of this new system is conserved. Since there is no larger system containing the universe, its total linear momentum is conserved.

We have seen that whenever we have a system of particles on which no net external force acts, we can apply the law of conservation of linear momentum to analyse their motion. In fact, the advantage is that this law enables us to describe their motion without knowing the details of the forces involved.

4.0 CONCLUSION

In this unit, you have learnt

- that impulse is a force of very short duration
- that linear momentum is given by the product of the force and velocity.
- that force is as a result of change of momentum of a particle
- that the principle of momentum change is applied in rocket propulsion
- that when two objects collide, their momentum must be conserved.

5.0 SUMMARY

What you have learnt in this unit are:

- that impulse = $F \ \ t$ where $\ \ t = t_2 - t_1$ is a very short time interval F, t_2 and t_1 have their usual meanings.
- that momentum, p = mv where m = mass of particle and v = velocity of particle
- that force = $mv_2 - mv_1 = \ \ mv = p$ or force $= \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$
- that the sum of the linear momentum p for a system of two particle p having mass m_1 and m_2 and linear momenta p_1 and p_2 is $p = p_1 + p_2$
- that linear momentum is always conserved ie if $\frac{dp}{dt} = 0$, then momentum is conserved.

Note
If the external force acting on a system is zero, then its total linear momentum is conserved.

6.0 TUTOR-MARKED ASSIGNMENT

(1) A ball of mass 0.4 kg is thrown against a brick wall. When it strikes the wall it is moving horizontally to the left at 3 ms^{-1}, and it rebounds horizontally to the right at 20m s^{-1}. Find the impulse of the force exerted on the ball by the wall.

(2) A ball moves with a velocity of 1.2m/s in the positive y-direction on a table and strikes an identical ball that was at rest. The rolling ball is deflected so that its velocity has a component of 0.80 ms^{-1} in the +ve y- direction and a component of 0.56m s^{-1} in the + x-direction. What are the final velocity and final speed of the struck ball?

7.0 REFERENCES/FURTHER READING

Sears, F.W., Zemansky M.W &Young, H.D (1975). *College Physics*. (4th ed.). Reading U.K: Addison - Wesley Pub. Co.

Indira Gandi National Open University School of Sciences. PHE- 01, Elementary Mechanics, Concepts in Mechanics 1

Fishbane, P.M., Gasiorowicz, S. & Thronton, S.T. (1999). *Physic for Scientists and Engineers*. (2nd ed.). New Jersey: Prentices Hall Pub.

Nelkon M & Parker, P. (1970). *Advanced Level Physics*. London: Heinemann Educational Books Ltd.

UNIT 5 LINEAR COLLISIONS

CONTENTS

1.0 INTRODUCTION

Attempts to understand collisions were carried out by Galileo and his contemporaries. The laws that describe collisions in one dimension were formulated by John Wallis, Christopher Wren and Christian Huygens is 1668. In this Unit you will learn about collisions between two objects moving along a straight line. You will find out what happens when objects collide. The interesting phenomena of their change in velocity, momentum and possibly change in kinetic energy will be discussed. This will lead us to the understanding of the phenomenon of explosions that is popularly applied in war fares. Relax and find out as you read how simple observations lead to important scientific discoveries.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define collision
- classify collisions
- apply the principles of conservation of energy and momentum in order to determine the energy lost by colliding particles
- use collision principle in explaining rocket propulsion
- explain what is meant by elastic, inelastic and perfectly inelastic collisions
- solve problems in collisions.

3.0 MAIN CONTENT

3.1 Classification of Collisions

In Unit 9 you learnt about impulse and momentum. You learnt that impulse is a force which acts for only a very short duration of time. This means that impulsive forces are the types of forces we experience during collisions. Have you ever collided with somebody or with some object unsuspectingly? Can you recall some actions that depict collisions? An example is the collision of two balls rolling on a table. Another is the popular pin-pong game popularly called table tennis. You can imagine the very short time of impact between the tennis ball and the bat used by the player.

Collision is the sudden impact felt between two objects. You may ask what happens during collisions? During collision there could be transfer of energy from one object to the other or energy could be transformed from one form to another. For example, some of the kinetic energy of the tennis ball is converted to sound energy on hitting the bat of the player while playing table tennis. Also during explosions, potential energy is converted to kinetic energy and sound energy. From the principle of conservation of momentum you studied in unit 9, you learnt that momentum of colliding particles must be equal before and after collision. This knowledge will be applied in this unit to determine the velocity of objects after collisions.

There are two types of collisions viz **elastic** and **inelastic** collisions. Elastic collision is a collision between two or more objects during which no energy is lost. That is, the total kinetic energy of the objects before collision is equal to the total kinetic energy of the objects after collision. In other words, kinetic energy is conserved. But if the kinetic energy is not conserved in a collision the collision is called inelastic collision. This implies that during inelastic collision, some of the kinetic energy is converted to heat or sound.

There is also a situation in which two bodies can collide and coalesce (i.e stick together). This kind of collision is referred to as perfectly inelastic collision because it corresponds to a situation where maximum kinetic energy is lost during collision.

3.2 Perfectly Inelastic Collision

We shall now discuss perfectly inelastic collision in one dimension because it is the simplest of the three types of collision we have identified. In this type of collision, the objects coalesce at impact. The collisions are described by the equatio $MV = m_1 v_1 + m_2 v_2$ 3.1

$M = m_1 + m_2$ Where that is the sum of the masses of the two colliding particles, V is the velocity of M after coalescing, v_1 and v_2 are the velocities of particles m_1 and m_2 respectively before collision.

$$V = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \qquad 3.2$$

Hence the velocity, V of the coalesced object is
Let us look at special cases:

Case 1: One of the objects is stationary and the other object runs into it. In this case $v_2 = 0$ so equation (3.2) becomes

$$V \quad = \frac{m_1 v_1}{m_1 + m_2} = \left(\frac{m_1}{M}\right) v_1 \qquad 3.3$$

Equation (3.3) shows that if $m_1 >> m_2$ the coalesced object will move with a velocity nearly equal to v_1

$\left({m_1}/{m_2} \right) v_1$ Conversely if $m_1 << m_2$ as is the case when a stationary goalkeeper catches a ball, the keeper will recoil only with a low velocity. This will be equivalent to just the fraction of the velocity of the ball i.e. .

Case 2: There is heed-on collision between two objects moving towards each other and having equal velocities.

$$V \quad = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^{V_1} \qquad 3.4$$

Here $v_2 = -v_1$, therefore equation (3.1) becomes

If $m_1 = m_2$ then their momenta are $-m_1v_1$ and m_1v_1 which means that their momenta are equal and opposite because substituting we have

$$m_1v_1 + m_2v_2 = m_1v_1 + m_1v_2 = m_1(v_1 + v_2) = 0 \qquad 3.5$$

If this is so, the final momentum must be zero and that V = 0.
Hence the objects collide and stay there.

3.2.1 Energy lost in perfectly inelastic collisions

The case under consideration here is to find the change in energy when two objects coalesce at impact.

$M = m_1 + m_2$ Let E_i be the sum of the kinetic energy K. E of the objects before collision. And let E_f be the final energy i.e. K.E. of the coalesced object (composite object) of mass

Hence the energy change ΔE is given by

$$\Delta E = E_f - E_i \qquad 3.6$$

To find this, we apply equation 3.2.

$$\Delta E = \frac{1}{2}Mv^2 - \left(\frac{1}{2}m_1 v_2^1 + \frac{1}{2}m_2 v_2^1\right) \qquad 3.7$$

$$= \frac{1}{2}\frac{(m_1 + m_2)(m_1 v_1 + m_2 v_2)^2}{(m_1 + m_2)^2}$$

$$= \frac{1}{2}\frac{\left[m_1^2 v_1^2 + 2m_1 m_2 v_1 v_2 + m_2^2 v_2^2 - (m_1 + m_2)(m_1 v_1^2 + m_2 v_2^2)\right]}{m_1 + m_2}$$

$$= \frac{1}{2}\frac{\left[(m_1 m_2)(-v_1^2 - v_2^2 + 2v_1 v_2)\right]}{m_1 + m_2}$$

$$\Delta E = -\frac{m_1 m_2}{2(m_1 + m_2)}(v_1 - v_2)^2 \qquad 3.8$$

Since kinetic Energy is ½mv^2, we have

Note that the expression in the right hand side of equation (3.8) is always negative. This is because energy is lost in such a collision. This means that the collision is inelastic.

This composite object is at rest in only one frame of reference. In this frame there is no final kinetic energy so the collision is known as perfectly inelastic collision. In this frame of reference, the total momentum is zero. The total kinetic energy of the system before collision goes into the coalesion of the objects.

SELF-ASSESSMENT EXERCISE 1

A dog running at a speed of 32km h^{-1} jumps into a stationary canoe on the river Niger at Lokoja. The dog's mass is 14kg and that of the canoe plus the rower is 160kg. Let us assume that the water surface is frictionless,

(i) what is the speed of the canoe after the collision.
(ii) what is the ratio of the energy loss to the initial energy
(iii) where did the energy go?

Solution:

The initial momentum is the momentum of the dog only. This is because the canoe is at rest. Given mass of dog as m and initial velocity of the dog (i.e. its velocity as it enters the canoe) as v_o then

Initial momentum $P_i = mv_o$
the final momentum $P_f = Mv$

where v is the unknown speed, and M is the sum of the masses of the canoe, rower and dog = 174kg.
Since
(i) Initial Momentum = final momentum, therefore

(ii) The initial energy is the K.E. of the dog, therefore

$$v = \frac{P_f}{M} = \frac{P_i}{M}$$

$$= \frac{mv_o}{M} = \frac{(14kg)(32kmh^{-1})}{174kg}$$

$$= 2.6kmh^{-1}$$

$$= 0.72ms^{-1}$$

$$K_i = \frac{1}{2}mv_0^2$$

(iii) The final energy is all in form of K.E. Therefore

$$K_f = \frac{1}{2}mv^2$$

$$K_f = \frac{1}{2}M\left(\frac{mv_0}{M}\right)^2 = \frac{1}{2}\left(\frac{m}{M}mv_0^2\right)$$

$$= \frac{m}{M}K_i$$

Hence, the loss in energy is

$$\Delta E = K_i - K_f = K_i - \frac{m}{M}K_i$$

$$= K_i\left(1 - \frac{m}{M}\right)$$

The ratio of the energy loss to the initial energy is given by

$$\frac{\Delta E}{K_i} = 1 - \frac{m}{M}$$

This means that $\Delta E/K_i$ is of value less than unity. The energy has decreased in value.

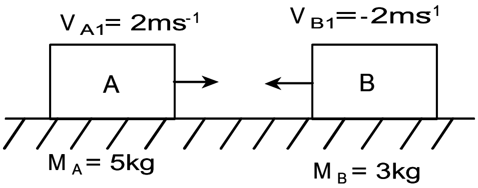
Substituting our values for m and M we get

$$\frac{\Delta E}{k_i} = 1 - \frac{14kg}{174kg}$$
$$= 0.92$$

Energy is lost as the rower 'gives' in order to bring the dog in.

SELF-ASSESSMENT EXERCISE 2

Suppose the collision in the Figure below is completely inelastic and that the masses and velocities have the values shown. Find the velocity after the collision. Find the K.E. of A and B before the collision (iii) The K.E after collision



Let V_{A2} and V_{B2} be the velocities of blocks A and B respectively after collision

Then $\quad (ii) \quad K.E.\ of\ mass\ A\ \ before\ collision\ is\ \frac{1}{2}m_A v_{A1}^2 = 10J$

$K.E.\ of\ mass\ B\ \ before\ collision\ is$

The $\quad \frac{1}{2}m_B v_{B1}^2 \quad = 6J$

The total K.E. before collision is 16J

Note that the kinetic energy of body B is positive but its velocity V_{B1} and its momentum MV_{B1} are both negative

Therefore the kinetic energy after collision is

$$\frac{1}{2}(m_A + m_B)v_2^2 = 1J$$

What has happened to the rest of the K.E. they had before collision?
For the same conditions above when

$$m_A = 5kg,\ m_B = 3kg$$
$$v_{A1} = 2ms^{-1},\ v_{B1} = -2ms^{-1}$$

The masses A and B travelling towards each other and under goes perfect elastic collision (i) what are the velocities of masses A and B after collision (ii) the kinetic energy before collision (iii) the total K. E. after collision.

$$(i) \quad v_2 = \frac{m_A v_{A1} + m_B v_{B1}}{m_A + m_B} = 0.5ms^{-1}$$

Since v_2 is positive, the system goes to the right after collision

Solution:
From the principle of conservation of momentum,

$$(i) \qquad (5kg)(2ms^{-1}) + (3kg)(-2ms^{-1})$$
$$= (5kg)v_{A2} + (3kg)v_{B2}$$
$$\therefore 5v_{A2} + 3v_{B2} = 4ms^{-1}$$

Since the collision is perfectly elastic. $V_{B2} - V_{A2} = -(V_{B1} - V_{A1}) = 4\text{m s}^{-1}$.

Solving these equations simultaneously we obtain

$$v_{A2} = -1ms^{-1}; \; v_{B2} = 3ms^{-1}$$

This implies that both bodies reverse their directions of motion. A now travels to the left at 1ms^{-1} and B goes to the right at 3ms^{-1}.

(ii) The total K.E. after collision is

$$\frac{1}{2}(5kg)\left(-1ms^{-1}\right)^2 + \frac{1}{2}(3kg)\left(-1ms^{-1}\right)^2 \qquad = 16J$$

We see that this is equal to the total K.E. before collision which confirms that the collision is perfectly elastic.

3.2.2 Explosions

$m_1 \, and \, m_2 + m_2$ Let us consider a case where two objects approach each other and merge in a frame of reference where the total momentum is zero. We also assume that these objects remain at rest after merging. When the opposite of this action occurs, that is, when an object at rest in such a frame of reference breaks up into two or more objects with an attendant sound, it becomes an explosion. The initial object of mass at rest breaks up into two objects, and they move with velocities such that the momentum is zero. That is their

$$m_1v_1 + m_2v_2 = 0 \qquad 3.9$$

From the law of energy conservation, once an object has initial potential energy U, then explosion is possible.

$$\therefore U = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \qquad 3.10$$

Explosives used during wars have potential energy stored in molecules. When the explosives are detonated, there is tremendous release of energy. Let us now use an example to explain this concept.

SELF-ASSESSMENT EXERCISE 3

Let us consider what happens during fission of an element. That is, the case in which an unstable atomic nucleus disintegrates. Let's use element Polonium, for example. It's symbol is ^{210}Pb which has mass 3.49 x 10^{-25} kg. This element can decay into an alpha particle (actually a Helium nucleus) of mass 6.64 x 10^{-27}kg and a type of lead nucleus (symbol ^{206}Pb) of mass 3.42 x 10^{-25}kg.

□

That is

$$^{210}P_0 \rightarrow \propto + ^{206}Pb$$

The products of the decay have K.E. of 8.65 x 10^{-13}J above any K.E. possessed by the polonium nucleus itself. For the decay of such a polonium nucleus at rest, Find the speeds of the □ particle and the lead nucleus?

Solution:

Let Q = the K.E. of the products of decay

Then by conservation of momentum law and

$$M_\propto v_\propto = M_{pb} v_{pb}$$

$$Q = \frac{1}{2} m_\propto v_\propto^2 + \frac{1}{2} M_{pb} v_{pb}^2$$

where v is the speed of the respective particles as indicated by the subscripts. We solve these two equations for the variables of interest and find that

$$v_\propto = \sqrt{\frac{2Q}{m_\propto (1 + m_\propto / M_{pb})}}$$

and

$$v_{pb} = \sqrt{\frac{2Q}{M_{pb}(1 + M_{pb} / m_\propto)}}$$

Given that Q = 8.65 x 10^{13}J, then computing the above gives

v_α = 1.60 x 10^7ms^{-1}

and

v_{pb} = 3.10 x 10^5 ms^{-1}

We observe that the speed of the heavier of the two products of decay is much less than that of the lighter one. This result is seen in the conservation of momentum equation.

3.3 Elastic and Inelastic Collisions

In an elastic collision in one dimension, there is no transfer of mass from one object to another. This implies that the total kinetic energy of the objects before collision is equal to the total kinetic energy of the objects after collision. If the final velocities of the two objects 1 and 2 are v_3 and v_4 then additionally

$$m_1v_1 + m_2v_2 - m_1v_3 + m_3v_4 \qquad 3.11$$

By the conservation of energy, it follows that

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_2v_3^2 + \frac{1}{2}m_2v_4^2 \qquad 3.12$$

We can find the final velocities of the colliding objects if we know the initial velocities. Rewriting equation (3.11) we have

$$m_1(v_1 - v_3) = -m_2(v_2 - v_3) \qquad 3.13$$

Now, applying our knowledge of mathematical algebra, we use.

$$v_1^2 - v_4^2 = (v_1 - v_3)(v_1 + v_3)$$
and
$$v_2^2 - v_4^2 = (v_2 - v_4)(v_2 + v_4)$$
to rewrite equation (12) *in the form*
$$\frac{1}{2}m_1(v_1 - v_3)(v_1 + v_3) = -\frac{1}{2}(v_2 - v_4)(v_2 + v_4) \qquad 3.14$$

We now divide both sides of equation (3.14) by the two sides of equation (3.13) to get

$$v_1 + v_3 = v_2 + v_4 \qquad 3.15$$

Let u be the relative velocity of the two colliding particles (objects) then,

$$u_i = v_1 - v_2$$
and
$$u_f = v_3 - v_4$$
subsituting these in eqn. (3.15) *we get*
$$u_i = -u_f \qquad 3.16$$

We conclude from Eqn. (3.16) that in an elastic collision, the relative velocity of the colliding objects change sign but does not change in magnitude.

As a rule of thumb always think of a perfectly elastic rubber ball hitting a brick well. Relative velocity behaves like the velocity of this rubber.

We now solve Eqn.(3.15) for one of the unknown variables like v_4
Thus

$$v_4 = v_1 - v_2 + v_3 \qquad 3.17$$

Substituting this in the momentum conservation equation (3.11) we get

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 (v_1 - v_2 + v_3)$$
regrouping terms, we get
$$(m_1 + m_2) v_3 = (m_1 - m_2) v_1 + 2 m_2 v_2 ;$$
$$v_3 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) + \left(\frac{2m_2}{m_1 + m_2}\right) v_2 \qquad 3.18$$
Similarly we can, solve for v_4 *to get*
$$v_4 = \left(\frac{2m_1}{m_1 + m_2}\right) v_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2 \qquad 3.19$$

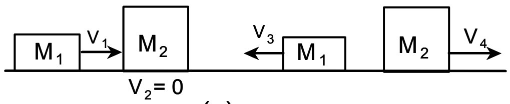
As an exercise, show the derivation of equation (3.19).Equations (3.18) and (3.19) seem complicated. Let us now simplify them by applying them to practical situations (3.19).

1. A Scenario where object 2 is initially at rest.
   Here V_2 = O so that equations (3.18) and (3.19) reduce to

$$v_3 = \frac{m_1 - m_2}{m_1 + m_2} \qquad 3.20$$
and
$$v_4 = \left(\frac{2m_1}{m_1 + m_2}\right) v_1$$

$m_1 = m_2$ 1a. If the two objects have equal masses i.e.

$v_3 = 0$ *and* $v_4 = v_1$ Here. This means that the moving object after collision stays at rest while the object formerly at rest now moves with the initial velocity of the first object. This effect can be seen vividly in hard billiard shots along a line.



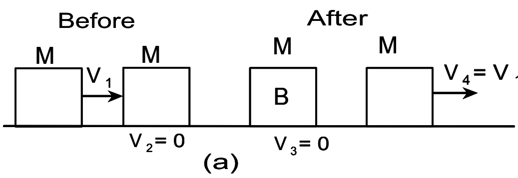
(c)

m_2 1b. If mass >> mass

$v_3 = v_1$ *and* $v_4 = 2v_1$ In this case eqns (3.20 a b) yield

It means that the velocity of the moving object decrease a little, while the object initially at rest picks up almost twice the velocity of the incoming object.

$m_2 >> mass m_1$ *as in the fig below* 1c. If mass



(a)

$v_3 = -v_1$ *and* $v_4 = (2m_1/m_2)v_1$ For these conditions Eqn. (3.20a and b) give

We see that the moving object very nearly reverses its velocity, while the object initially at rest recoils (i.e. moves back) with a very small velocity.

m_2 In the limit that approaches infinity, we neglect the velocity of recoil and the final velocity of the first object is equal and opposite to its incident velocity. A practical example is what happens when a tennis ball bounces off a wall.

Can you suggest more practical phenomena that demonstrate this case?

2. Scenario where the initial total momentum is zero.

The two objects under discussion approach each other with velocities such that the initial and total momentum is zero.
That is,

$$m_1 v_1 + m_2 v_2 = 0 \qquad 3.21$$

$$Thus,\ v_2 = \left(\frac{m_1}{m_2}\right) v_1$$

Putting this value for v_2 in Eqn. (3.18) we find

$$\begin{aligned} v_3 &= \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_1 + \left(\frac{2m_2}{m_1 + m_2}\right)\left(-\frac{m_1}{m_2}\right) v_1 \\ &= \left(\frac{m_1 - m_2 - 2m_1}{m_1 + m_2}\right) v_1 \\ &= -v_1 \qquad 3.22 \end{aligned}$$

$\left(m_1 v_3 + m_2 v_4\right)$ To solve for, we apply the conditions set out in this scenario that the initial total momentum was zero. Therefore, by the conservation of liner momentum, the final total momentum must also be zero and

$$v_4 = \left(\frac{m_1}{m_2}\right) v_3 = \left(\frac{m_1}{m_2}\right) v_1 = -v_2 \qquad 3.23$$

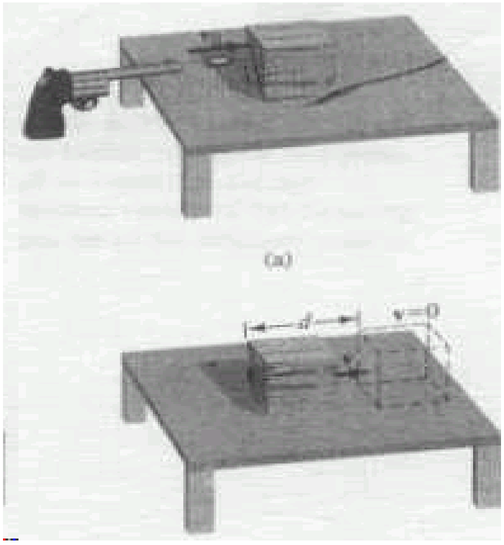
We conclude that for the case where total momentum is zero, the velocities of each of the objects are unchanged in magnitude but they change in sign. We conclude that in each of these cases each of the objects behave as if it hit an infinite massive brick wall. We now do some examples

SELF-ASSESSMENT EXERCISE 4

A bullet is fired in the + x-direction into a stationary block of wood that has a mass of 5kg. The speed of the bullet before entry into the block is $V_0 = 500ms^{-1}$. What is the speed of the block just after the bullet has become embedded? What distance will the block slide on a surface with coefficient of friction equal to 0.50?

Solution:

Let m = mass of the bullet
V_0 = bullet's velocity before it enters the block of wood.
M = combined mass of bullet and wood.



The initial momentum $P_i = Mv_O$
if v = velocity of bullet combined with wood after the bullet has entered the wood then by conservation of momentum

$$mv_o = Mv$$

$$\therefore v = \left(\frac{m}{M}\right)v_o = \frac{10g}{5010g}\left(500ms^{-1}\right) = 1ms^{-1}$$

Note that if we ignore the mass of the bullet compared to the mass of the wood, we shall still get almost the same value for velocity.

Given that the frictional force acting is

:N = -:Mg

(here, the R.H.S. is minus because friction points to the left opposing motion of the block). The frictional force has constant magnitude and leads to a constant acceleration, a, of the block. Therefore, applying Newton's second law we have

-:Mg = Ma
a = -:g

The negative sign implies that the block slows down travelling a distance d before it stops.
Since acceleration is uniform, we use the relation

$$v_f^2 - v_i^2 = 2ad \ (\textit{from the fact that } v - v_o = at).$$

Now with $v_f = 0$ (that's when the block stops moving) we have

$$d = \frac{v_i^2}{2a} = \frac{1}{2}\frac{v_i^2}{\mu g} = \frac{1}{2}\frac{(1ms^{-1})^2}{(0.5)(9.8ms^{-2})}$$
$$= 0.1m$$

SELF-ASSESSMENT EXERCISE

An empty freight car of mass 10,000kg rolls at $2ms^{-1}$ along a level track and collides with a loaded car of mass 20,000kg, standing at rest with brakes released. If the cars couple together:

1. find their speed after the collision.
2. find the decrease in kenetic energy as a result of the collision
3. with what speed should the loaded car be rolling toward the empty car in other that both shall be brought to rest by the collision?

Solution: Recall that,

(a) The momentum before collision = momentum after collision i.e.

$$m_1 v_1 + m_2 v_2 = Mv$$
$$\therefore \quad v = \frac{m_1}{M} v_1$$
$$= \frac{10000kg}{30000kg} \; x \; 2ms^{-1}$$
$$v = 0.67ms^{-1}$$

(b) Now, K.E. before collision - K.E. after collision = loss K.E.

$$\frac{1}{2}m_1 v_1 + \frac{1}{2}m_1 v_2^2 - \frac{1}{2}mv^2$$
$$\therefore \frac{1}{2_1}10^{5000},000 \; X \; 2^2 - \frac{1}{2_1}30^{15000},000 \; (.67)^2$$
$$i.e.\; 20000 - 6666.65 = 13{,}333J.$$

(c) Answer is $1ms^{-1}$

4.0 CONCLUSION

In this unit, you have learnt

- that collision is the sudden impact felt between two objects that there are two types of collisions viz: elastic and inelastic collisions
- how to determine the energy lost in perfectly inelastic collisions
- how to distinguish between elastic and inelastic collisions
- how to apply the principles of the conservation of energy and momentum in the solution of collision problems.
- how to apply the collision principle in the study of rocket propulsion.

5.0 SUMMARY

What you have learnt in this unit are:

- that collision is the sudden impact felt between two objects.
- that collisions can be classified into elastic collisions
    - inelastic collisions
    - erfectly inelastic collisions
- that during elastic collision no energy is lost, that is,
    - that the total K.E. of the colliding particles before and after collision are equal
- that during inelastic collision kinetic energy is not conserved.
- that when bodies collide and coalesce, the phenomenon constitutes perfectly inelastic collision.
- that collisions are described by the equation

$$m_1v_1 + m_2v_2 = Mv$$

where the symbols have their usual meaning

- that for perfectly inelastic collision when the objects coalesce on impact that before collision one of the objects was at rest and the other runs into it, then,

$$v = \frac{m_1 v_1}{m_1 + m_2} = \left(\frac{m_1}{M}\right) v_1$$

$m_1 >> m_2$, that if the coalesced object will move with a velocity nearly equal to v_1

$m_2 << m_2$, • that if then, will move with velocity $\left(\frac{m_1}{m_2}\right) v_1$

- that for head-on collision of two objects moving towards each other with equal velocities

$$velocities,\ v_2 = -v_1$$

$$hence,$$

$$v = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_2$$

- that the energy change is given by

$$DE = E_f - E_i$$

$$DE = -\frac{m_1 m_2}{2(m_1 + m_2)}\left(v_1 - v_2\right)^2$$

- that the negative sign in the R.H.S. of the equation above shows that energy is lost in such a collision.
- that the ratio of the energy loss to the initial energy is given by

$$\frac{\Delta E}{K_i} = 1 - \frac{m}{M}$$

- that $\Delta E/K_i$ is of value less than unity.
- that when a stationary object disintegrates with attendant sound, it becomes, an explosion. Here, after explosion, particles move but their momentum is conserved, hence,

$$m_1 v_1 + m_2 v_2 = 0$$

$$also,\ Potentialenergy,\ U = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2$$

that for elastic collision

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4$$

$$and$$

$$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_3^2 + \frac{1}{2}m_2 v_4^2$$

6.0 TUTOR-MARKED ASSIGNMENTS

1. An object of mass 2kg is moving with a velocity of $3ms^{-1}$ and collides head on with an object B of mass 1kg moving in the opposite direction with a velocity of $4ms^{-1}$.
   (i) After collision both objects coalesce, so that they move with a common velocity, v. Calculate v.

2. A 14,000kg truck and a 2000kg car have a head-on collision. Despite attempts to stop, the truck has a speed of 6.6 ms^{-1} in the + x-direction when they collide and the car has a speed of $8.8ms^{-1}$ in the - x-direction. If 10% of the initial total kinetic energy is dissipated through damage to the vehicle, what are the final velocities of the truck and the car after the collision? Assume that all motions take place in one dimension.

3. Two spheres with masses of 1.0kg and 1.5kg hang at rest at the ends of strings that are both 1.5 long. These two strings are attached to the same point on the ceiling. The lighter sphere is pulled aside so that its string makes an angle 2= 60^0 with the vertical. The lighter sphere is then released and the two spheres collide elastically. When they rebound, what is the largest angle with respect to the vertical, that the string holding the lighter sphere makes?

7.0 REFERENCES/FURTHER READING

Duncan, T. (1982). A textbook for Advanced Level students. London: John Murray Publ. Ltd.

Grounds Stephen, & Kerby, E. (1994). Longman revise guides A -Level and As-Level PHYSICS (7th Impression.) Longman Group UK Ltd.

Sears, F.W. Zemansky, M. W. & Young, W.D. (1975)College Physics Addision-Wesley Publishing Company, London

Nelkon, M. & Parker, P. (1971).Advanced level physics(3rd ed}, London: Heinemann Educational Books Ltd