MODULE 3

UNIT 1 GRAVITATIONAL MOTION

CONTENT

1.0 INTRODUCTION

In units 6 to 8 you studied linear, circular and projectile motion as well as forces. We restricted our study to motion of objects on the earth and we touched slightly on acceleration due to gravity as a pull the earth exerted on objects. In this and the subsequent two units, you will study gravitation in more details.

We shall begin here by developing the concept of gravitation, introduce Kepler's laws and see how Newton used kepler's law to test his universal law of gravitation. We shall also discuss the concept of mass and weight and solve problems partaining to gravity.

In the next two Units we shall apply the concepts of mechanics developed here to orbital motion under gravity and to gravitation and extended or heavenly bodies.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define Newton's law of universal gravitation
- describe the experiment used in the determination of the magnitude of the gravitational
- constant, G.
- apply the law of gravitation
- state keplers laws
- differentiate between weight and mass
- determine the mass, volume and density of the earth
- differentiate between inertial and gravitational mass.

3.0 MAIN CONTENT

3.1 Law of Universal Gravitation

Sir Isaac Newton deduced the law of universal gravitation in 1686 from speculations concerning the fall of an apple toward the earth. His proposal, **the principia** (mathematical principles of natural knowledge) was, that the gravitational attraction of the sun for the planets is the source of the centripetal force which maintains the orbital motion of the planets round the sun. Newton also affirms that this was similar to the attraction of the earth for the apple. **Thus, gravity-the attraction the earth has for an object** - which you are already familiar with, was a particular case of gravitation. According to Newton also, there is a gravitational force between all objects in the universe. It is this universal gravitational force that is responsible for the orbital motion of the heavenly bodies.

So, what is this universal law of gravitation? This Newton's law of universal gravitation may be stated thus:

> *Every particle of matter in the universe attracts other particles with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distances apart.*

What do you say to this. This means there is gravitational attraction between you and any object in the room where you are.

The gravitational attraction, F between two bodies of masses M_1 and M_2 which are a distance r apart is given by

$$F \propto \frac{m_1 m_2}{r_2} \qquad 3.1$$

That is

$$F = G\frac{m_1 m_2}{r_2} \qquad 3.2$$

where G is a constant called the universal gravitational constant. It is assumed to have the same value every where for all matter.

Newton believed that the force was directly proportional to the mass of each particle because the force in a falling body is proportional to its mass (F = ma = mg = m x constant, therefore F α m), that is, the mass of the attracted body. From the stand point of his third law, Newton also argued that a falling body exerts an equal and opposite force that is proportional to the mass of the earth. Then it was concluded that the gravitational force between the bodies must also be proportional to the mass of the attracting body. The moon test to be discussed later justified the use of an inverse square law relation between force and distance.

Newton law of gravitation refers to the force between two particles. It can also be shown that the force of attraction exerted on or by a homogeneous sphere is the same as if the mass of the sphere were concentrated at its centre. The prof of this will be treated in a latter course. We shall simply state here the fact that the gravitational force exerted on a body by a homogeneous sphere is the same as if the entire mass of the sphere were concentrated in a point at its centre. Thus if the earth were a homogeneous sphere of mass M_E, the force exerted by it on a small body of mass m_1 at a distance r from its centre, would be

$$F_g = G\frac{mM_E}{r^2}$$

A force of the same magnitude would be exerted on the earth by the body.

The magnitude of the gravitational constant G can be found experimentally by measuring force of gravitational attraction between two bodies of known masses m and m^{l} , at a known separation. For bodies of moderate sizes, the force is extremely small, but it can be measured with an instrument invented by the Rev. John Michell and first used for this by Sir Henry

Cavendish in1798. the same type of instrument was also used by Coulomb for studying forces of electrical magnetic attraction and repulsion which you will study later.

The Cavendish balance, Fig. 3.1 consists of a light rigid T-shaped member, supported by a fine vertical fibre such as a quartz thread or a thin metallic ribbon. Two small spheres of mass m are mounted at the ends of the horizontal portion of the T, and a small mirror M, fastened to the vertical portion, reflects a bean of light onto a scale. To use the balance, two large spheres of mass $m^|$ are brought up to the positions shown. The forces of gravitational attraction between the large and small spheres result in a couple which twists the system through a small angle, thereby moving the reflected light bean along the scale. By using the extremely fine fibre, the deflection of the mirror may be made sufficiently large so that the gravitational force can be measured quite accurately. The gravitational constant, measured in this way, is found to be

$$G = 6.670\,x\,10^{-11}\,Nm^2kg^{-2}\text{ "}$$

[*Searsetal*, 1975]

Example

The mass m of one of the small spheres of a Cavendish balance is 0.001kg, the mass m^1 of one of the large spheres is0.5kg, and the centre-to-centre distance between the spheres is 0.05m. Find the gravitational force on each sphere?

Solution:

We apply the law of universal gravitation which stated mathematically is

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = 6.67\,x\,10^{-11}\,Nm^2kg^{-2}\frac{(0.001kg)(0.5kg)}{(0.05m)^2}$$

$$= 1.33\,x\,10^{-11}\,N$$

SELF-ASSESSMENT EXERCISE 1

Two spherical objects of masses 0.001kg and 0.5kg are placed 0.05m from each other in space far removed from all other bodies. What is the acceleration of each relative to an interial system? [where $F_g = 1.33 \text{ x } 10^{-11}$N]

Solution:

Applying Newton's third law of motion F=ma, the acceleration, a of the smaller sphere is

$$a = \frac{F_g}{m} = \frac{1.33\,x\,10^{-11}\,N}{10\,x\,10^{-3}\,kg}$$

$$= 1.33\,x\,10^{-8}\,ms^{-2}$$

$a^|$ The acceleration a for the larger sphere is

$$a' = \frac{F_g}{m^|} = \frac{1.33 x 10^{-11} N}{0.5kg}$$

$$= 2.67 x 10^{-11} ms^{-2}$$

We see that the accelerations are not constant since the gravitational force increases as the spheres approach, each other.

3.2 Kepler's Laws of Planetary Motion

Planetary motion excited the interest of earliest scientists, Babylonian and Greek astronomers. They attempted to predict the movements of planets to some degree of accuracy. Before Nicolaus Copernicus, it was considered that the earth was the centre of the universe but about 1543 Copernicus introduced a heliocentric frame, with the sun at the centre of the solar system. He suggested that the planets revolved round the sun in circular motion with the construction of more refined instruments no telescopes still existed. Tycho Brahe, towards the end of the sixteenth century improved on the knowledge of planetary orbits to an accuracy of less than half a minute of arc.

Brahe died in 1601 and his assistant Johannes Kepler continued his work. Kepler inherited Brahe's accumulated data and spent over twenty years analyzing them. He finally came up with the idea of elliptical orbits for planetary motion. This was a crucial break through in the data analysis and the idea of circular orbits was discarded. Kepler thus enunciated three laws known by his name These laws state:

- During equal time intervals, the radius vector from the sun to the planet sweeps out equal areas (Fig. 3.2b)
- If T is the time that it takes for a planet to make one full revolution round the sun, and if R is half the major axis of the ellipse (R reduces to the radius of the planet's orbit if that orbit is circular), then

$$\frac{T^2}{R^3} = C \qquad 3.4$$

Where C is a constant whose value is the same for all planets. Kepler's second law follow from the conservation of angular momentum which we shall treat in Unit 19. It is also consequent on the fact that the gravitational force between the sun and the planet is a central force. This means that the force acts along the line joining the sun and the planet. In fact, kepler's second law can be taken as evidence that the gravitational law is central. Conservation of angular momentum also means that the path of the planets must lie in a plane that is perpendicular to the direction of the fixed angular momentum vector.

Newton was led to the discovery of his law of gravitation by considering the motion of a planet moving in circular orbit round the sun S (Fig 3.3a). Let the force acting on the planet of mass M be mrT^2, where r is the radius of the circle and T is the angular velocity of the motion. But T = 2Bπ/T, where T is the period of the motion, then,

The force on the planet

$$= mr(\frac{2\pi}{T})^2$$

$$= \frac{4\pi^2 mr}{T^2} \qquad 3.5$$

This being equal to the force of attraction of the sun on the planet. If we assume an inverse square law where K is a constant, thenforce on planet

$$= \frac{km}{r^2} \qquad 3.6$$

Therefore,

$$\frac{km}{r^2} = \frac{4\pi\pi mr}{T^2} \qquad 3.7$$

$$\therefore\ T^2 = \frac{4\pi}{k} r^3 \qquad 3.8$$

Hence

$T^2 \propto r^3$

Since k, 2 are constants Kepler having announced that the square of the periods of revolution of the planets are proportional to the cubes of their mean distances from the sun (as stated in his laws above), Newton used this law to test the inverse square law by applying it to the case of the moon's motion round the earth referenced above (Fig. 3.3b).

The period of revolution, T of the moon above the earth is 27.3days. The force on the moon is mRπ^2, where R is taken to be the radius of the moon's orbit and m, its mass

$$\therefore force = mR(\frac{2\pi}{T}) = \frac{4\pi^2 mR}{T^2} \qquad 3.9$$

If the planet were at the earth's surface, the force of attraction in it due to the earth would be mg, where mg is the acceleration due to gravity (Fig. 3.3b). If we assume that the force of attraction varies as the inverse square of the distance between the earth and the moon, then

$$\frac{4\pi mR}{T^2} : mg = \frac{1}{R^2} : \frac{1}{r^2}$$

$$\therefore \frac{4\pi R}{T^2 g} = \frac{r^2}{R^2} \qquad 3.10$$

$$\therefore g = \frac{4\pi R^3}{r^2 T^2} \qquad 3.11$$

where r is the radius of the earth
Substituting the known values of R, r and T, the result for g was very close to 9.8m s^{-2}.

Thus the inverse square law was justified.

SELF-ASSESSMENT EXERCISE 3

State Kepler's laws of planetary motion

3.3 Mass and Weight

The weight of a body can be defined more generally as the resultant gravitational force exerted on the body by all other bodies in the universe. The earths attractive force on an object on its surface is much greater than all other gravitational forces on the object so we neglect all these other gravitational forces. The weight of the object for practical purposes then results solely from the earth's gravitational attraction on it. Similarly if the object is on the surface of the moon or of another planet, its weight will result solely from the gravitational attraction of the moon or the planet on it. Thus, assuming the earth to be homogeneous sphere of radius R and mass of M_E, the weight w of a small object of mass M in its surface would be

$$W = F_g = G\frac{mm_E}{R^2} \qquad 3.12$$

Note that the weight of a given body or object varies by a few tenths of percent from location to location on the earths surface. Do you know why this is so? It is partly because there could be local deposits of ore, oil or other substances, with differing densities or partly because the earth is not a perfect sphere but flattened at its poles. It is known that the distance from the poles to the centre of the earth is shorter than that from the equator to the earth's centre, so, the acceleration due to gravity varies at these locations. Also the weight of a given body decreases inversely with the square of the distance from the earth's centre. For example, at a radial distance of two earth radii, the weight of a given object has decreased to one quarter of its value at the earth's surface. This means that if you are taken far away into the space, your weight will be far much less than it is here. At a certain distance you might even become weightless. We shall discuss this phenomenon later in Unit 2 of this course.

The rotation of the earth about its axis is also part of what causes the apparent weight of a body to differ slightly in magnitude and direction from the earth's gravitational force of attraction. For practical purposes we ignore this slight difference and assume that the earth is an inertial reference system. Then, when a body is allowed to fall freely, the force accelerating it is its weight, w and the acceleration produced by this force is that due to gravity, g. The general relation

F = ma

therefore becomes, for the special case of freely falling body,

$$w = mg \qquad 3.13$$

now,

$$w = mg = \frac{Gmm_E}{R^2}$$

it follows that,

$$g = \frac{Gm_E}{R^2} \qquad 3.14$$

M_E This shows that the acceleration due to gravity is the same for all bodies or objects (because m cancelled out). It is also very nearly constant (because G and are constants and R varies only slightly from point to point on the earth)

The weight of a body is a force and its unit is the Newton, N in mks system. In cgs system, it is the dyne and in the engineering system it is the pound (lb). So Eqn. (3.3) gives the relation between the mass and weight of a body in any consistent set of units.For example, the weight of the object of mass 1kg at a point where g=9.80ms^{-2} is

$$w = mg = 1kg \ x \ 9.80ms^{-2}$$
$$= 9.80N$$

at another place where g = 9.78ms^{-2}
its weight is w = 9.78N

Thus, we see that weight varies from one point to another. Mass does not.

You can now answer the question,
What is your weight? Will your weight be the same on the surface of the earth as on the surface of the moon?
The 1kg mass placed on the surface of the moon will weigh,

$w = mg = 1kg \times 1.67m\ s^{-2}$
$= 1.67\ N$

This is so, because $g = 1.67m\ s^{-2}$ on the moon. This will help you determine what your own weight would be if you were placed at the surface of the moon.

3.3.1 Mass of the Earth

Applying Newton's law of gravitation we have that

$$w = mg = G\frac{mm_E}{R^2}$$

This gives the mass of the earth as

$$m_E = \frac{R^2 g}{G} \qquad 3.15$$

where R is the earth's radius . Since all the quantities on the R.H.S of the Eqn. (3.15) are known, we can calculate the mass of the earth.

Hence,
for R = 6370km, $G = 6.37 \times 10^{6}$ m and $g = 9.80\ ms^{-2}$
$:M_E = 5.98 \times 10^{24}kg$

The volume V_E of the earth is

$$\vee_E = \frac{4}{3}\pi R^3 = 1.09\, x\, 10^{21} m^3$$

Thus the average density of the earth is
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$$\rho_E = \frac{M_E}{v_E} = 5500kgm^{-3}$$

or

$$= 5.5gcm^{-3}$$

(The density of water is 1g cm^{-3} = 1.000kgm^{-3}). The density of most rock near the earth's surface, such as granites and gneisses, is about 3g cm^{-3} = 3000kg m^{-3}. We see that the interior of the earth have higher density than the surface.

SELF ASSESSMENT EXERCISE 3.3

In an experiment using Cavendish balance to measure the gravitational constant G, it is found that sphere of mass 0.8kg attracts another sphere of mass 0.004kg with a force 13 x 10^{-11} N when the distance between the centres of the spheres is 0.04m. The acceleration of gravity at the earth's surface is 9.80 ms^{-2}, and the radius of the earth is 6400km, compute the mass of the earth from these data.

Solution

The gravitational force between the objects of mass m_1 and m_2 is

$$F = G\frac{m_1 m_2}{r^2}$$

where r is the distance between the centres of the spheres as given . Substituting other given values we have

$$G = \frac{13\,x\,10^{-11}\,N\,x\,(0.04)^2\,m^2}{0.8\,kg\,x\;0.004\,kg}$$

$$= \;6.5\,x\,10^{-11}\,Nm^2\,kg^{-2}$$

mass of the earth will be given by

$$m_E = \frac{R^2 g}{G} = \frac{(6.40\,x\,10^6)^2\,m^2\,x\,9.8\,ms^{-2}}{6.5\,x10^{-11}\,Nm^2\,kg^{-2}}$$

$$= \;61.7551\,x10^{23}\,kg$$

$$= \;6.2\,x\,10^{24}\,kg$$

SELF ASSESSMENT EXERCISE 3.4

The mass of the moon is about one eighty-first, and its radius one fourth, that of the earth. What is the acceleration due to gravity on the surface of the moon?

Solution

$$M_m = \frac{1}{81}x\;6.2\;x\;10^{24}\;kg$$

We are given that mass of moon is, radius of mass

$$r_m = \frac{1}{4} x\, 6.4\, x\, 10^6\, m$$

But

$$g_m = \frac{M_m G}{r_m^{\ 2}}$$

$$= \frac{\frac{1}{18} x\, 6.2\, x\, 10^{24}\, kg\, x\, 6.67\, x\, 10^{-11}\, Nm^2 kg^{-2}}{(\frac{1}{4} x\, 6.4\, x\, 10^6\, m)^2}$$

$$= 1.98 ms^{-2}$$

I would want you to note that the mass m in F = ma = mg is known as the inertial mass of the body. It is a measure of the opposition or resistance of the body to change of motion. That is, its inertia. When considering the law of gravitation, the mass of the same body is regarded as gravitational mass. From experiments, the two masses are seen to be equal for a given body and so, we can represent each by m (be it inertial or gravitational mass).

4.0 CONCLUSION

In this unit, you have learnt

- that the universal law of gravitation was stated by Sir Isaac Newton as the force ofattraction every object in the universe exerts on each other which is proportional to theproduct of their masses and inversely proportional to the square of the distance between them.
- how to describe the experiment to determine the magnitude of the gravitational constant,G .
- how to apply the law of universal gravitation.
- the three Kepler's laws of planet motion
- the general definition of the weight of a body
- how g and G are related.
- how to determine the mass volume and the density of the earth
- what inertial and gravitational masses mean.

5.0 SUMMARY

What you have learnt in this unit are:

- that every particle in the universe attracts every other particle with a force which is directly proportional to the product of their masses and inversely proportional to the square of their distances apart. Hence

$$F = G\frac{m_1 \; m_2}{r^2}$$

- that in the expression for gravitational force above, that G is the universal gravitationalconstant and is the same everywhere
- that the Cavendish balance is used to determine G experimentally.
- that G has value 6.67 x 10^{-11} Nm^2 kg^{-2}
- that acceleration due to gravity is not constant since the gravitational force increases as
- the spherical bodies approach each other.
- that astronomical observations led Kepler to three laws of planetary motion.

1. Planets move in planar elliptical paths with the sum at one focus of the ellipse.
2. During equal time intervals, the radius vectors from the sun to the planet sweepsout equal areas.
3. If T is the time it takes for a planet to make one full revolution around the sun, and if R is half the major axes of the ellipse, (R reduces to the radius of the orbit of the planet if that orbit is circular) then.

$$\frac{T^2}{R^3} = C$$

where C is a constant whose value is the same for all planets.

- Newton showed that these Keplers laws are a consequence of a law of universal gravitation.
- that the masses that exert gravitational forces are not always point like.We can have an object with spherical mass distribution like the earth or sun. In this case, the gravitational force is the same as if all the mass of the extended object were concentrated at centre of the spherical distribution.
- that the Newtonian theory of gravity is a limiting case of a more accurate and
- fundamental theory of gravity.
- that the weight of a body can be defined more generally as the resultant gravitational force exerted on the body by all other bodies in the universe.

i.e.

$$w = F_g = \frac{GmM_E}{R^2}$$

It follows that,

$$g = \frac{GM_E}{R^2}$$

where the symbols have their usually meaning.

- that weight varies from location to location
- that the mass of the earth is given by

$$m_E = \frac{R^2 g}{G}$$

where R = radius of the earth, G = the universal constant and g is the acceleration due to gravity.

6.0 TUTOR-MARKED ASSIGNMENT

1. State Newton's law of Gravitation. If the acceleration due to gravity, g_m at the surface of the moon is 1.70 ms^{-2} and its radius is 1.74 x 10^6m, calculate the mass of the moon.

2. Calculate the mass of the sun, assuming the Earth's orbit around the Sun is circular, with radius r = 1.5 x 10^8 km.

3. Explain what is meant by the gravitation constant and describe an accurate laboratory method of measuring it. Give an outline of the theory of your method.

   (i) The weight of a body on the surface of the earth is 900N. What will be its weight on the surface of mars whose mass is 1/9 and radius ½ that of theearth

   (ii) Mass of the moon M_m is given by

7.0 REFERENCES/FURTHER READING

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UNIT 2 ORBITAL MOTION UNDER GRAVITY

CONTENTS

1.0 INTRODUCTION

In this Unit, we shall continue our discussion on gravitation, commenced in unit 11. Particularly shall we focus on orbital motion under gravity beginning with motion in a vertical circle. We shall then discuss motion of a satellite and identify possible trajectories a satellite can have. You will learn to determine the velocity of a satellite in its orbit as well as its period of revolution by applying the knowledge of gravitational force on the satellite. We shall end with the introduction of concept of parking orbit and weightlessness. This unit will let you have a feel of what astronauts experience when they are projected into space. The next Unit will enlighten you as to the velocity an object or a satellite can have before it could be able to escape from the surface of the earth. You will see that Science stimulates one to take giant steps and do giant things to move the world forward. Positioning telecommunication satellite in space is an example of how science has turned this vast world into a global village whereby communication has been successfully trivialised.

2.0 OBJECTIVES

At the end of this unit, you should be able to

- determine normal radial and tangential accelerations of a body in vertical circular motion
- describe the motion of a satellite in an orbit in terms of the velocity and period.
- state at least one application of a parking orbit.
- explain the concept of weightlessness
- calculate the magnitude and direction of an impulse needed to launch a satellite in space given all necessary requirements

3.0 MAIN CONTENT

3.1 Motion in a Vertical Circle

Figure 3.1 represents a small body attached to a cord of length R and whirling in a vertical circle about a fixed point 0 to which the other end of the cord is attached. The motion, though circular is not uniform because the speed increases on the way down and decreases on the way up. The forces on the body at any point are its weight w=mg and

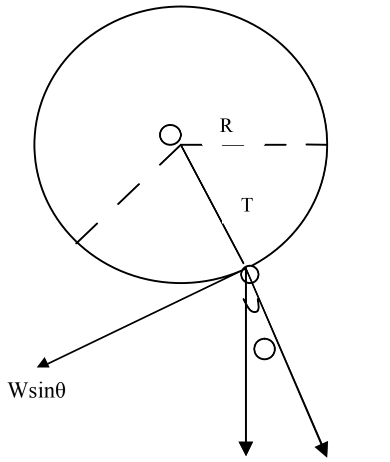


Fig 3.1

the tension T in the cord. Resolving the weight of the body into its components we have magnitude of normal component = w cosθ
Magnitude of tangential component = w sin θ

The resultant tangential and normal forces are:

$$F_{11} = w\ Sin\theta\ and\ F_{\perp} = T - w\cos\theta$$

From Newtons second law then, we get the tangential acceleration a_{11}

$$a_{11} = \frac{F_{11}}{m} = gSin\theta. \qquad 3.1$$

This is the same as that of a body sliding down a frictional inclined plane of slope angle θ.

The normal radial acceleration $a_{\perp} = \frac{V^2}{R}$ is

$$a_{1\perp} = \frac{F_{\perp}}{m} = \frac{T - w\ \ Cos\theta}{m} = \frac{V^2}{R}. \qquad 3.2$$

$$T = m\left(\frac{V^2}{R} + g\,Cos\theta\right) \qquad 3.3$$

at the lowest point of the path, θ = O, ∴ Sin θ = 0 and cosθ = 1. Therefore at this point F_{11} = 0 and a_{11} = 0 and the acceleration is purely radial (upward). The magnitude of the tension, from Eqn.., (3.3) is $T = m\left(\frac{V^2}{R} + g\right)$

At the highest point, θ = 180° ∴ Sin θ = O and Cos θ = - 1, and the acceleration once more is purely radial (downward). The tension for this case is

$$T = m\left(\frac{V^2}{R} - g\right). \qquad 3.4$$

For this kind of motion, there is a certain critical speed V_C at the highest point below which the cord slacks and the path will no longer be circular. To find this critical speed, we set T = 0 in Equation (3.4) i.e.

$$m\left(\frac{V_C{}^2}{R} - g\right) = 0$$

$$\therefore V_C = \sqrt{Rg}$$

Example

(a) Fig 3.2

In Fig. 3.2 above, a small body of mass m = 0.10kg swings in a vertical circle at the end of a cord of length R = 1.0m. If the speed V = 2.0ms^{-1} when the cord makes an angle θ = 30° with the vertical. Find
(a) the radial and tangential components of its acceleration at this instant.
(b) the magnitude and direction of the resultant acceleration and (c) the tension T in the cord.

Solution:
The radial component of acceleration is

$$a_\perp = \frac{V^2}{R} = \frac{(2.0ms^{-1})^2}{1.0m}$$

$$= 4.0ms^{-2}$$

The tangential component of acceleration due to the tangential force mg Sin θ, is

$$\alpha_{11} = g\,Sin\theta = 98ms^{-2}x0.50$$
$$= 4.9ms^{-2}$$

The magnitude of the resultant acceleration as shown in Fig. 3.2 above is

$$a = \sqrt{a_{\perp}^2 + a_{11}^2} = 6.3ms^{-2}$$

The angle Φ is

$$\phi = \tan^{-1}\frac{a_{11}}{a_{\perp}} = 51°$$

The tension in the cord is given by

$$F_{\perp} = Ma_{\perp} : T - mgCos\theta = \frac{mV^2}{R}$$
$$\therefore T = m\left(\frac{V^2}{R} + g\,Cos\theta\right)$$
$$= 1.3N$$

Note that the magnitude of the tangential acceleration is not constant. It is proportional to the sine of the angle θ. So, we cannot use the equations of motion to find the speed at other points. Later on, we shall show how we determine the speeds at other points using energy considerations.

3.2 Motion of a Satellite

In our discussion of the trajectory of a projectile in Unit 8 we assumed that the gravitational force on the projectile (its weight w) had the same direction and magnitude at all points of its trajectory. These conditions are satisfied to a certain degree provided the projectile remains near the surface of the earth as compared to the earth's radius. We saw that for these conditions, the trajectory is a parabola.

Note that in reality the gravitational force is directed toward the centre of the earth and it is inversely proportional to the square of the distance from the center of the earth, which means that it is not constant in magnitude and direction. Under an inverse square force directed to a fixed point, it can be shown that the trajectory turns out to be a conic section (ellipse, circle, parabola or hyperbola).

Let us assume that a tall tower could be constructed as in Fig. 3.3 below and that a projectile were launched from point A at the top of the tower in the "horizontal" direction AB.

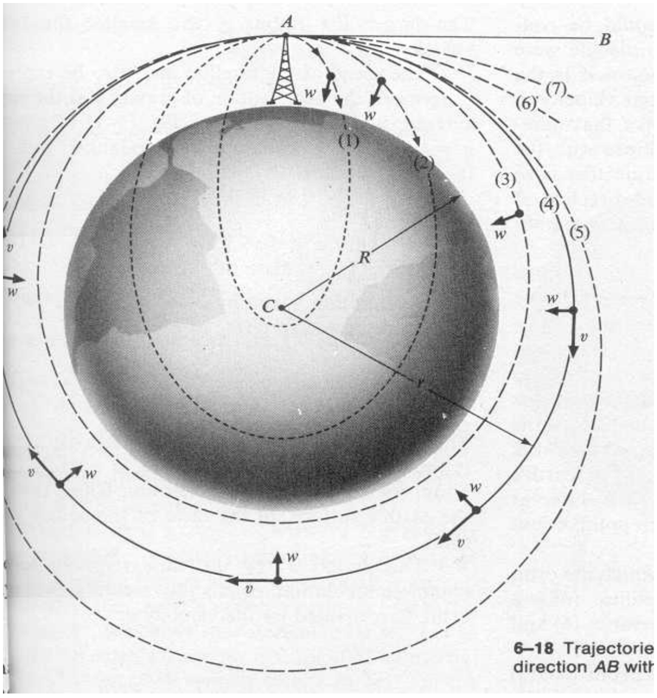


Fig. 3.3 Trajectories of a body projected from point A in direction AB with different initial velocities

The trajectory of the projectile will be like that numbered (1) in the diagram if the initial velocity is not too great. We see that this trajectory is an ellipse with the centre of the earth at one focus. If the trajectory is short so that we can neglect changes in magnitude and direction of ω then, the ellipse approximates a parabola.

The trajectories resulting from increasing the initial velocity of the projectile are shown as numbers (2) to (7). Note that the effect of the earth's atmosphere has been neglected. Trajectory (2) is also a portion of an ellipse. Trajectory (3) just misses the earth. It is a complete ellipse, so the projectile has become satellite revolving round the earth. Its velocity on returning to point A is same as the initial velocity. It can repeat this motion indefinitely if there are no retarding forces acting on it. Due to the rotation of the earth about its axis, the tower would have moved to a different point by the time the satellite returns to point A. This earth's rotation does not affect the orbit. Trajectory (4) is a special case in which the orbit is a circle. Trajectory (5) is an ellipse while (6) is a parabola and (7) is a hyperbola. We remark that trajectories (6) and (7) are not closed orbits.

All artificial satellites have trajectories like (3) and (5) though some are very nearly circles. We shall, for the sake of simplicity, consider only circular orbits. Let us now calculate the velocity required for such an orbit and the time taken for one complete revolution. To help us to achieve our objective, let us recall that the centripetal acceleration of the satellite in its circular orbit is produced by the gravitational force on the satellite. This force is equal to the product of the mass and the centripetal (radial) acceleration (i.e. $F = Ma_I$). We may compute the acceleration from the velocity of the satellite and the radius of the orbit thus:

$$W = F_g = G\frac{MM_E}{r^2} = M\left(\frac{V^2}{r}\right) \qquad 3.5$$

From which we get

$$V^2 = \frac{GM_E}{r}; \ V = \sqrt{\frac{GM_E}{r}} \qquad 3.6$$

We deduce from equation.. (3.6) that the larger the radius r, the smaller the orbital velocity.

We can also express the speed of the satellite in terms of the acceleration due to gravity g at the surface of the earth which is given by $g = GM_E/R^2$. Combing this with equation (3.6) we get

Since $GM_E = gR^2$

$$V = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{gR^2}{r}}$$

$$V = R\sqrt{\frac{g}{r}} \qquad 3.7$$

The acceleration given by $a_R = V^2/r$ can also be expressed in terms of g thus

$$a_R = \frac{r^2}{R^2} g \qquad .3.8$$

Equation (3.8) gives the acceleration of gravity at radius r. The satellite, like any projectile is a freely falling body. The acceleration is less than g at the surface of the earth in the ratio of the square of the radii.

The period T or the time required for one complete revolution is Equal to the circumference of the orbit divided by the velocity, V:

$$T = \frac{2\pi r}{V} = \frac{2\pi r}{R\sqrt{g/r}} = \frac{2\pi}{R\sqrt{g}} r^{\frac{3}{2}}. \qquad 3.9$$

We see that the longer the radius of the orbit the longer the period. R is the radius of the earth here.

Example
An earth satellite revolves in a circular orbit at a height 300km above the earth's surface (a) What is the velocity of the satellite, assuming the earth's radius to be 6400km and g to be 9.80 ms^{-2}? (b) What is the period T? (c) What is the radial acceleration of the satellite?

(a) Solution: Recall that

$$V = R\sqrt{\frac{g}{r}}$$

$$= \left(6.40x10^6 m\right)\left(\frac{9.80mS^2}{6.70x10^6 m}\right)^{\frac{1}{2}}$$

$$= 7740ms^{-1}$$

(b) The period, T is given by

$$T = \frac{2\pi r}{V} = 90.6\min.$$

(c) The radial acceleration of the satellite is

$$a_R = \frac{V^2}{r} = 8.94ms^{-2}$$

This is equal to the free fall acceleration at a height of 300km above the earth.

SELF-ASSESSMENT EXERCISE 1

An earth satellite rotates in a circular orbit of radius 6600km (about 600km above the earth's surface) with an orbital speed of 425km $\min^{-1}$

(a) Find the time of revolution
(b) Find the acceleration of gravity at the orbit

Solution
(a) Recall that period T is given by

$$T = \frac{2\pi r}{V} = \frac{2\pi 6600km}{425km\min^{-1}}$$

$$\therefore T = 97.6\min$$

(b) Recall that $a_R = \frac{V^2}{r}$

$$i.e\ a_R = \frac{(425 km m^{-1})^2}{r}$$

$$= 27.36 km m^{-2}$$

$$= \frac{27.36 x 1000m}{60 x 60}$$

$$= 7.6 mS^{-2}$$

3.3 Parking Orbit

Consider a satellite of mass m revolving round the earth in the plane of the equator in an orbit 2 concentric with the earth as represented in the Figure 3.4 below

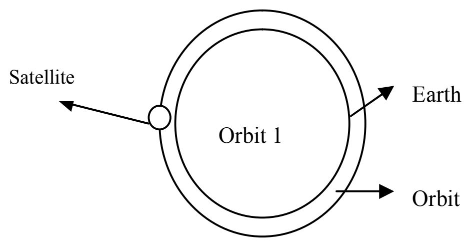


Fig 3.4

Let us suppose the direction of rotation is the same as the earth and the orbit is at a distance R from the centre of the earth. Assume V to be the velocity of the satellite in orbit, then
Centripetal force = F_g

$$\therefore \frac{mV^2}{R} = \frac{GMm}{R^2} \qquad 3.10$$

but GM = gr^2, where r is the radius of the earth.

$$\frac{mV^2}{R} = \frac{mgr^2}{R^2} \qquad 3.11$$

This reduces to

$$V^2 = \frac{gr^2}{R} \qquad 3.12$$

Now, if T is the period of the Satellite in its orbit, then $V = \frac{2\pi R}{T}$

$$\therefore \frac{4\pi^2 R^2}{T^2} = \frac{gr^2}{R} \qquad 3.13$$

Yielding

$$T^2 = \frac{4\pi^2 R^2}{gr} \qquad 3.14$$

Note that if the period of the satellite in its orbit is exactly equal to the period of the earth as it turns about its axis, that is 24 hours, then the satellite will stay over the same place on the earth while the earth rotates. When this is the situation, the orbit is called a `**parking orbit**'.

One application of a parking orbit is that relay satellites can be located there to aid transmission of television programmes continuously from one part of the world to another. It has also aided other forms of communications. Have you experienced any of them?

Now, since the period T of the satellite is 24 hours, the radius R can be found from Equation 3.14.

$$i.e. R = \sqrt[3]{\frac{T^2 gr^2}{4\pi^2}} \qquad .3.15$$

Given g = 9.8 mS^{-2}, r = 6.4 x 10^6m
Then

$$R = \sqrt[3]{\frac{(24x3600)^2 x 9.8 x (6.4x10^6)^2}{4\Pi^2}}$$
$$= 42400km$$

SELF-ASSESSMENT EXERCISE 2

At what distance (or height) is the parking orbit located above the surface of the earth?
Let h be the height above the earth's surface where the parking orbit is located

$$\therefore h = R - r =$$

where R is radius of satellite and r is radius of the earth

$$\therefore h = (42400 - 6400)km$$
$$= 36000km$$

SELF-ASSESSMENT EXERCISE 3

What is the velocity of the satellite in the parking orbit?
The velocity of the satellite here is

$$V = \frac{2\pi R}{T} = \frac{2\pi\, x\, 42400km}{24\, x\, 3600s}$$
$$= 3.1kms^{-1}$$

3.4 Weightlessness

To fire a rocket in order to launch a space craft and an astronaut into orbit round the earth we require that initial acceleration be very high. This is because large initial upwards thrust is required. This acceleration, a is of the order fifteen times the acceleration due to gravity g at the earths surface (i.e. 15g).

Suppose S is the reaction of the couch to which the astronaut is initially strapped as represented in Figure 3.5a. Then, from Newton's law of motion, we have

F = ma, S – mg = ,ma = m.15g,

Where m is the mass of the astronaut. Thus S = 16mg. This means that the reaction force S is 16 times the weight of the astronaut so he or she experiences a large force on take off.

Once they are in orbit the sceriario (changes). Here, the acceleration of the space craft and the astronaut becomes g^1 in magnitude where g^1 is the acceleration due to gravity at the particular height of the orbit outside the space craft. Now, if S^1 represents the reaction of the surface of the space craft in contact with the astronaut, the circular motion gives

$$F = mg^1 - S^1 = ma = mg^1$$
$$\therefore S^1 = 0$$

Hence the astronaut becomes "weightless' because he or she experiences no reaction at the floor when he walks about. At the surface of the earth, we are conscious of our weight because we experience the reaction at the ground where we are standing or on the chair where we are sitting. Do you feel your weight as you are reading this Unit? Think of it and you become conscious of it. When you jump up, what happens?

What do you feel when you are inside a lift (or an elevator} that takes people up and down a many storey building?). If the lift descends freely, the acceleration of objects inside it is the same as that outside. So the reaction on them is zero. The people inside it then experience a sensation of `weightlessness". In orbit as shown in Figure 3.5 b, objects inside a space craft are also in `free fall'. This is because they have the same acceleration g′ as the space craft so they feel the sensation of `weightlessness'.

Do you now understand what brings about the phenomenon of `weightlessness'? Read this section again, thoroughly well. Aim to take a trip to a building with an elevator and get a ride in it. `Weightlessness' is an experience worth feeling. `Good luck'!

Example

A satellite is to be put into orbit 500km above the earth's surface. If its vertical velocity after launching is 2000ms^{-1} at this height, calculate the magnitude and direction of the impulse required to put the satellite directly into orbit, if its mass is 50kg. Assume g = 10ms^{-2}, radius of earth, R = 6400km.

Solution

Suppose u is the velocity required for an orbit, of radius r. Then with our usual notation,

$$\frac{mu^2}{r} = \frac{GmM}{r^2} = \frac{gR^2m}{r^2}, \ as\ \frac{GM}{R^2} = g$$

$$\therefore u^2 = \frac{gR}{r}$$

Given that R = 6400km, r = 6900km, g = 10ms^{-2}

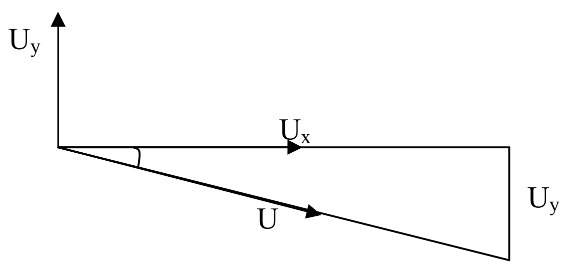
$$\therefore u^2 = \frac{10x(6400x10^3)^2}{6900x10^3}$$

$$U = 7700ms^{-1}$$

At this height, vertical momentum is

$$U_y = mV = 50x2000$$

$$= 100{,}000kg\,ms^{-1}$$



Horizontal momentum required U_x is

U_x = mu = 50 x 7700 = 385000 kg m^{-1}

Therefore impulse needed, $U = \sqrt{U_y^2 + U_x^2}$

$$= \sqrt{10000^{2`} + 385000^2}$$

$$= 4.0x10^5 kgms^{-1}$$

Direction: The angle θ made by the total impulse with the horizontal or orbit tangent is given by

$$\tan\ \theta = \frac{U_y}{U_x} = \frac{10{,}000}{385000}$$

$$\tan \theta = 0.260$$

$$\theta \quad = \ \tan^{-1} 0.260 = 14.6°$$

4.0 CONCLUSION

In this unit, you have learnt

- about motion in a vertical circle,
  - that it is not a uniform motion
  - that here, speed increases on the way down but decreases on the way up for a particle undergoing such a motion.
- that the resultant tangential and normal forces are F = ω Sinθ and F = T- ω cosθ where T is the tension in the string holding the particle
- that satellites revolve round the sun in orbit, that turn out to be conic sections (ellipse, circle, parabola or hyperbola).
- that the centripetal acceleration of the satellite in its circular orbit is produced by the gravitational force on the satellite.
- that this force is Equal to the mass times the radial acceleration $F = ma_{\perp}$

$$i.e\ \omega = F_g = \frac{GmM_E}{r^2} = m\left(\frac{V^2}{r}\right)$$

$$where\ a_{\perp} = \frac{V^2}{r}$$

- that the velocity of the orbiting satellite is $V^2 = \sqrt{\frac{GM_E}{r}}\ and\ a_R = \frac{r^2}{R^2}\cdot g$
- that the period of revolution is

$$T = \frac{2\pi r}{V} = \frac{2\pi r^{\frac{3}{2}}}{R\sqrt{g}}$$

- that a parking orbit is the orbit of a satellite whose period of revolution is approximately Equal to the period of rotation of the earth about its axis which is 24 hours
- that satellites in parking orbit are used as relay satellites for TV and other forms of communications.
- that great acceleration is needed to fire a rocket in order to launch a satellite or space craft with astronauts.

- that when there is no reaction force to an object's weight, the object feels weightless.

5.0 SUMMARY

What you have learnt in this unit are:

- that in a vertical motion, the tangential force is F_{11} = w Sinθ and the radial force is F_1 = w cosθ. Where T is the tension in the string and w = mg is the weights of the object in circular motion. The resultant tangential and normal forces are

$$F_{11} = w\,Sin\,\theta \; and \; F_{\perp} = T - w\cos\theta$$

- that the path described by a satellite round the sun is a conical section (ellipse, circle, parabola or hyperbola). That the normal radial acceleration is
-

$$a_{\perp} = \frac{V^2}{R} = \frac{F_1}{m} = \frac{T - w\cos\theta}{m}$$

$$\therefore \quad T = m\left(\frac{V^2}{R} + g\,Cos\,\theta\right)$$

At the lowest point of the path, θ = 0 Therefore Sin θ = 0 and Cos θ = 1 and the acceleration is purely radial (upwards). The magnitude of the tension is then

$$T = m\left(\frac{V^2}{R} + g\right)$$

At the highest point θ = 180^0, Sin θ = 0 and Cos θ = -1, and the acceleration is once more purely radial (downwards).

$$T = m\left(\frac{V^2}{R} - g\right)$$

For this kind of motion, there is a critical point below which the cord slacks and the path will no longer be circular. This happened at T = 0

$$m\left(\frac{V^2}{R}-g\right)=0$$

$$\therefore \qquad V_C = \sqrt{Rg}$$

- that the gravitational force on a satellite produces the centripetal (radial) acceleration that keeps the satellite in orbit
- that the velocity of the orbiting satellite is given by $V=\sqrt{\frac{GM_E}{r}}$

Where M_E is mass of the earth, G is the gravitational constant and r is the radius of the satellite.

- that its acceleration is $a=\frac{V^2}{r}$ *or* $a_R=\frac{r^2}{R^2}g$
- that the period of revolution of a satellite is

$$T=\frac{2\pi r}{V}=\frac{2\pi r^{\frac{3}{2}}}{R\sqrt{g}}$$

- that if a satellite is in its parking orbit round the earth, it will remain at the same place while the earth rotates because in its parking orbit its period of revolution is same as the period of revolution of the earth. That is why it is called the parking orbit for the satellite
- the radius of the satellite in its parking orbit is given by

$$R = \sqrt[3]{\frac{T^2gr^2}{4\pi^2}}$$

where r is the radius of the earth, T = 24h

- the velocity of the satellite in its parking orbit is $V=\frac{2\pi R}{T}$
- the height of the parking orbit above the surface of the earth is $h=R-r$
- that when the reaction force to the force of gravity is zero, the object feels weightless.

6.0 TUTOR-MARKED ASSIGNMENT

1. A satellite is to be sent to the position between the moon and Earth where there is no net gravitational force on an object due to those two bodies. Locate that point.
2. What is the period of revolution of a manmade Satellite of mass m which is orbiting the earth in a circular path of radius 8000km? (mass of earth = 5.98×10^{24}kg)

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UNIT 3 GRAVITATION AND EXTENDED BODIES

CONTENTS

1.0 INTRODUCTION

In this unit, our study of the universal gravitation will be concluded with a look at gravitation and extended bodies. You will learn how to determine gravitational potential energy and escape speed or velocity of a satellite to be projected into space, we shall discuss the variation of gravitational force with height, depth and latitude.

Finally we shall visualise the gravitational force as a fundamental force in nature. This will bring us to what opposes motion of an object in the next Unit titled Friction.

2.0 OBJECTIVES

At the end of this Unit, you should be able to:

- compute the gravitational potential
- derive expression for escape speed
- solve problems related to the variation of acceleration due to gravity with the height, depth and latitude of a place
- distinguish between the fundamental forces in nature.

3.0 MAIN CONTENT

3.1 Gravitational Potential Energy

We have seen in Unit 12 that gravitational force is a central force and depends only on the distance of the influenced object from the force center. Since it is a conservative force it can be derived from a potential energy function. We shall show here that the potential energy of a system of two point masses interacting with each other through the gravitational force is $U(r) = -(GmM)\ r$.

Let the potential energy at infinity be zero. Now, potential energy is defined as

$$U(r) - U(\infty) = \int_{\infty}^{r} \vec{F}(\vec{r})dr \qquad .3.1$$

Where we have chosen one point to be at infinity. The force points from the location of mass m to the origin (location of M), and we also chose the path to go directly along a radial direction so that the magnitude $\hat{r}.d\vec{r}^{\,1} = dr^1$.

Thus we obtain from

$$\vec{F} = \frac{GmM}{r^2} d\hat{r} \qquad 3.2$$

that

$$U(r) - U(\infty) = -\int_{\infty}^{r} -\frac{GmM}{r^2} dr^1$$

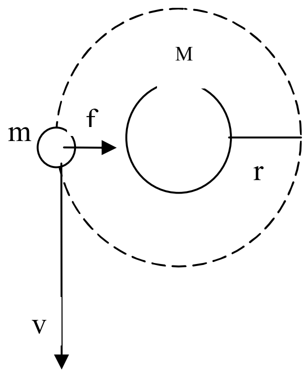
$$= -\frac{GmM}{r^1} \Big|_{\infty}^{r}$$

But U(∞) = 0

$$\therefore \qquad U(r) = -\frac{GmM}{r} \qquad 3.4$$

SELF-ASSESSMENT EXERCISE 1

A particle of mass m moves in a circular orbit of radius r under the influence of the gravitational force due to a point object of mass M>>m. Calculate the total energy of the particle as a function of r.



The sketch above will help you to understand the problem,

Solution:
The total Energy E is given by

E = KE + Pot Energy, U

$$KE + U = \frac{1}{2}mv^2 + \left(-\frac{GmM}{r}\right)$$

We see it's a function of both V and r. We want to eliminate the speed. We achieve this by applying F = ma. For a circular orbit, the acceleration is centripetal and is of the form $a = \frac{V^2}{r}$ and directed towards the centre. The force has the magnitude $\frac{GmM}{r^2}$ and is also directed to the centre. Newton's second law therefore has the form

$$\frac{GmM}{r^2} = \frac{mv^2}{r}$$
$$or\, v^2 = \frac{GM}{r}$$

We use this expression for v^2 to eliminate the speed in the expression for total energy to get

$$E = \frac{1}{2}mv^2 - \frac{GmM}{r} = \frac{1}{2}m\frac{GM}{r} - \frac{GmM}{r}$$
$$= \quad -\frac{1}{2}\frac{GmM}{r}$$

This means that the total energy is just one-half the potential energy for a circular orbit. The value is negative. This is appropriate because the orbit is closed.

3.2 Escape Speed

What happens when you throw a ball vertically upward? Does it continue going up forever? We notice that the faster a ball is thrown upwards, the higher it rises before falling backwards. It falls backwards due to the pull of gravity on it. This concept you studied in Units 11 and 12. In this unit, we shall find out the value of an initial velocity an object can have in order to be able to escape from the surface of the earth into space. That is the velocity or speed we refer to as escape speed or escape velocity.
To project an object (satellite) and land it say, on the moon, it could first be projected to land on an orbit whose period of revolution is same as time taken for the earth to rotate about its axis i.e. 24 hours (this orbit is referred to as parking orbit for the satellite) with a speed of $8kms^{-1}$ and then subsequently firing the rocket again to reach escape speed in the appropriate direction to land on the moon.

To obtain the escape speed we use the following analysis. We know that a certain amount of energy is required to escape from the earth. The escape speed will be determined considering the fact that the potential energy gained by the satellite will be equal to the kinetic energy lost if we neglect air resistance.

Let m be the mass of the escaping body and M the mass of the earth. The force F exerted on the body by the earth when the distance separating them is x from the earth's center is given by

$$F = G\frac{Mm}{x^2} \qquad 3.5$$

Work done, δW by gravity when the body moves a distance dx upwards is

$$\delta W = - Fdx = - G\frac{Mm}{x^2}dx. \qquad 3.6$$

The negative sign shows that the force acts in the opposite direction to displacement therefore,.

Total work done while body escapes =

$$\int_r^\infty - G\frac{Mm}{x^2}dx \qquad 3.7$$

where r = radius of the earth

$$\therefore \quad Total\,Work\,done = - GMm\left[-\tfrac{1}{x}\right]_r^\infty = GMm\left[\frac{1}{x}\right]_r^\infty$$

$$= \frac{GMm}{r} \qquad 3.8$$

If the body leaves the earth with speed v and just escapes from its gravitational field then, KE = Potential Energy.

$$i.e. \quad \frac{1}{2}mv^2 = \frac{GMm}{r} \qquad 3.9$$

$$\therefore \quad v = \sqrt{\frac{2GM}{r}} \qquad 3.10a$$

But $g = \frac{GM}{r^2}$

Substituting, we get

$$v = \sqrt{2gr} \qquad 3.10b$$

Eqn. (3.10) gives the expression for the velocity of escape. Substituting the values of g = 9.8mx, s^{-2} and r = 6.4 x 10^{6} m the escape speed is calculated to be

$$V \approx 11 km s^{-1}$$

We conclude that with an initial velocity of about 11km s^{-1}, a rocket will completely escape from the gravitational attraction of the earth. It can be directed to land on the moon so that it eventually will be under the influence of the moons gravity. At present `soft' landings on the moon have been achieved by firing retarding retro rockets.

Possible paths for a body projected at different speeds from the earth have already been given in Fig 3.3 of Unit 12.

Summarising we note that with a velocity of about 8km s^{-1}, a satellite can describe a circular orbit close to the surface of the earth. When the velocity is greater than 8kms^{-1} but less than 11km s^{-1}, a satellite describes an ecliptical orbit round the earth. We note that its maximum and minimum height in the orbit depends on its particular velocity.

Air molecules at standard temperature and pressure posses an average speed of about 0.5kms^{-1}. This is much less than the escape speed so the earth's gravitational field is able to maintain an atmosphere of air round the earth. On the other hand, hydrogen molecules are rare in the earth's atmosphere because their average speed is three times that of air molecules. The moon has no atmosphere.

(i) Can you suggest why it is so?
(ii) Why does the earth retain its atmosphere?

SELF-ASSESSMENT EXERCISE 2

Find the velocity or speed of escape on the surface of the moon?

Solution
The speed of escape on the moon V_{em} is

$$V_{em} = \sqrt{\frac{2GMm}{r_m}}$$

$$If\ M_m = 7.65x10^{22} kg\ and\ r_m = 1.6x10^6 m$$
$$G = 6.67x10^{-11} N m^2 kg^{-2}$$
$$V_{em} = \sqrt{\frac{2x6.67x10^{-11}x7.65x10^{22}}{1.6\ x\ 10^6}\ ms^{-1}}$$
$$= 2.53x10^3 ms^{-1}$$

3.3 Variation of g With Height and Depth

Let us assume that g is the acceleration due to gravity at a distance a from the centre of the earth where a >r. r is the radius of the earth. Then from our studies on weight in Unit 11 we had that

$$g = \frac{GM_E}{r^2}$$

$$Hence\ g' = \frac{GM_E}{a^2} \quad 3.11$$

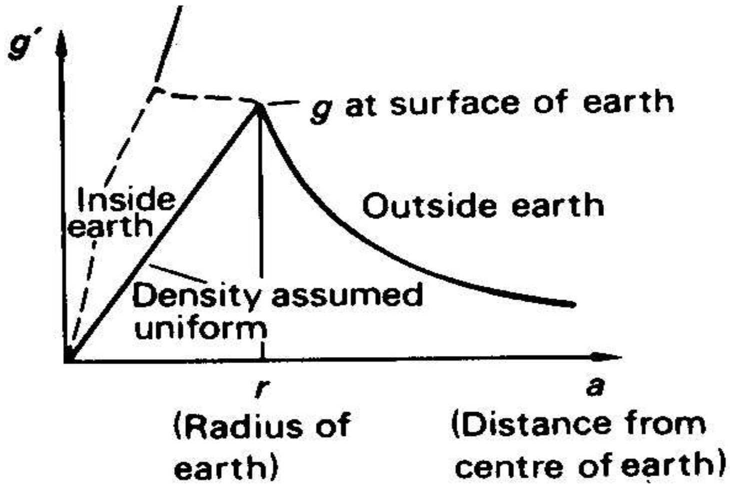
where M_E is the mass of the earth and G is the universal gravitation constant.

Dividing $$\frac{g'}{g} = \frac{r^2}{a^2}. \qquad 3.12$$

Or

$$g' = \frac{r^2}{a^2} g \qquad 3.13$$

From Eqn 3.9, we conclude that, above the earth's surface, the acceleration due to gravity g′ varies inversely as the square of the distance, a between the object and the center of the earth. Note that in the same equation r and g are constants. g′ thus decreases with height as shown in Fig 3.1 below.



At height h above the earth's surface, a = r + h

$$\therefore \quad g' = \frac{r^2}{(r+h)^2} g = \frac{1}{\left(1 + \frac{h}{r}\right)^2} g \qquad 3.14$$

$$g' = \left(1 + \frac{h}{r}\right)^{-2} g. \qquad 3.15$$

We see that if h is very small compared to r (where r is 6400km) we neglect the powers of $\frac{h}{r}$ higher than the first

Hence

$$g' = \left(1 - \frac{2h}{r}\right) g \qquad 3.16$$

$g - g^!$ = reduction in acceleration due due to gravity

$$g' = \frac{2h}{r} g \qquad .3.17$$

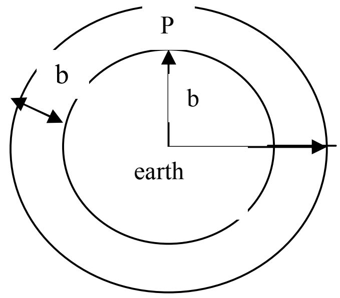


Fig.3.2 Variation of g with Depth.

At a point say p below the earth's surface it can be shown that if the shaded spherical sheet in Fig 3.2 is of uniform density, it produces no gravitational field inside itself. The gravitational acceleration g, at point p is then due to the sphere of radius b. If we assume this sphere to be of uniform density, then from our knowledge of the relation between g and G we have

$$g_1 = \frac{GM_1}{b^2} \ and \ g = \frac{GM}{r^2} \qquad 3.18$$

where M_1 is the mass of the sphere of radius b. The mass of a uniform sphere is proportional to its radius cubed, hence

$$\frac{M_1}{M} = \frac{b^3}{r^3} \qquad 3.19$$

But

$$\frac{g_1}{g} = \frac{M_1}{M} \frac{r^2}{b^2}$$

$$\therefore \quad \frac{g_1}{g} = \frac{b}{r}. \qquad 3.20$$

or

$$g_1 = \frac{b}{r} g \qquad 3.21$$

Thus, assuming the earth has uniform density, the acceleration due to gravity g is directly proportional to the distance b from the center. That is, it decreases linearly with depth, Fig. (3.1). At depth h below the earth's surface, b = r – h

$$\therefore \quad g_1 = \left(\frac{r-h}{r} \right) g = \left(1 - \frac{h}{r} \right) g \qquad 3.22$$

But because the density of the earth is not constant, g, actually increases for all depths now obtainable as shown by part of the dotted curve in Fig. 3.1

SELF-ASSESSMENT EXERCISE 3

If r is the radius of the earth and g is the acceleration at its surface, what is the expression for the acceleration of g^1 of a satellite at an orbit a distance R from the Centre of the earth. R>>r

Solution

$$\frac{g^1}{g} = \frac{r^2}{R^2}$$

$$\therefore \quad g^1 = \frac{r^2}{R^2} g$$

SELF-ASSESSMENT EXERCISE 4

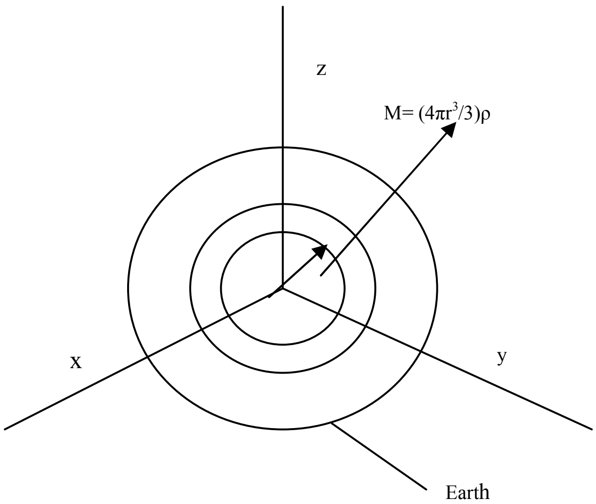
Suppose that a tunnel is drilled through our planet along a diameter. Assume the earth's mass density is uniform and is given by ρ. Describe the force on a point mass m dropped into the hole as a function of the distance of the mass from the centre.

Solution

The gravitational force on the point mass m is due only to the mass of the material contained within a radius r, where r is the distance from the point mass in to the center of earth. The force is attractive, towards Earth's centre and it is given by

$$F = \frac{GmM^1}{r}$$

where the mass M^1 that attracts the point mass as the total mass inside radius r (see diagram below). M^1 is given by (volume) x (density ρ)



$$\therefore \qquad M^1 = \left(\frac{4\pi r^3}{3}\right)\rho$$

Thus substituting for M^1

$$F = \left(\frac{4\pi G m \rho}{3}\right) r$$

We see that F is proportional to r. This result shows that, inside Earth, the point mass acts as if it were moving under the influence of a spring with spring constant $K = 4\pi G m\rho / 3$. This motion is oscillatory and the point mass moves from one end of the tunnel to the other and back.

3.4 Variation of g with Latitude

The acceleration due to gravity has been observed to vary from location to location. This is as a result of the following:

(i) the equatorial radius of the earth exceeding its polar radius by about 21km hence making g greater at the poles than at the equator because, a body is far from the center of the earth here.

(ii) the effect of the earth's rotation.

Let us look at how the earth's rotation affects acceleration due to gravity. Recall that a body of mass m at any point on the surface of the earth (except the poles) must have centripetal force acting on it. Part of this centripetal force is due to the force of gravity on the body. If the earth were stationary, the pull of gravity on m would be mg where g is the acceleration due to gravity. But due to the earth's rotation the observed gravitational pull is less than this and is equal to mg_o where g_o is the observed acceleration due to gravity. Hence,

$$Centripetal\ force\ on\ body = mg - mg_0. \qquad 3.23$$

At the equator, the body moves in a circle of radius r where r is the radius of the earth and it has the same angular velocity as the earth. Here, the centripetal force is mw^2r, so we have

$$mg - mg_O = m\omega^2 r \qquad 3.24$$

$$\therefore \qquad g - g_0 = \omega^2 r \qquad 3.25$$

When we substitute the values $r = 6.4 \times 10^6$m,
w =1revolution in 24hours = $2\pi/(24 \times 3600)$ rad s^{-1} we get

$$g - g_o = 6.4x10^6 m\,x\left(\frac{2\pi}{24x3600} rad\,s^{-1}\right)^2$$

$$= 3.4x10^{-2} m\,s^{-2}$$

Assuming the earth is perfectly spherical, the result above is also the difference between the polar and equatorial values of g. Note that at the poles ω = 0 and so g = g_o. The observed difference is 5.2 x $10^{-2}ms^{-2}$, of which 1.8 x $10^{-2}ms^{-2}$ arises from the fact that the earth is not a perfect sphere.

At altitude θ if we assume a spherical earth, the body describes a circle of radius r cos θ, Fig.3.3.

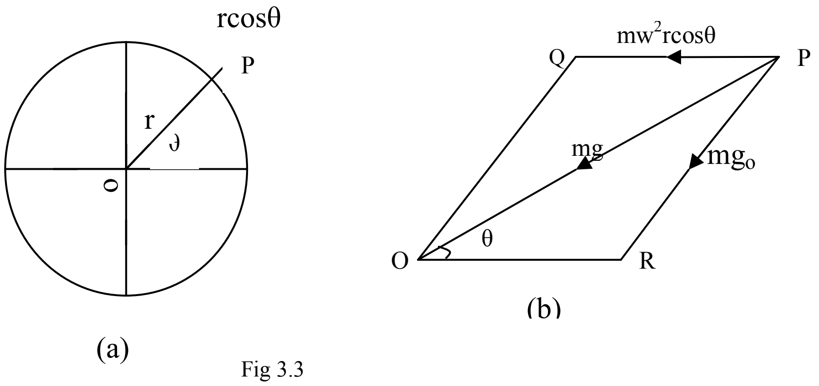


Fig 3.3

The magnitude of the required centripetal force at this latitude is $m\omega^2 r\cos\theta$ which is smaller than that at the equator since ω has the same value. Its direction is along PQ in the diagram but mg acts along PO towards the center of the earth. The observed gravitational pull mg_o is therefore less than mg by a factor $m\omega^2 r\cos\theta$ along PQ and is in a different direction from mg. We remark that the direction and value of g_0 must be such that its resultant with $m\omega^2 r\cos\theta$ along PQ will give mg in a parallelogram law diagram as shown in Figure (3.3b). The direction of g_o as shown by a falling body or plumb line is not exactly towards the center of the earth except at the equator and poles.

3.5 Fundamental Forces in Nature

So far we have dealt with the phenomenon of gravitation and some of its applications. Newton's law of gravitation was the forehead of all the discussion. But now we raise the question – why at all there is a force of attraction between any two material bodies? Does Newton's law provide an answer? It can not because the gravitational force between two bodies exists naturally. Such a force is called a `Fundamental force of Nature'. There are three different kinds of fundamental forces in nature. We shall discuss them briefly now.

Fundamental or basic forces are those for which we cannot find an underlying force from which they are derived. It then stands to reason that those forces resulting from the operation of some underlying fundamental force are known as derived forces.

This concept is similar to the concept of fundamental and derived units of measurement which we discussed in Unit 2 of this course.

There are three kinds of fundamental forces. These are (i) **gravitationa**l (ii) **electroweak** and (iii) **strong**. You have read in detail in Units 11, 12 and this present one about gravitational force, which acts on all matter as you have seen so far. You recall that it varies inversely as the square of the distance but its range is infinite. This force is responsible for holding together the planets and stars and in fact, in overall organization of solar system and galaxies.

The electro weak force includes **electro-magnetism** and the so called weak nuclear force. Electromagnetic forces include the force between two charged particles at relative rest (electrostatics) or in relative motion (electro-dynamics). The electrostatic between two charges obeys the inverse square law like gravitational force between two masses. [You will learn more about that in your electro magnetism course in the second semester]. The dissimilarity here is that charges are of two kinds – positive and negative. If the charges are of opposite kind the force between them is attractive but if they are of the same kind, the force is repulsive. It can be shown that the gravitational force between an electron and a portion in a hydrogen atom is 10^{39} times weaker than the electrostatic force between them. Thus we get a comparative estimate of the strengths of gravitational and electrostatic force.

In the case of moving charges, we know that charges in motion give rise to electric current. You also learnt in the secondary school that a current carrying conductor is equivalent to a magnet. This is the meeting point of electricity and magnetism and hence the word `electromagnetic' got associated with this field of force. The force that one comes across in daily life, like friction, tension etc. can be explained from the standpoint of the electromagnetic force field. An estimate of the relative strengths of the repulsive electrostatic and the attractive gravitational force between two protons in a nucleus shows that the former is 10^{36} times larger than the latter. So, how is it that the protons in an atomic nucleus, stay together instead of flying away? The answer lies in the third kind of fundamental force known as the **strong (nuclear**) force that exists between the protons inside the nucleus, which is strongly attractive, much stronger than the electrostatic force between them. Strong nuclear force also exists between neutrons in the nucleus as well as between neutrons and protons. The nuclear force decrease rapidly with distance so it is a short range force. You will study in detail about the nuclear forces in a nuclear physics course.

The nuclear force as we have seen accounts for the binding of atomic nuclei. But this cannot account for processes like radioactivity beta decay about which, once again, you will read in the Nuclear Physics course. This can be explained from the point of view of the so-called weak nuclear force. It is much weaker than the electromagnetic force at nuclear distance but still greater by a factor of 10^{34} than the gravitational force. Just a few years ago, this weak force was listed separately from the electromagnetic force. However, a theory was proposed which led to the unification of the weak forces and the electromagnetic forces and hence the name `electroweak' forces.

4.0 CONCLUSION

In this unit, you have learnt

- that the potential energy of a system of two point masses interacting with each other through the gravitational force is $U_{(r)} = -GmM/r$.
- That escape sped I e speed an object can have in order to escape from the surface of the earth into space
- tat the acceleration due to gravity, a decreases the farther away an object is far away from the center of the earth outside the earth's surface.
- tat there is no gravitational field inside the shell beneath the earth's surface if the shell is of uniform density.
- That g decreases linearly with depth below the earth's surface,
- That the acceleration due to gravity g increases with latitude.
- That fundamental forces or basic forces are forces for which we cannot find underlying forces from which they are derived.
- That gravitational force is one of the fundamental forces in nature. Others are electro weak and strong nuclear forces.

5.0 SUMMARY

What you have learnt in this Unit are.

- That $U(r) - U(\infty) = \int_{\infty}^{r} F(r)dr$
  where U(r) – U(∞) is the potential energy of a system of two point masses interacting with each other. If U(∞) = O then $U(r) = -\frac{GmM}{r}$
- That the escape speed or escape velocity is the minimum velocity needed by an object to be projected into space from the surface of the earth.
- That the potential energy gained by the satellite is equal to the kinetic energy lost (neglecting air resistance). Therefore from

$$\frac{1}{2}mv^2 = \frac{GMm}{r}$$

We get

$$v = \sqrt{\frac{2GM}{r}}$$

but

$$g = \frac{Gm}{r^2}$$

$$v = \sqrt{2gr}$$

- That the escape speed is calculated to be 11kms^{-1}

- That the gravitational acceleration g at a distance a from the center of the earth of radius r where a > r is given by

$$g^1 = \frac{r^2}{a^2} g$$

- From the above we conclude that, above the earths surface, g varies inversely as the square of the distance a between the object and the center of the earth
- That when the point mass is placed inside the sphere, it experiences force of attraction only due to a concentric spherical mass on whose surface it lies. The matter contained in the shells external to this point mass does not contribute at all to the force of attraction.
- That the mass of a uniform sphere is proportional to its radius cubed hence from eqn. 3.18 and 3.19

$$g_1 = \frac{b}{r} g$$

  Where b is radius of the sphere and r the radius of the earth
- That g varies with latitude – greater at the poles than at the equator because the earth bulges at the equator
- Gravitational force is a fundamental force in nature. There are two other kinds of fundamental forces – the **electro weak** and the **strong nuclear** forces.

6.0 TUTOR-MARKED ASSIGNMENT

1. A satellite moves in a circular orbit around earth, taking 90 minutes to complete 1 revolution. The distance from the moon to earth is $d_{ME} = 3.84 \times 10^8$m; the moon's orbit is circular, the speed of the moon's rotation about Earth is $T_M = 27.32d$□. Earth's radius is $R_E = 6.37 \times 10^6$ m and Earth's gravitational force acts as if all of Earth's mass were concentrated at its center. With this information, calculate the height of the satellite above Earth.
2. By what percentage of its value at sea-level does g increase or decrease when one gets to (i) an altitude of 2500km and (ii) Kolar Gold Field at a depth of 3000m.

7.0 REFERENCES/FURTHER READING

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UNIT 4 FRICTION

CONTENTS

1.0 INTRODUCTION

In Units 1, 5, 6 and 7 we saw that every object stays in relative rest or motion unless it is impinged upon by an applied force. In this unit we shall discuss friction. Friction is a type of force we experience everyday without giving it a thought. Have you ever considered why you walk without slipping unless you unknowingly step on a banana peel or on smooth slippery floor or on a thin film of water on a smooth film? We do not slip and fall down when we walk because of the frictional forces acting between our feet and the ground. Friction allows cars to move on the roads without skidding and it even holds nails and screws in place etc. The study of friction, wear and lubrication is called tribology and it is very important to industry. In studying frictional forces, you will draw from your knowledge of conditions for equilibrium of forces treated in Units 3 and 7. We shall limit our discussion in this course to solid friction. Friction also exists in liquids and gases but you will learn about that in your course on Thermal Physics and Properties of Matter next semester.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- describe an experiment to determine the coefficient of static or dynamic friction
- state where frictional forces act
- define the coefficients of static and kinetic friction
- state the laws of friction
- apply the laws of friction in solving problems
- differentiate between static and dynamic friction
- explain the nature of friction.

3.0 MAIN CONTENT

3.1 Laws of Friction

Frictional forces act along the surface between two bodies when one tries to move or succeeds in moving over the other. So Friction is a contact force. It is that force that tries to or opposes motion. Rubbing surfaces in machinery need to be lubricated to reduce friction so that their life span could be extended. Yet we need friction because it enables us to walk without slipping. It enables us to keep things in standing positions. But note that wherever there is friction, you expect some surface wear of the materials in contact.

Coefficient of Friction

There are different apparati one can use to study the friction between two solid surfaces. We shall limit our discussion here to the use of the apparatus described in Figure 3.1 below.

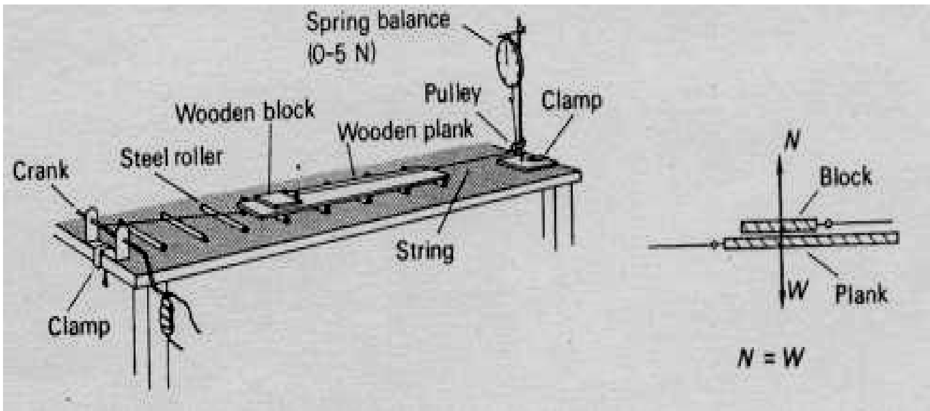


Fig. 3.1

The set up is described in the diagram. Initially the plank is at rest, but when some force is applied to the crank, the plank will tend to move or moves depending on the amount of force applied. All this while, the block remains at rest. The spring balance set as shown, measures the frictional force between the block and the plank.

As the crank is wound slowly, the spring balance reading increases until it reaches a maximum value. This maximum value is the value of the frictional force when the plank is just about to move, and it is called the limiting frictional force. It is observed that when the plank starts moving, the spring balance reading decreases slightly. This shows that the kinetic or dynamic frictional force is smaller than the limiting frictional force. To check if friction depends on area of contact between the two surfaces, the block can be positioned at the edge.
The normal force N exerted by the plank on the block is equal to the weight w of the block. So we can then vary the weight of the block by putting standard weights on it, and recording the corresponding frictional forces as indicated by the spring balance. Thus the effect of frictional force of varying N can be found.

The results of such an experiment are summarized in what we call the laws of friction which state that:

1. The frictional force between two surface opposes their relative motion.
2. The frictional force does not depend on the area of contact of the surfaces
3(a) When the forces are at rest the limiting frictional force F is directly proportional to the normal force N
(b) When motion occurs the kinetic (dynamic) frictional force F is directly proportional to the normal force N i.e. F_R α N (or F_R/N = constant) and is reasonably independent of the relative velocity of the surfaces.

Hence the coefficient of limiting static friction μ_s is

$$\mu_s = \frac{F}{N} = conata\eta t \qquad 3.1$$

and that coefficient of kinetic (dynamic) friction is

$$\mu_K = \frac{F_K}{N} = Consta\ \eta t \qquad 3.2$$

Note that for two given surfaces, μ_k is less than μ_s, though occasionally they may be assumed to be equal. For sliding over wood, μ is about 0.2 to 0.5.

Generally, when a surface exerts a frictional force the resultant force in a body on the surface has two components. It has a normal force N which is perpendicular to the surface and a frictional force F along the surface with direction opposite to the direction and motion. This is illustrated in Figure (3.2) below

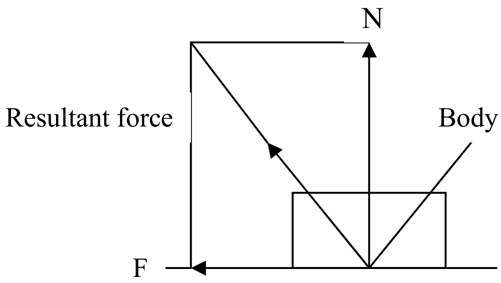


Fig 3.2

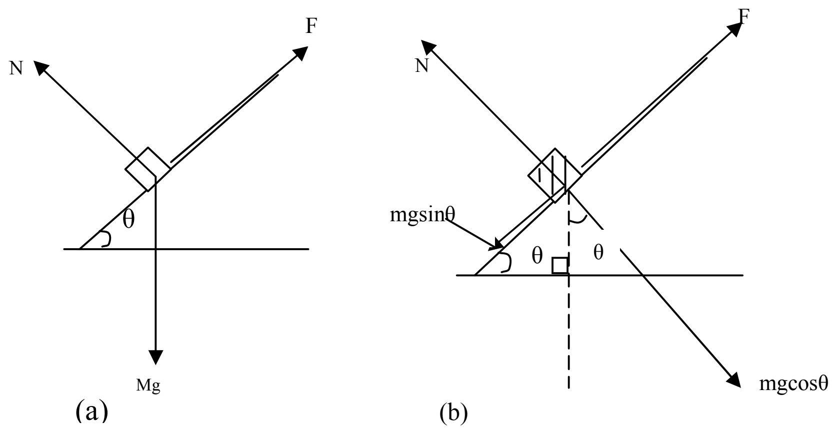
Note that if the surface is smooth, then μ = 0 and so F = 0. We conclude then that a smooth surface will only exert a force at right angles to itself,that is, only the normal force survives here.

Can you think of any other way by which we can find the coefficient of limiting friction?

Yes. Another possible way is by placing a block of mass m on the surface of say a horizontal plank and tilting the plank gradually. The angle of tilt is slowly increased until the block is just about to slip as shown in Figure 3.3a. The forces acting on the block are

(i) its weight mg
(ii) the normal force N of the surface and
(iii) the limiting frictional force F – μN.

These three forces are in equilibrium.



Let mg be the weight of mass m. When mg is resolved into its components, we might get mgcosθ along the surface and mgcosθ perpendicular to the surface as shown in Fig 3.3b. Then we have that

$$F = \mu N = mg \sin\theta \qquad 3.3$$
$$N = mg\cos\theta \qquad 3.4$$

Dividing and Eqn. (3.3) by (3.4) gives

$$\mu = \tan\theta \qquad 3.5$$

Thus if we measure angle θ, then μ can be computed.

Example
A uniform ladder 4.0m long, of mass 25kg, rests with its upper end against a smooth vertical wall and with its lower end on rough ground. What must be the least coefficient of friction between the ground and the ladder for it to be inclined at 60° with the horizontal without slipping?

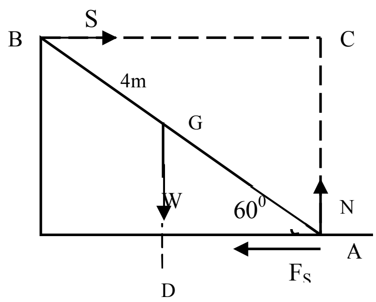


Fig 3.4

Solution

Mg = wt. of ladder = 250N

The forces acting are as shown in the diagram. The wall is smooth so the force S is normal to it. We assume the weight of the ladder to be acting from the mid point G because it is of uniform cross section. When the ladder is just about to slip, the force exerted on it by the ground could be resolved into its vertical (normal and horizontal components i.e. its normal force N and its limiting frictional forces $F_s = \mu_s N$ correspondingly. Now μ_s is the expected coefficient of limiting friction, we are to find.

So, for equilibrium

W = 250 Newtons = N for vertical forces and
$F_S = \mu_S N = S$ for horizontal components
If we now take moments about point A then,

$$S\,x\,AC = W\,x\,AD$$
$$S\,x\,4.0\cos 30^o = 250\,x\,2\,Sin\,30^o$$
$$= 250\ Newtons$$
$$\therefore\ S = \frac{125}{\sqrt{3}}\ Newtons$$

Hence,

$$\mu_s = \frac{S}{N} = \frac{125}{250\sqrt{3}}$$
$$\mu_S = 0.29$$

SELF ASSESSMENT EXERCISE 3

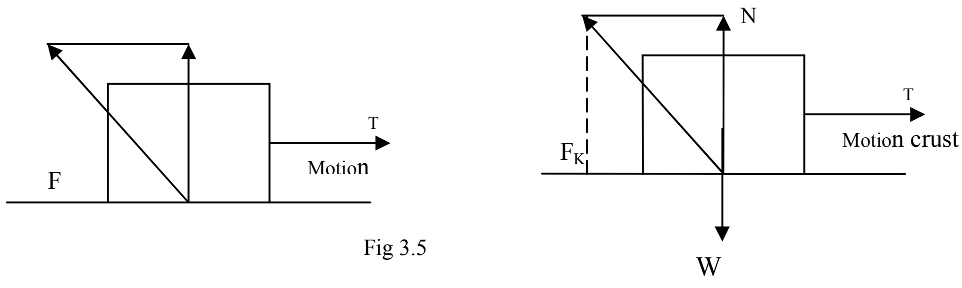


Fig 3.5

Suppose that the block in the figure above weighs 20 Newtons and that the tension T can be increased to 8 Newtons before the block starts to slide, and that a force of 4

Newtons can keep it moving at constant speed once it has been set in motion. Find the coefficients of static and kinetic (dynamic) friction.

Solution
Resolving the forces horizontally and vertically we have

$$\sum F_y = sum\ of\ vertical\ forces$$
$$\sum F_x = sum\ of\ Horizontal\ foreces$$
$$\sum F_y = N - W = N - 20\ Newtons = 0$$
$$\sum F_x = T - \mu_S N = 8\ Newtons - \mu_s N = 0 \quad \}First\ Law$$

where μ_S is the coefficient of limiting friction, f_s. Note: and $f_s = \mu_s N$

$$\therefore \mu_S = \frac{f_s}{N} = \frac{8}{20} = 0.40$$

For the same condition except that a force of 4 newtons keeps the block in motion we have

$$\sum F_y = N - W = N - 20\ Newtons = 0 \quad \}\ First\ Law$$
$$\sum F_x = T - f_K = 4 Newtons - \mu_K N = 0\}\ First\ Law$$

Since μ_k is the coefficient of kinetic friction, motion exists, $\mu_K N = f_K$

Hence

$$\mu_k = \frac{4\ Newtons}{20\ Newtons} = 0.20$$

Example
A professor with a light eraser in her hand leans against a blackboard. Her hand makes an angle of 30° with the horizontal and the Force F exerted by her hand on the eraser has magnitude F = 50N. The coefficient of prof static friction between the eraser and the blackboard is $\mu_s = 0.15$. Does the eraser slip?

Solution:

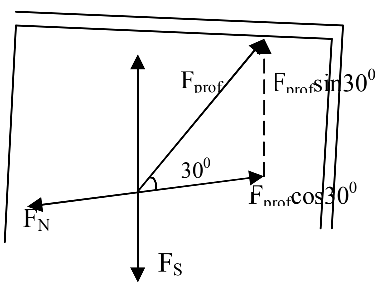


Fig 3.6

We have tried to represent the forces on the eraser on the diagram above. It also indicates a useful co-ordinate system. Under equilibrium, Newtons first law applies that is

$$F_N + F_s + F_{prof} = 0$$

Component wise then, we have

$$-F_N i - F_s j + F_{prof}\ Cos\, 0 i + F_{prof}\ Sin\ \theta\ j = 0$$

The unit vectors are included to show they are forces in component form.

Separating the x-component from the y-components we have,
For x-component, $-F_N + F_{prof}\ Cos\,\theta = 0$
For y-component: $-f_s + F_{prof}\ Sin\,\theta = 0$

The x-component equation determines F_N from the requirement that it balances the perpendicular component of the force the professor exerts.

$$F_N = F_{prof}\ Cos\,\theta$$

The maximum value of the static friction is thus

$$f_{s\,\max} = F_{prof}\ Sin\,\theta \ = \ \mu_S F_N$$

Note that the eraser can only begin to slip if this maximum limiting frictional force is exceeded. Thus, when we substitute this maximum value of static friction into the y-component equation, we find a condition for the critical angle θ_C for which the eraser begins to slip.

$$-\mu_S F_{prof}\ Cos\,\theta_C + F_{prof}\ Sin\,\theta_C = 0$$

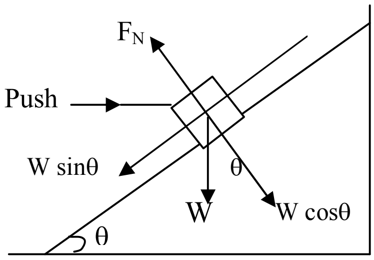
$$\therefore \frac{Sin\,\theta_C}{Cos\,\theta_C} = ten\ \theta_C = \mu_S$$

Note the striking feature that the critical angle is independent of the force the professor exerts. When numerical values are substituted, the equation yields tan θ_C = 0.15 or θ_c is less than the 30° angle made by the professor's arm, so the razor slips down

SELF-ASSESSMENT EXERCISE 3

The figure below shows a person applying a horizontal force in trying to push a 25kg block up a frictionless plane inclined at an angle of 15°

(a) Calculate the force needed just to keep the block in equilibrium
(b) Suppose that she applies three times that force. What will be the acceleration of the block?



For Equilibrium Note. Plane is frictionless $\therefore \mu_S = 0$
x-component: $F_{push} - w\,Sin = 0$ } *First Law*
y-Component: $F_N - wCos\,\theta = 0$

(a) Therefore force needed o keep the block in equilibrium is

$$F_{push} = W\,Sin\theta$$
$$= 250\,x\,0.259 = 64.7N$$

(b) If she applies three times the force then F_{push} becomes

$$F_{push} = 3x\,64.7N$$
$$= 194.1N$$

But the weight of the block acting in the negative x axis is w Sinθ
F_{net} for push is 194.1 – 64.7 = 129.4N

But F_{net} I = m x a
Where F_{net} is the net or effective force pushing up the black

$$\therefore a = \frac{F_{net}}{m} = \frac{129.4N}{25kg}$$
$$a = 5.17ms^{-2}$$

3.2 Nature of Friction

The coefficients of static and kinetic (dynamic or sliding) friction depend on the nature of surfaces in contact between two bodies. Coefficient of friction is large for rough surfaces than for smooth ones. The coefficient of kinetic friction varies with the relative velocity but for the sake of simplicity we assume it to be independent of velocity.

Close examination of the flattest and most polished surfaces reveals that there still exist hollows and humps which are more than one hundred atoms stacked one on top of the other. This means that when two solid surfaces are placed one on the other, or are made to touch, their actual area of contact is very small. An example is shown in Figure 3.7 below

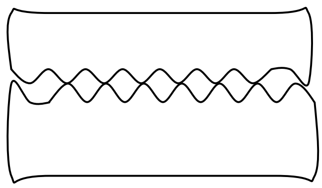


Fig 3.7

Electrical resistance measurements of two metals in contact reveal that the true area of contact between the surfaces is extremely very small. It is estimated that in the case of steel, the actual area that is touching many be just about one ten thousandth ($1/10,000^{th}$) of the apparent area actually placed together. Two metal surfaces thus sit on each other's projections when they are placed one on top of the other. This goes for non-metallic objects too. Look around your room where you are now and examine surfaces in contact with each other. But note that you can not see all we are saying with the naked eyes. Yet, the concept of frictional force is easy to experience when you try to push or pull a heavy table.

Pressures at the points of contact between two metals are extremely high and cause the bumps to flatten until the increased area of content enables the upper solid to be supported. It is presumed that at the point of contact small, cold welded joints' are formed by the strong adhesive forces between molecules that are very close together. These have to be broken before one surface can slide over the other. This phenomenon accounts for the first law of frictional force.

Experiments like the ones made by Leonardo da Vinci some 200 years before Newton's work on dynamics (Fishbane et al) with a set of blocks of varying sizes sliding on table tops show that changing the apparent area of contact of the bodies has little effect on the actual area for the same normal force. This explains the second law of friction. It is also found that the actual area is proportional to the normal force and since this theory suggests that frictional force depends on the actual area, we might expect the frictional force to be proportional to the normal force – as the third law states.

4.0 CONCLUSION

In this unit, you have learnt

a) that friction is a contact force which acts along the surface between two bodies in contact when one tries to move or succeeds in moving.
b) that friction opposes motion

c) that the coefficient of friction is the maximum limiting force just before a body starts sliding over another surface.
d) the three laws of friction.
e) how to apply the laws of friction to solve relevant problems pertaining to friction.
f) about the nature of friction.

5.0 SUMMARY

What you have learnt in this unit concerns frictional force. You have learnt

- what frictional force is
- where it acts
- how it is determined
- the laws of frictional force
- to differentiate between static and dynamic friction
- how to apply the laws of friction in solving problems
- the nature of friction that

  $\mu_S = \frac{F_S}{N}; \mu_k = \frac{F_K}{N}$

  where the symbols have their usual meaning
- that friction is important to life because it allows us to walk, drive cars etc. and place things in steady positions, etc.
- that friction between two surfaces in contact leads to wearing off of such surfaces hence such matter needs lubrication.
- that since friction is important in the industry it is essential that we study about it.

6.0 TUTOR-MARKED ASSIGNMENT

1. An automobile with four wheel drive and a powerful engine has a mass of 1000kg. Its weight is evenly distributed on its four wheels whose coefficient of static friction with dry road is $\mu_S = 0.8$. If the car starts from rest on a horizontal surface, what is the greatest forward acceleration that it can attain without spinning its wheels?

2. What is the friction force if the block weighing w = 20N in the figure above is at rest on the surface and a horizontal force of 5N is exerted on it.

3. What force T at an angle of 30° above the horizontal is required to drag a block weighing 20N to the right at constant speed, if the coefficient of kinetic friction between block and surface is 0.20?

4. Two blocks of masses M and m are connected by a light rope which passes over a frictionless pulley. Mass M sits on an inclined plane with an angle of inclination of 30°. The coefficient of static friction between mass M and the

inclined plane is 0.20, while m = 30kg. Determine the smallest and largest possible values of M for which the system remains in equilibrium.

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UNIT 5 WORK AND ENERGY

CONTENTS

1.0 INTRODUCTION

Work and energy is central to life. We do it and experience it every day. We can thus say that the notion of energy is one of the most basic concepts in physics and indeed in all sciences. Energy takes many forms and in this unit we shall focus on energy contained in moving objects which we call kinetic energy and also in energy a body possesses by virtue of its position called potential energy. The work done on an object involves the force acting on it as it moves. We can relate the change in kinetic energy of an object to the work done on it as it moves. This relation is called work-energy theorem. In this unit you will learn how to calculate the work done by an object which will serve as a powerful tool for the understanding of motion.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define work in the scientific sense
- distinguish between positive and negative work
- determine the work done by a varying force
- explain the terms energy, potential and kinetic energy
- state the principle of conservation of mechanical energy
- apply the work-energy equation in solving energy related problems.
- state the fundamental law of conservation of energy.

3.0 MAIN CONTENT

3.1 Work

3.1.1 Work Done by a Constant Force

The term work is erroneously used in everyday life as applied in any form of activity where we exercise muscular or mental effort. But in physics the term work is used in a specific sense. **So, in the scientific sense work is done when a force moves its point of application along the direction of its line of action.**

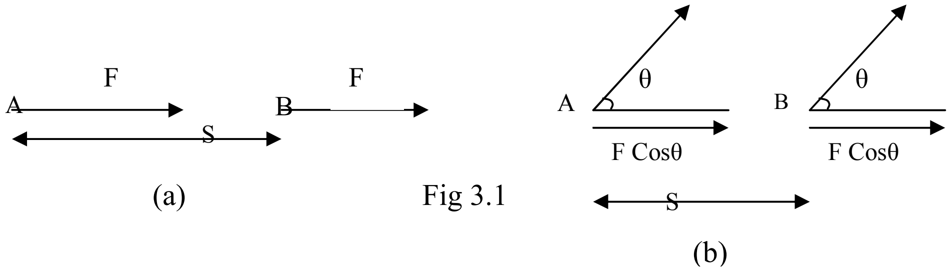

Fig 3.1

For example, in Figure 3.1 (a) if constant force F moves from point A to point B a distance of s in a constant direction, then the work done by this force is defined as

Work = Force x distance moved by force

$$W = Fs \qquad 3.1$$

If the force acts at an angle θ to the direction of motion of the point of application of the force as shown in Figure 3.1b then the work is defined as the product of the component of the force in the direction of motion and the displacement in that direction. That is

$$W = (F \cos \theta)s \qquad 3.2$$

We note that when $\theta = 0$, $\cos \theta = 1$ and so, $W = Fs$. This agrees with equation (3.1). When $\theta = 90^{o}$, $\cos \theta = 0$ and we see that F has no component in the direction of motion and so, no work is done. This means that if we relate this to the force of gravity, it is clear that for horizontal motion, no work is done by the force of gravity. You remember we saw this situation during our discussion on projectile motion in Unit 8.

Now, get a big textbook and place it on the table where you are reading. Apply a push force horizontal to it. What do you observe? You have now seen that work is done only when a force is exerted on a body while the body at the same time moves in such a way that the force has a component along the line of motion of its point of application. I would want you to pay special attention to this: If the component of the force is in the same direction as the displacement, the work done W is positive. If it is opposite in direction to the displacement, then the work is negative. If the force is perpendicular to the displacement, it has no component in the direction of the displacement and the work is zero. Can you give some examples where work done in some activities is positive and negative? Think of the work done when a body is lifted up. It is positive work. The work done by a stretching spring is also positive. On the other hand, the work done by the force of gravity on a body being lifted up is negative. Why is this so? This is because the force of gravity is opposite to the upward displacement. When a body slides on a fixed surface, the work of the frictional force exerted on the body is negative since frictional force is always opposite to the displacement of the body. Because the fixed surface does not move, the frictional force does no work on it.

3.1.2 Unit of Work

The unit of work is the unit of force multiplied by the unit of distance in any particular system of measurement. Recall the systems of measurement you studied in unit 2 of this course.

In the SI system, the unit of force is the Newton and the unit of distance is the meter; therefore in this system the unit of work is one Newton meter (I Nm). This is called the joule (IJ).

In the cgs system, the unit of work is one dyne centimeter (1 dyn cm) and it is called one erg. Note that since 1m = 100cm and 1N = 10^5dyn, then

1Nm = 10^7dyn cm or 1J = 10^7erg.

In the engineering system, the unit of work is one foot pound (1ft lb):
Note: IJ = 0.7376 ft 1b
And 1 ftlb = 1.356 J

We remark that when several forces act on a body, we resolve them into their components and find the algebraic sum of the work done by the effective component forces. This follows because work is a scalar quantity.

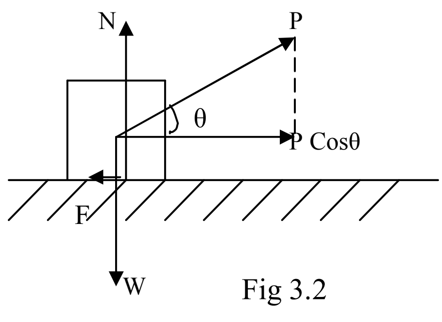

Fig 3.2

SELF ASSESSMENT EXERCISE 3.1

The diagram above shows a box being dragged along a horizontal surface by a constant force P making a constant angle θ with the direction of motion. The other forces on the box are its weight w, the normal upward force N exerted by the surface and the friction force f. What is the work of each force when the box moves a distance s along the surface to the right. Given w = 100N, P =50N, f = 15N, $\theta = 37^o$ and s = 20m

Solution
The component of p, in the direction of motion is
$w_P = (P\cos\theta)s$
$= (50N)(0.8)(20m) = 800Nm$

The forces w and N are both perpendicular to displacement hence,

$w_N = 0$ and $w_N = 0$

The frictional force f is opposite to the displacement so its work is

$w_f = -f_s = (-15N)(20m)$
$= -300$ Nm

Therefore, the total work done W is

$$W = W_p + W_f = (800 - 300) \text{ Nm}$$
$$= 500 \text{ Nm}$$
$$= 500 \text{ J}$$

SELF ASSESSMENT EXERCISE 2

A box of books of mass 100kg is pushed with constant speed in a straight line across a rough floor with a coefficient of kinetic friction $\mu_k = 0.2$. Find the work done by the force that pushes the box if the box is moved a distance d = 3m
(Take g = 9.8 m s^{-2}).
Solution
We approach this problem by drawing a force diagram Fig. (3.2b) below

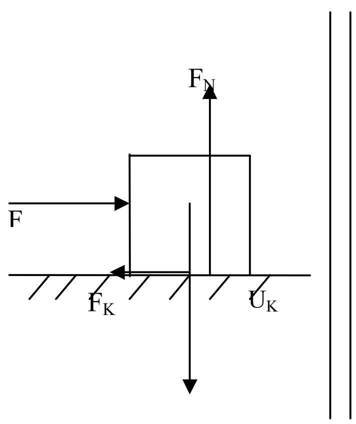

Fig .3.2b Force diagram of a crate being pushed across the floor.

With no vertical displacement, no work is done by gravity or by the normal force. The forces in the vertical direction must therefore cancel each other so $F_N = mg$.

Now, because the box moves with a constant velocity, the net horizontal force must vanish. Thus the pushing force F must be equal in magnitude but opposite in direction to the force of friction f whose magnitude is given by $f = \mu_k F_N = \mu_k mg$. Hence, the magnitude of F is also $\mu_k mg$. The direction of F is the same direction as the displacement d.

Thus, the work done by the pushing force is positive. This work is then given by

$$W = Fd = \mu_k mgd$$
$$= (0.2)(100kg)(9.8m\ s^{-2})(3m)$$
$$= 6 \times 10^2\ J$$

You see how easy the solution of this problem is. Once you try to understand and analyse the problem before you start computing, the work is half done. So never be mesmerized with verbose questions.

3.1.3 Work Done by A Varying Force

We started this Unit by defining the work done by a constant force. We shall now consider the work done by a varying force because this is also encountered in the practical world. Here work could be done by a force, which varies in magnitude or direction during the displacement of the body. For example on stretching a spring slowly, the force required to do this increases steadily as the spring elongates. Also the gravitational force pulling an upward vertically projected particle downward decreases inversely as the square of the distance from the centre of the earth.

We can find the work done by a varying force graphically as follows: With reference to Figure 3.3 suppose the force is F when the displacement is x, then for a further small displacement dx is Fdx (i.e. if we take dx to be so small that Fis considered constant).

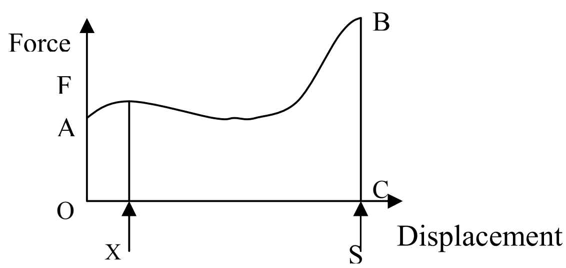

Fig 3.3

If the whole area under the curve AB is divided into small narrow strips, the total work done during a displacement S will be given by the area under the curve AB i.e. Area OABC.

3.2 Energy

A body is said to expend energy when it does work on another body. For example if body A does work by exerting a force on body B, then body A is said to lose energy. This energy lost by body A is equal in amount to the work it performed on body B. thus we can define energy as that which enables a body to do work. So when we say that you have some energy in you, we mean that you are capable of doing some work. Energy is measured in joules just like work. Work done can be taken to be a measure of the quantity of energy transferred between two bodies. That is, if for example, body P does 10 joules of work on body Q then the energy transfer from P to Q is 10 joules.

Power
When we talk about the power of equipment we mean the rate at which it does work. This is the same as the rate at which the machine or appliance converts energy from one form to another. The unit of power is the watt (W). When one joule of work is done in one second it is known as the watt or that energy expended is IW

$$\therefore\ 1W = 1\ J\ s^{-1}$$

The two basic reasons why bodies have mechanical energy will be considered now.

3.2.1 Kinetic Energy

Kinetic energy is the energy a body passes by virtue of its motion. For example a moving hammer does work against the resistance of the wood into which a nail is being driven. We obtain the expression of kinetic energy by computing the amount of work done by a body while the body is being brought to rest. Consider a body of constant mass, m moving with velocity u. A constant force F acts on it to bring it to rest in a distance s (Fig. 3.4)

Fig 3.4

When it comes to rest, its final velocity, v is zero. Then from the equation of motion you studied in Unit 5 we have

$$V^2 = u^2 + 2as \qquad 3.3$$

where a is the acceleration

$\therefore O = u^2 + 2as$

and

$$a = -\frac{u^2}{2s} \qquad 3.4$$

the negative sign in equation 3.4 shows that acceleration is in the opposite direction to the motion of the body hence the body decelerates. We expect the acceleration in the direction of the force F to be + $u^2/2s$. Now, the kinetic energy of the body is equal to the work ,W the body does against F, Therefore,

Kinetic energy, K.E of the body = W = Fs

But Fs = mas

$\therefore$ K.E = mas 3.5

Putting $a = \frac{u^2}{2s}$

we have

$$K.E = \frac{1}{2} mu^2 \qquad 3.6$$

You now see how we derive the popular expression for K.E. Conversely if work is done on a body the gain of kinetic energy when its velocity increases from zero to u can be shown also to be $\frac{1}{2} mu^2$.

We now generalise. If a body of mass, m with an initial velocity of u moves when work is done on it by a force acting over a distance s and if its final velocity is v then the work done Fs is given by

$$Fs = \frac{1}{2} mv^2 - \frac{1}{2} mu^2 \qquad 3.7$$

Eqn. (3.7) is called the work- energy equation. It may be stated in words as follows:
Work done by the forces (Acting on the body) = change in kinetic energy of the body.

3.2.2 Potential Energy

The potential energy of a system of bodies is the energy the body has by virtue of the relative position of the parts of the body of the system. Potential energy P.E arises when a body experiences a force in a region or field. An example is the gravitational field of the earth. In this case, the body occupies a position with respect to the earth. The P.E is then taken to be a joint property of the body–earth system and not of either body separately. Thus the P.E is determined by the relative position of the body and the earth. It is seen that the greater the separation, the greater the P.E. The P.E of a body on the surface of the earth is always taken to be zero. But for a body of mass m at a height h above ground level, the P.E. is equal to the work that will be done against gravity, to raise the body to this height. This means that a force equal and opposite to mg is needed to be applied to the body to raise it to the required height. This is because we have assumed g to be constant near the surface of the earth. Hence ,

Work done by external force (against gravity)
= Force x displacement
= mgh
:. P.E = mgh 3.8

When the body returns straight to the ground level an equal amount of potential energy is lost.

Example:
A car of mass 1x 10^3 kg traveling at 72 km h^{-1} on a horizontal road is brought to rest in a distance of 40m by the action of the brakes and frictional forces. Find (a) the average stopping force (b) the time taken to stop the car.

Solution:
A speed of 72km h^{-1} = 72 x10^3m/3600s
= $20ms^{-1}$

(a) If the car has mass m and initial speed u, then
K.E lost by car = ½ mu^2
If F is the average stopping force and s the distance over which it acts, then

Work done by car against F = Fs
But Fs =½mu^2
:. F x 40m = ½ x (1 x10^3kg) x $(20ms^{-1})^2$

$$F = \frac{1.0x10^3 x4.00}{2x40} \quad \frac{ky \ \ m2 \ \ s^{-2}}{m}$$
$$= 5.0x10^3 N$$

(b) Assuming constant acceleration and substituting
v = 0, u =20m s^{-1} and s= 40m in
v^2 = u^2 + 2as
we have 0 = 20^2 + 2a x 40
:. a = -5.0 ms^{-2}

the negative sign indicates the acceleration is in the opposite direction to the displacement. Using v = u +at we have
0 = 20 – 5. 0t
:. t = 4.0s

SELF ASSESSMENT EXERCISE 3

What is the kinetic energy of a body of mass 10kg moving with an initial velocity V_1 = 4m s^{-1} If the force applied to the body is 25N. What is its acceleration and final K.E if the body covered a distance of 20m

The initial K.E = ½ $MV,^2$ = ½ (10kg) $(4ms^{-1})^2$
= 80J

To find the final K.E we need to know the acceleration and final velocity. Hence from F = ma We have

$$a = \frac{F}{m} = \frac{25N}{10.00} = 2.5m\,s^{-2}$$

$$hence \quad v_2^2 = v_1^2 + 2as$$

$$= \left(4ms^{-1}\right)^2 + 2x\left(2.5ms^{-1}\right)\left(20m\right)$$

$$= 116 \; m^2 s^{-2}$$

$$\therefore \quad Final \quad K.E. = \frac{1}{2} mv_2^2$$

$$= \frac{1}{2} = \left(116 \; m^2 s^{-2}\right) x 10 \; kg$$

$$= 580 \; J$$

3.2.3

If the increase in K.E is needed, it is found thus:

$$Increase \quad in \; K.E = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

$$= 580 \; J - 80 \; J$$

$$= 500 \; J.$$

Conservation of Energy

The word conserve could be taken to mean preserve so that nothing is lost. So in this section we are going to find out that as energy is transformed from one form to another that no part of it is lost. For example if body of mass m is projected vertically upwards and if its initial velocity is u at point of projection A say, it will do work against the constant force of gravity, Figure (3.5).

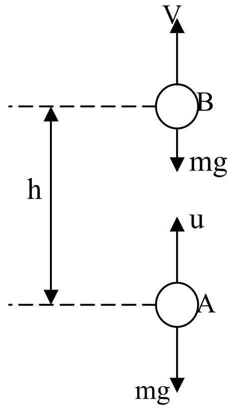

Fig 3.5

Let the velocity of body at a higher point B be V and the height attained at this point be h. Now, by definition

K.E lost between points A and B = work done by body against mg
Also by definition of P.E
Gain of P.E between A and B = work done by the body against mg

Therefore, we have that

loss of K.E. = gain of P.E

:. $\frac{1}{2} mu^2 - \frac{1}{2} mv^2 = mgh$ 3.9

This is what we call the principle of conservation of mechanical energy. This principle is stated as follows;

> the total amounts of mechanical energy (K.E + P.E) which the bodies in an isolated system possess is constant.

This applies only to frictionless motion i.e. to conservative system. Also, the gain in P.E will depend on the path taken but it does not in a conservative system.

Note that work done against frictional forces is often accompanied by a temperature rise. Therefore in our energy account we have to take this into consideration.

By so doing our energy conservation principles will be extended to include non-conservative systems and it becomes

loss of K.E = gain of P.E + gain of internal energy.

Thus the mechanics of a body in motion has been related to a phenomenon which is not clearly mechanical and in which motion is not directly detected. But we know that the internal energy is as a result of random molecular kinetic and potential energy of the particles of the system. In the same way energy has been extended to other parts of physics and it is now a unifying theme. Physics is at times referred to as the study of energy transformations, measured in terms of the workdone by forces created in the transformation. We thus see that the principle of conservation of mechanical energy is a special case of the more general principle of conservation of energy, which is one of the fundamental laws of science.

Energy may be transformed from one form to another, but it cannot be created or destroyed, ie. The total energy of a system is constant.

SELF-ASSESSMENT EXERCISE 4

Early in the nineteenth century, James Watt wanted to market his newly discovered steam engine to a society that until then had relied heavily on horses. So Watt invented a unit that made it clear how useful a steam engine could be. He conducted a demonstration in which a horse lifted water from a well over a certain period of time and called the corresponding power expended "one horse power".

Assume that water has a mass density of 1.0 x 10^3kg/m^3, the well was 20m deep, and the horse worked for 8 hours. How many litres of water did the horse raise from the well?

Solution:
Let the mass density of water be ρ.
Then a volume V of water has mass,
M = ρV. So, the work done in lifting a mass m of water from the bottom of the well is,

$$W = F\Delta Y = mg\Delta y$$

Where Δy is the depth of the well. Thus the work done in lifting a volume V from the well in a time t is ρ VgΔy and the power is

$$p = \frac{Work}{time} = \frac{\rho Vg\Delta y}{t}$$

We notice that the only unknown term here is volume V and we now solve for it:

$$V = \frac{pt}{\rho g\Delta y}$$

Since
1 horse power = 746W

$$V = \frac{(746\ W)(8.0\,h x\ 3600\ sh^{-1})}{(1.0\,x10^{3}\,kgm^{-3})(9.8\,ms^{-1})(20\ m)}$$

$$= 1.1\,x10^{2}\,m^{3}$$

but because there are 10^3L in Im^3
the number of litres lifted by the horse is
1.1 x10^5L.

4.0 CONCLUSION

In this unit you have leant

a) how work is defined in the scientific sense
b) to distinguish between positive and negative work depending on the sign of the force, which does the work.
c) that the unit of work is the joule.
d) how to determine the work done by a varying force
e) the forms of energy and the principle of conservation of mechanical energy
f) how to apply the work-energy equation in solving problems related to energy.

5.0 SUMMARY

What you have learnt in this unit concerns work and energy.

- that work is done when a force moves a distance in the direction of the line of action of the force.
  $Fs = W$ or $(F \cos \theta)\, s = W$
- that the unit of work is the joule,
- that work done by a varying force could be represented graphically and it is equal to the area under the curve of a force-displacement graph.
- that work is a measure of the quantity of energy transferred between two bodies.
- that power is the rate of doing work.
- that energy could be in the form of kinetic energy → $\frac{1}{2} mv^2$ Potential energy → mgh or internal energy due to molecular vibrations and P.E
- that the work-energy equation is given by $Fs = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$ where the symbols have their usual meanings.
- that the total amount of mechanical energy (K.E + P.E) which the bodies of an isolated system possesses is constant.
- that energy may be transformed from one form to the other but can never be created or destroyed i.e. total energy of a system is always constant.

6.0 TUTOR-MARKED ASSIGNMENT

1. A bullet of mass 10g traveling horizontally at a speed of $1.0 \times 10^2 \text{m s}^{-1}$ embeds itself in a block of wood of mass 9.9×10^2g suspended by strings so that it can swing freely. Find
   (a) the vertical height through which the block rises
   (b) how much of the bullet's energy becomes internal energy.
   ($g = 10\text{ms}^{-2}$).

2 A car of mass 1200kg falls a vertical distance of 24m starting from rest what is the work done by the force of gravity on the car? Use the work-energy theorem to find the final velocity of the car just before it hits the water. (Treat the car as a point like object).

3 A crate of mass 96kg is pushed across a horizontal floor by a force F. The coefficient of kinetic friction between the crate and the floor is $\mu_k = 0.27$. The crate moves with uniform velocity. What is the magnitude of force F? Suppose that at some point the crate passes on to a new section of floor, where μ_k 0.085. The pushing force on the crate is unchanged.After 1.2m on the new section of the floor, the crate moves with a speed of$v_i = 2.3$ m s^{-1}. What was the original speed of the crate v_i?

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