MODULE 1

Unit 1 Space and Time
Unit 2 Units and Dimensions
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UNIT 1 SPACE AND TIME

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1.0 INTRODUCTION

Have you had the chance of reading through the course guide yet? If yes, it means you have an idea of what we shall be discussing in this unit. This unit is very important because it sets the stage for understanding that branch of Physics that deals with motion, which we call mechanics. Everything in the universe is in constant motion including the tree or the rock which you probably think is not moving. The topics we shall cover in this unit which includes frame of reference, space and time will help you to understand that all motion is relative. This means that objects in the universe move relative to one another.

2.0 OBJECTIVES

At the end of this unit, you will be able to:

- explain the terms relative motion and absolute motion.
- define a frame of reference
- explain the concept of time
- draw and specify the position of a point in a two dimensional space with reference to a fixed origin, O
- list the two polar coordinates of point, P a distance r from the origin of a fixed frame of reference.

3.0 MAIN CONTENT

3.1 Frame of Reference

Under the frame of reference, we shall discuss rest and motion, relative motion, inertial and non-inertial frame of reference and related issues.

3.1.1 Rest and motion

To help us to understand the concept of frame of reference we need to note certain observations that have been made by physicists about this physical world we are living in. One of such observations is that a body is said to be at rest when it does not change its position with time. It is said to be in motion when it changes its position with time. But to know if the position of an object changes with time or not, we require a point absolutely fixed in space to be known. Such a fixed or stationary point is not known to exist in the universe. This is because physicists have observed that everything in the universe is in constant motion including this earth we are living in. The earth revolves round the sun and at the same time rotates round its polar axis. The sun itself together with the planets bound to it is in constant whirling motion among the galaxy of stars. The planets are also in motion with respect to each other. We now see that even if a wrist watch you place on the surface of the earth seems to be at rest it is actually in motion because the earth in which it rests is in motion. We say that the wrist watch is in motion relative to the earth. This means that there is nothing like absolute rest position for any object. It will interest you to know that this is true, about you, whether you are now sitting or standing. Everything in the room where you are only seems to be at rest. They are not actually at rest because they are actually moving relative to the earth. We can then conclude that absolute rest has no meaning in reality. When we say that the wrist watch you placed on the ground is at rest we mean that it does not change position with respect to the earth. Rest here means relative rest. It is always important for you to remember that a body is at relative rest with respect to another when it does not change its position relative to the latter. To help you appreciate this concept of relative rest better, think of passengers seated in a luxury bus moving along the road. The passenger is at relative rest with respect to other passengers in the same luxury bus while he or she is actually moving with respect to the objects along the road side.

3.1.2 All Motion is Relative

Now, let us go back to our discussion on relative motion. Since change in position is involved for motion to take place, then to be able to measure the distance travelled, we need a fixed point we can refer to as the reference point. From this fixed point, the change in position (i.e. motion) can be known or measured. But as explained earlier, no such fixed point is realistic in nature because every object is in constant motion in the universe. This means that every moving object is changing position with respect to some known object. All bodies in our earth move with respect to the earth. Hence we say that all motion is relative.

3.1.3 Specifying Frame of Reference

Since we now know that every object is at rest or in motion relative to another object, it means that the position or motion of the object can be designated with reference to a fixed point in a rigid frame work. This so called fixed point is called the ORIGIN, O. At this point, which is the origin, we draw three mutually perpendicular axes to represent the X, Y and Z axes respectively. So the initial position of the object or the final position of the object can be designed with reference to this fixed frame work X, Y and Z axes at the origin. This applies to all types of objects be it a particle, or a system of particles or a rigid body. We therefore define the FRAME of REFERENCE as the rigid or fixed frame work, relative to which the position and movements of a particle, or of a system of particles, or of a rigid body may be measured. If coordinates of the object remain fixed despite the elapse of time, we say that the object is at rest. But if a change occurs in one, or two, or all three coordinates with time, then the object is said to be in motion.

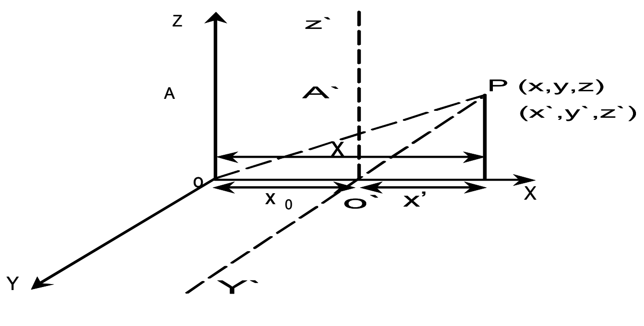


Fig. 1.1 The reference frame.

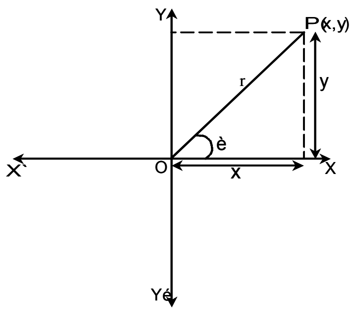
3.1.4 Inertial and Non-Inertial Frame of Reference

Figure 3.1 as drawn will help you conceptualize what we are saying. In this figure, let P be the position of a particle with reference to a rectangular coordinate system. Here, 0 is the origin of the system and X, Y, Z are its coordinates. A new system of reference with 0^1 as the origin is drawn as shown where, for convenience, 0^1 is taken along the X axis of the first system. Let 0^1X, 0^1Y^1 and 0^1Z^1 be the corresponding axes of the new system. 0^1Y^1 and 0^1Z^1 are evidently parallel to 0Y and 0Z. The point, P has coordinates in the new system indicated as $x^1 y^1$ and z^1 where $y = y^1$ and $z = z^1$ but x coordinate only undergoes a change. So, P is a fixed point in both systems. But if x, y, z change with time and P is moving, then x^1, y^1, z^1 will also change with time and P will also posses a similar motion with respect to the second system. That is, both systems are within the same frame of reference though the origins of the different coordinate systems may be different and their axes may also be inclined to one another. But if, there is any relative motion between these two systems, their frames of reference will be different. **The co-ordinate system in which the motion of any object depends only on the interactions of the constituent particles among themselves is called an inertial frame of reference.**

In such frames, Newton's laws of motion holds good. In a non-inertial frame, the motion of the objects is partly due to interactions among constituents particles and partly due to the movement of the frame with respect to an inertial frame.

At this point, I would like to call your attention to the fact that in nature inertial frames do not exist. This is because, on prolonged observation all motions, including the motions of the earth, planets and even the stars, are found to be non-inertial. But for most of the ordinary purposes any system of coordinates situated on the earth's surface may be regarded as an inertial system.

Also note that any co-ordinate system which moves with constant velocity with respect to an inertial frame is also inertial. Any one of them may be considered to be at rest because the motions are relative. This is known as a moving frame of reference.



SELF-ASSESSMENT EXERCISE 1

Explain the statement that, in reality, there is no absolute position of rest.

SELF-ASSESSMENT EXERCISE 2

What do you understand by the statement that the speed of a car is 100km per hour? This means that the car is changing its position relative to the earth and covers a distance of 100km in one hour.

3.2 Concept of Space

This concept deals with the Cartesian coordinates and polar coordinates.

3.2.1 Cartesian Coordinates

There are various ways you can specify a point in space. In one of the ways to specify a point in space, we need to know its coordinates along two or three mutually intersecting straight lines fixed at some rigid point called the origin. These intersecting straight lines are called the axes of reference. The distances from the point in space to the axes are found by drawing parallel lines from it to the axes. When the axes of reference are mutually perpen-dicular to each other for example, in a two dimensional plane, they are called rectangular axes. When they are inclined to each other at an angle, other than a right angle, they are called oblique axes. The rectangular axes are more commonly used because they are more convenient to draw. The coordinates referred to either rectangular or oblique axes are called Cartesian co-ordinates. Let us now give a diagrammatic example of a point in space in a two dimensional rectangular co-ordinate system. This is shown in Figure 3.2.

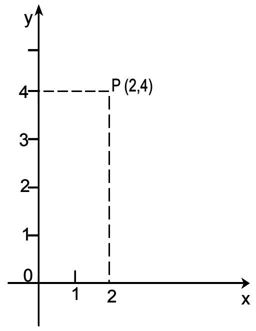


Fig. 4

The horizontal and vertical lines XX^1 and YY^1 in Figure 3.2 represent the rectangular axes fixed at origin, 0. The coordinates of any point in space for example P referred to the axes XX^1 and YY^1 are respectively given by x and y. The former is called the abscissa and the latter, the ordinate. The distance r = 0P of the point from the origin can be evaluated in terms of the coordinates X and Y as follows.

$$0P = r = \sqrt{x^2 + y^2} \qquad 3.1$$

This follows from our knowledge of the properties of a right angled triangle which you did at the senior secondary school level.

3.2.2 Polar Coordinates

We see that just as the position of any point on a given plane can be found when its coordinates with reference to two given axes in the plane are given, the position can also be traced if the distance r from the point of the origin and the angle □ by which the line joining the point with the origin is inclined to either of the given axes of reference are known. In this case, r and θ are known as polar coordinates.

Here $r \sin \theta = y$ 3.2
And $r \cos \theta = x$ 3.3

So, we get the same relation

$$r^2 = r^2(\sin^2 \theta + \cos^2 \theta) \quad 3.4$$
$$= y^2 + x^2$$

or that

$$r = \sqrt{x^2 + y^2} \quad 3.6$$

These two methods of specifying a point in a rectangular plane are used in our daily life. Furthermore, to find the position of a point in space, its coordinates referred to three mutually perpendicular axes meeting at a common fixed origin must be known. Thus to locate a point in space requires a three-dimensional rectangular co-ordinate system having three axes x, y, and z.

SELF-ASSESSMENT EXERCISE 3

Draw a diagram showing the Cartesian co-ordinates of a point P(2,4) in a plane surface.

The Cartesian coordinates of a point P (2,4) is as shown in the diagram Fig 3.3 .It means that with reference to some fixed origin O , the location of the point is 2 units along X-axis from the origin ,O and 4 units from O along the y-axis.

3.3 Concept of Time

3.3.1 Setting the Standard of Time

You remember that from our knowledge of Geography the earth rotates round its polar axis. It completes one rotation in what we call a complete day. This complete day consists of the day time and night time segments of the earth's rotation. This is because during the day time segment we see the sunlight but during the night time segment the sunlight is obscured from us and we see just darkness. The sun appears to us to move across the sky because of this diurnal rotation of the earth about its polar axis. The meridian at a place is an imaginary vertical plane through it. The sun is said to be in the meridian when it reaches the highest position in the course of its apparent journey in the sky. The interval of time between two successive transitions of the centre of the sun's disc across the meridian at any place is called a solar day. The length of this solar day varies from day to day because of many reasons but the same cycle of variations repeats after a solar year which is 365½days, approximately. The mean of the actual solar days averaged over a full year is called the mean solar day. A clock, watch or chronometer keeps the mean solar time. These are regulated against standard clocks and chronometers controlled under specific conditions. So, this

periodic appearances of the sun overhead, averaged over a year and called the mean solar day had helped us to capture the concept of time. The time interval between successful appearances gives the standard of time. This was the situation before 1960. With developments in science, the standard of time was changed to the periodic time of the radiation corresponding to the transition between the two energy levels of the fundamental state of the atom caesium-133.
The mean solar day is divided into 24 hours. An hour is divided into 60 minutes and a minute is divided into 60 seconds.

Therefore,
The mean solar day = 24hrs x 60min x 60secs = 86,400 mean solar Seconds ….. 3.7

This means that a mean solar second is 86,400th part of the mean solar day. This gives the unit of time known as the second.

Using the standard of time as the periodic time associated with a transition between two energy levels of cesium-133 atom,

1 second = 9, 192,631,170 cesium periods. ….. 3.8

What has helped us to understand the concept of time? Anything that happens periodically. For example, the periodic appearance of the sun over a particular location on the earth.

4.0 CONCLUSION

In this unit you have learnt that every object in space is in motion.

a) that a body is at relative rest with respect to another, so there is nothing like absolute rest.
b) that the Cartesian co-ordinates and the polar coordinates are used to locate a point in space with reference to a fixed origin.
c) that the periodic appearance of the sun at a particular location on earth or any other periodic happening has helped us understand the concept of time.

5.0 SUMMARY

What you have learnt in this unit:

- concerns frame of reference which helps us locate any point or object in space.
- you have learnt that rest and motion are all relative.
- you have learnt how time is determined.

6.0 TUTOR-MARKED ASSIGNMENT

1. Explain the terms ‘absolute motion’ and relative motion. Which one of them is more important to man, and why?
2. Explain what is meant by frame of reference. What is the significance of coordinates of a point in a three dimensional Cartesian system.

7.0 REFERENCES/FURTHER READING

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UNIT 2 UNITS AND DIMENSIONS

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1.0 INTRODUCTION

Today, we shall learn about units and measurements. Our minds will go readily to traders in the market who sells grains like rice, garri or others who sell clothing materials. These traders do one form of measurement or the other, depending on what they are selling. For example, the garri seller measures the garri with a specific type and size of measuring cup. The particular type and size of cup has been accepted by the grarri traders union as the unit of measurement. In this way, they set their own standard of measurements. Another example is that when you measure the height of a man, you are comparing him to a meter stick. Science takes note of what is around us and tries to explain it. Therefore, we say that science speculates, observes and analyses, etc. The whole basis of science is rooted in measurement. This is why this unit of our course is very important.

There are always two aspects to measurement. When you say that a person's height is 1.4m, you notice that in the expression of the height of the person, you have a number (that is, 1.4) and a unit (that is, m for metres). You immediately see that the measurement of a physical quantity consists of a pure number and a unit.

2.0 OBJECTIVES

At the end of this unit, you will be able to:

- explain what is meant by a unit of measurement
- state the different systems of measurement in physics
- list the fundamental units
- distinguish between a fundamental unit and a derived unit
- determine the units of a physical quaintly given the dimensions

3.0 MAIN CONTENT

3.1 Units of Measurement

Fundamental and derived units are discussed and some common units are discussed. And some common units of measurements are enumerated.

3.1.1 Definition of the Standards for Length, Time and Mass

Important because it makes for uniformity in experiments in physics no matter where it is carried out in the world as we saw in the introduction to this section of the course.

A very long time ago, people used what was available as standards for measurement. Measurement of length using the "foot" came into use in this manner. Here, the foot is defined as:

The average length of the feet of 20 Gerrman men.

Now, just as the union of garri traders accepted a specific type and size of measuring cup as their standard for the sake of uniformity, in 1791 French scientists established the forerunner of the international system of measurements. They defined the meter, the second and the kilogram.

- The metre was defined as one ten-millionth (10^{-7}) of the distance along Earth'ssurface between the equator and the North pole.
- The second was defined as 1/86,400 of a mean solar day.
- The kilogram was defined as the mass of a certain quantity of water.

In 1889, an International organization called the General conference on weights and measures was formed. Their mission was to periodically meet and refine these units of measurement. Therefore, in 1960, this organisation named the system of unit s based on the metre, kilogram and second the International System abbreviated SI (meaning in French -Systeme International). This system is also known as the metric system or mks system (after metre, kilogram and second). Other systems of measurement exist. This includes the cgs system (meaning-centimeter-gram-second). The F.P.S. system (British system) [meaning foot (ft),pound (lb) and second(s)]

The metre, the second and the kilogram are the units we use in measuring length, time and mass. Hence we define the unit as

- The convenient quantity used as the standard of measurement of a physical quantity.

To explain this further I can say that the numerical measure of a given quantity is the number of times the unit for it is contained in the quantity.

To illustrate this,

Get a long stick and measure it with a metre rule. Assuming you measured out five lengths of the metre. It means that the length of the stick you brought is 5 times the length of the metre rule which is 1metre. Hence the value of the length of the stick is 5metres (written 5m). Can you try this?

I would like to draw your attention to the fact that every physical quantity requires a separate unit for its measurement. For example, the unit of area is the square metre (m^2).

3.2 Fundamental and Derived Units

Fundamental and derived units are discussed and some common units of measurements are enumerated.

3.2.1 What is a Fundamental Unit?

These physical quantities, length, time and mass are known as the fundamental quantities. What this means is that length, time or mass can not be derived from any other quantity in physics and are independent of each other. So these three quantities are called the fundamental units. Recall that the unit of measurements of length is the metre, m. The unit of measurement of time is the second and the unit of measurement of mass is the kilogram.

3.2.2 What is a Derived Unit?

Definition: The units of all physical quantities which are based on the three fundamental units are termed derived units. This is how to get derived unit from fundamental unit. The unit of area is the area of a square each side of which is of one unit length.

Fig 3.1 shows the area of a square-the shaded portion. From our knowledge of mathematics, we know that area = length x width. The sides of a unit area have lengths 1m each. Therefore the value of the unit area is one square metre. The mathematical expression for it is

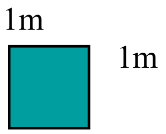

Fig. 3.1

Area = 1m x 1m = $1m^2$

This shows that the unit area is the square metre (written m^2).

Also the unit volume is the volume of a cube, each side of which is of unit length. We see that the unit of area or that of the volume is derived from the unit of length which is a fundamental unit. Velocity is another example of a physical quantity with a derived unit. A body has unit velocity when it moves over a distance of unit length in unit time in a constant direction or straight line. Therefore, the unit of velocity is derived from the units of length and time.

Mathematically, we write

$$\text{Velocity} = \frac{\text{distance (metres, m)}}{\text{time (in seconds, s):}} \quad ....3.1$$

:. The unit of velocity is metres per second written as ms^{-1} (or m/s).

SELF-ASSESSMENT EXERCISE 1

Can you now determine or derive the unit of force. Recall the definition of force. This will help you derive the unit of force.

We conclude that area, volume, velocity etc are all derived units. All the mechanical units, and units of all non-mechanical quantities like magnetism, electric, thermal, optical, etc can, with the help of some additional notions be derived from the three fundamental units of length, time and mass. This shows the true fundamental nature of these three units.

3.2.3 Some Units of Length, Mass and Time in Common Use.

Some units of length in common use in science are:

1 angstrom unit = 1A = 10^{-10} m (used by spectroscopists)...................3.2
1 nanometer = 1nm = 10^{-9}m(used by optical designers).......................3.3
1 micrometer = 10^{-6}m (used commonly in Biology)............................3.4
1 millimeter = 1mm = 10^{-3}m and 3.5

1 centimeter = 1cm = 10^{-2}m (used most often)...................................3.6
1 kilometer = 1km = 10^{3}m (a common unit of distance).....................3.7

The device used to subdivide the standard of mass, the kilogram, into equal Submasses is called the equal arm balance. The frequently used units of mass are:

1 microgram = 10g = 10^{-9} kg 3.8
1milligram = 10g = 10^{-6} kg 3.9
1gram = 1g = 10^{-3} kg 3.10
1pound mass = 1lb m = 0.45359237 kg …………….. 3.11

Units of length for very large distances:

Some objects are very far apart from each other. The Astronomical unit is the unit used in measuring such very large distances.

- 1Astronomical unit = 1.495 x 10^{8} km = 9.289 x 10^{7} miles 3.12
- 1 Astronomical unit, abbreviated 1 Au is taken to be the mean distance from earth to sun.

Other units for measuring long distances are:

- 1 Parsec = 3.083 x 10^{13} km = 1.916 x 10^{13} miles 3.13
- Light-year = Distance traveled by light in one year = 0.31 parsec = 5.94 x 10^{12} miles 3.14

The unit of time as we discussed in unit 1 of this module is the mean solar second. This applies to both the C.G.S and F.P.S systems of measurement. It is based on the mean solar day as the standard of time. If you recall from our discussions in unit 1, the solar day is divided into 24 hours, an hour into 60 minutes, and a minutes into 60 seconds.
Therefore, recall that,

The mean solar day = 24hrs x 60 minutes x 60 seconds = 86,400 mean solar seconds 3.15

That is the mean solar second is 86, 400^{th} part of the mean solar day.

The mean solar second is taken to be the unit of time (i.e 1s).

3.3 Dimensional Analysis

This section takes us through the definition of dimensional analysis and dimensional equations.

3.3.1 What is Dimension?

Three basic ways to describe a physical quantity are the space it occupies, the matter it contains and how long it persists. All descriptions of matter, relationships and events are combinations of these three basic characteristics. We have also found that all measurements ultimately reduce to the measurement of length, time and mass. From our discussion on derived units above, we saw that any physical quantity, no matter how complex, can be expressed as an algebraic combination of these three basic quantities.

For example we saw that velocity is length per time

The relation of the unit of any physical quantity to the fundamental units (length, mass and time) is indicated by what is known as the dimensions of the unit concerned.

Example *[Area] = [L x L].* Length, time and mass specify three primary dimensions. We use the abbreviations [L], [T] and [M] for these primary dimensions.

Definition: The dimension of a physical quantity is the algebraic combination of [L], [T] and [M] from which the quantity is formed.

Let us explain this further using the example of volume. The numerical value of volume, the unit volume is indicated by [V]. The dimensions of volume will therefore be given by $[L^3.M^0.T^0]$ or simply $[L^3]$. For a unit volume it is [unit length x unit width x unit height] that is [L x L x L] or just $[L^3]$. Thus, we say that volume has three dimensions in respect of length. Volume is not dependent of the units of mass and time.

Another example to determine the dimensions of a physical quantity, velocity is as follows:

$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}} = \frac{L}{T} \qquad 3.16$$

:. The dimensions of velocity is given by
$[\frac{L}{T}]$ or $[LT^{-1}]$.

3.3.2 What is a Dimensional Equation?

The equation such as $[V] = [L^3\ M^0\ T^0]$ or $[v] = [LT^{-1}]$ is called dimensional equation. These dimensional equations tell us the relation between the derived units(Volume, Velocity, etc) and the fundamental units, length, mass and time of any system of measurement.

The general expression for the dimension of any physical quantity is of the form $[L^q\ T^r\ M^s]$ of the primary dimensions. The superscripts q, r, and s refer to the order (or power) of the dimension. For example, the dimension of area is $[L^2\ T^o\ M^o]$. It simply reduces to $[L^2]$. So, if all the exponents q, r, and s are zero the combination will be dimensionless. Note that the exponents q, r and s can be positive integers, negative integers, or even fractional powers.

The study of the dimensions of an equation is called dimensional analysis. Any equation that relates physical quantities must have consistent dimensions i.e, the dimensions on one side of an equation must be the same as those on the other side. One use of dimensional analysis is that it provides a valuable check for any calculations. The second use is that dimensional analysis helps us convert the units of a physical quantity from one absolute system to another absolute system.

SELF-ASSESSMENT EXERCISE 2

Using dimensional analysis, determine the units of acceleration.

Further examples:

$$[\text{Acceleration}] = \frac{[\text{Velocity}]}{[\text{Time}]} = \frac{[\text{distance}]}{[\text{Time x time}]} \quad 3.17$$

$$= \frac{[L]}{[T^2]} = [LT^{-2}] \quad 3.18$$

Your answer shows that the units of acceleration is ms^{-2}

SELF-ASSESSMENT EXERCISE 3

[Coefficient of Linear Expansion]

$$= \frac{[\text{Change in Length}]}{[\text{Original Length x Change of Temperature}]} \quad .....3.19$$

$$\frac{[L]}{[L] \text{ x } [\text{degrees}]} = [\text{degree}^{-1}] \quad ......3.20$$

4.0 CONCLUSION

In this Unit you have learnt that in making a measurement of any physical quantity, some definite and convenient quantity of the same kind is taken as the standard in terms of which the quantity as a whole is expressed. You have learnt also that this convenient quantity used as the standard of measurement is called a unit. You also learnt that some physical quantities are known as fundamental quantities. These are length, time and mass and their units of measurement are the metre, the second and the kilogram respectively. You also learnt that there are different systems of measurement. You learnt that the fundamental quantities are used to derive the units of all other physical quantities by using dimensional analysis.

5.0 SUMMARY

What you have learnt in this unit concerns the

- meaning of a fundamental quantity
- meaning of the unit of a fundamental quantity
- different systems of measurement.

This unit has helped you to be able to derive the units of any physical quantity in nature using dimensional analysis.

You have also learnt some units of measurement in common use.
The knowledge you have acquired in this unit will help you to do correct calculations and measurements in the whole of your physics and mathematics courses. In short, the whole of science hinges on measurement. So, you can see how important this Unit is.

6.0 TUTOR-MARKED ASSIGNMENT

Newton's law of universal gravitation gives the force between two objects of mass, m_1, and m_2, separated by a distance r, as

$$F = G\left(\frac{m_1 m_2}{r}\right)$$

Use dimensional analysis to find the units of the gravitational constant, G.

7.0 REFERENCES/FURTHER READING

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UNIT 3 VECTORS

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    3.4.1 Components of a Vector in terms of Unit Vectors
    3.4.2 Components of a Vector in terms of Plane Polar Coordinates
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

When you read the topic of this unit which is 'Vectors', I know that in your mind you may be wondering why you have to study vectors. Questions like, of what use are they in physics? can also crop up in your mind. You may perhaps know the answers to these questions from your secondary school physics courses. It is interesting to know that vectors are used extensively in almost all branches of physics. In order to understand physics, you must know how to work with vectors, how to add, subtract and multiply vectors.

You are, already familiar with some physical quantities such as velocity, acceleration and force. These are all vector quantities. What you have learnt in Units 1 and 2 will definitely aid your quick understanding of this Unit.

In this Unit, we shall look afresh at vectors and build upon what you knew before now. We shall begin by defining vectors in a precise manner. You will learn how vectors are denoted and represented in the literature. You will also learn how to add and subtract vectors because, these will be applied in our study of motion, forces causing motion etc.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define a vector
- express a vector in terms of its components in two dimensional coordinate denote system
- Add and subtract vectors
- define the NULL vector
- multiply a vector by a scalar quantity
- express a vector in terms of unit vectors in a plane.

3.0 MAIN CONTENT

3.1 Definition and Examples of Vector Quantities

3.1.1 Definition

In the secondary school science courses, you must have studied scalar and vector quantities. You have learnt about physical quantities like mass, length, time, area, frequency, volume and temperature etc. You recall that a scalar quantity is completely specified by a single number (with a suitable choice of units). Many more examples of scalar quantities in physics exist. For example, the charge of an electron, resistance of a resistor, specific heat capacity of a substance, etc are all scalars.

You also learnt about physical quantities like displacement, velocity, acceleration, momentum, force etc. As you know, these are all vector quantities. The *definition of a vector is* as follows.

Any physical quantity which requires both magnitude and direction for it to be completely specified is called a vector.

Before we proceed to learn how vectors are represented, let us refresh our minds about vector notation.

$\vec{A}$, $\tilde{A}$, $\overline{A}$, $or \underset{\sim}{A}$, $\underline{A}$

3.1.2 Vector Notation

When you read different books on vectors you will notice that writers denote vectors differently. Generally, vectors are denoted by a letter in bold face type [**A, B, C**, etc] or by putting an arrow mark or a curly or straight line above the letter, or a curly or

$\vec{A}$ straight line below the letter, thus, . The magnitude of a vector is simply denoted by the letter without an arrow mark as In this course, we shall use the notation to denote a vector.

3.1.3 Representation of a Vector

In Figure 3.1 below, vector is represented by the line. But if the direction of another vector be opposite but has the same magnitude as vector then it will be represented as vector shown in Figure 3.1.

Now draw a vector P along a horizontal axis going from left to right from point P to point Q. Draw another vector equal to vector P and opposite in direction.

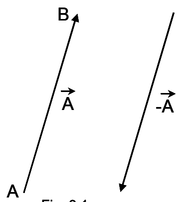


Fig. 3.1

$[-\vec{A}]$

You see that the vector language is not a jargon. Opposing vectors are always represented by a minus sign before the letter denoting the vector .

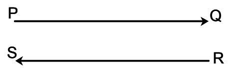


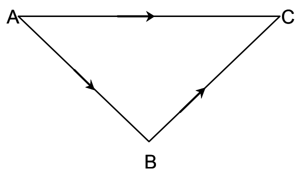
Fig. 3.2

$0.5\,\vec{AB}$

Take note also that if a vector has the same direction of another vector, say, AB its magnitude is 0.5 AB, then it will be written as 0.5 AB

You noticed that when you were doing the exercise above, you started drawing from somewhere and ended at another place. This shows you that there are three things you must consider while representing a vector. These are:

(i) a starting point also called the point of application



$\vec{AB} + \vec{BC} = \vec{AC}$

Fig. 3.5

(ii) a direction

(iii) a magnitude

Now that we have reached this point, let us proceed to study the composition of vectors.

3.2 Composition of Vectors

It is possible to have different vectors representing the same physical quantity (e.g. three forces). When these three vector act at the same point, a resultant vector can be obtained by the composition of these different vectors. Vector composition is done by the method of vector addition. Let us now look at one of the laws that guide us in vector composition.

3.2.1 Parallelogram law of vector composition

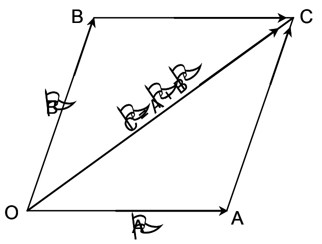


Fig. 3.3

$\vec{OA}$ *and* $\vec{OB}$ In Figure 3.3 above, let us assume that two vectors, act at point O. Now, represent these two vectors respectively. Using these two straight lines as adjacent sides draw a parallelogram OACB. The resultant of these two vectors acting at point O is given by which is the diagonal of the parallelogram through O. If we choose to represent the resultant vector by a letter C, then it is written as.

This method of composition of vectors is known as the parallelogram law. This law is normally stated as:

$$\vec{C} = \vec{A} + \vec{B}$$

(i) If two vectors acting at a point are represented by two adjacent sides of a parallelogram drawn from the point, then the resultant vector will be represented both in magnitude and direction by the diagonal of the parallelogram passing through that point.

Once you have this law always at the back of your mind you will be able to do addition and subtraction of vectors. Another rule that will aid your composition capabilities is this

(ii) If two vectors acting at a point are represented in magnitude and direction by the two sides of a triangle taken in order, then the resultant vector will be represented in direction and magnitude by the third side taken in the reverse order. $\vec{OC}$

The diagram in Figure 3 . 4 will help you understand the rule better.

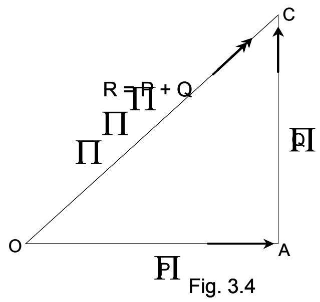


Fig. 3.4

$\Delta\, OAC$ $\vec{r} = 50N$

In Figure 3.4, represents the vector and O and A are its starting and end points respectively. At A, the starting point of the vector Q is placed and it is drawn in proper magnitude and direction as , C being the terminal point. This is completed. Then the side taken in the reverse order i.e. represents the resultant vector.

$\vec{OA}$ $\vec{OC}$

3.3 Addition and Subtraction of Vectors

3.3.1 Addition of vectors

$\vec{AC}$ and $\vec{BC}$

We have seen that the resultant of two vectors is give by the sum of the two vectors. Let us look at further examples. If I tell you that the sum of two vectors is defined as the single or equivalent or resultant vector, what it means is that when I draw the vectors as a chain, starting the second where the first ends, the sum is got by drawing a straight line from the starting point of the first vector to the end point of the second vector as shown below in Figure 3.5.

SELF-ASSESSMENT EXERCISE 2

$\vec{q} \equiv$

If a force of 40N, acting in the direction due East and a force of 30N, acting in the direction due North. Then, the magnitude of the resultant or sum of these two forces will be

= 50N. This is because applying our knowledge of Pythagoras theorem

$$r^2 = P^2 + q^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.1$$

$$= 1600 + 900$$

$$r^2 = \sqrt{2500} \qquad = 50N \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 3.2$$

When there are more than two vectors acting, the resultant can also be found. Here are some illustrations.

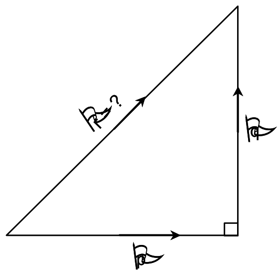


Fig. 3.6

Let us sum up the vectors

$\vec{p} + \vec{q} + \vec{r} + \vec{s} + \ldots\ldots..$

$$\text{Then} \quad \vec{p} + \vec{q} = \vec{PR} \qquad 3.3$$

$$(ii) \quad \text{and} \quad \vec{PR} + \vec{r} = \vec{PS} \qquad 3.4$$

$$\text{and} \quad \vec{PS} + \vec{s} = \vec{PT} \qquad 3.5$$

Firstly, we draw the vectors as a chain (Fig. 3.7)

$\vec{p}, \vec{q}, \vec{r}, \vec{s}$ We see that the sum of the vectors is given by the single vector joining the starting point of the vector to the end point of the last vector.

Self Assessment Exercise 1. 3

$\vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}$ Now, find the sum in Fig. 3.8

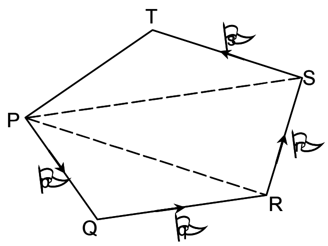
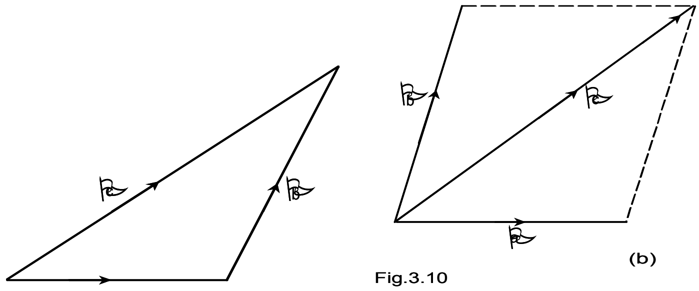


Fig. 3.7

$\vec{m}, \vec{n}, \vec{o}, \vec{p}, \vec{q}$ I would want you to pay particular attention to this. Suppose in another case we draw the vector diagram to find the sum of say ,and discover that it is a closed figure, what does that mean? It tells us that the sum or resultant of those vectors is zero.

That is, for example

Right. What about this one?



(a) (b)

Fig.3.10

$\vec{AB} - \vec{CB} + \vec{CD} - \vec{ED}$ Find the sum of you notice that some vector terms here are negative. This means there can be negative vectors or forces acting in opposite direction to other vectors or forces. Remember

$-\vec{CB} = \vec{BC}$ that i.e. the same magnitude and direction but in the opposite sense.

Also,

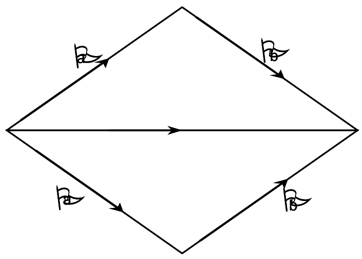
$$\vec{AB} + \vec{BC} - \vec{DC} - \vec{AD} \qquad 3.6$$

$$\therefore \vec{AB} - \vec{CB} + \vec{CD} - \vec{ED} = \vec{AB} + \vec{BC} + \vec{CD} + \vec{DE}$$

$$= \vec{AE} \qquad 3.7$$

Now, do this one immediately

Find the vector sum



(a) Commutative law

Are you finished? If so, did you get, the answer zero? Then you are correct, BRAVO!

I also want to draw your attention to the fact that vector addition is not an algebraic sum. For example:

$\vec{c} = \vec{a} + \vec{b}$

$\vec{c}$ *and* $\vec{b}$ As you recall, two vectors can be added graphically using either the triangular law or the parallelogram law. Now, in Figure 3.10b you may assume the forces are acting simultaneously at a point O, then the vector

represented by the diagonal of the parallelogram through the point of action of the two forces is the sum of the vectors We cannot add the magnitudes of to get the magnitude of .

$\vec{b} + \vec{a}$ From the definition of vector addition it follows that
= (This we refer to as commutative law for addition).3.9

(This we refer to as the associative law of addition)

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \qquad 3.10$$

Thus, the order in which you add vectors does not matter as shown in figure 3.11.

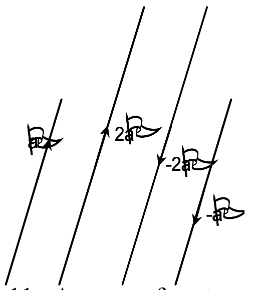


Fig 3.11: A group of vectors can be added in any order.

3.3.1.1 Multiplication of a Vector by a Scalar

$\vec{a}$ *a or m* $|\vec{a}|$ If I asked you the question, 'What is the vector'. From the methods of vector addition you can see that it is a vector three times as long as vector and is in the same direction as vector giving. So, we can generalise by saying that the product of vector, say by a positive scalar quantity m is. The product is a vector in the same direction as vector but its magnitude is i.e. *m* times the magnitude of vector (Fig. 3.12).

Note that if m is less than zero, is acting in the opposite direction to vector but its magnitude is . So, for m = -1, the new vector is and it is equal and opposite in direction (meaning antiparallel) to. We readily find a practical example of this in physics where it is depicted in Newton's second law, F = ma. Here, force is expressed as product of mass (which is a scalar) and acceleration (vector).

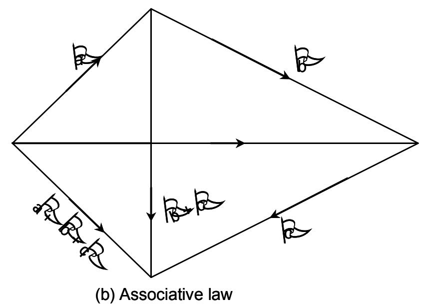


Fig. 3.12 Multiplication of a vector by a scalar

SELF-ASSESSMENT EXERCISE 4

Can you think of more examples?
Other laws which follow the above discussion are.

$$m(n\vec{a}) = (mn)\vec{a} \qquad 3.11$$

$$(m+n)\vec{a} = m\vec{a} + n\vec{a} \qquad 3.12$$

$$m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b} \qquad 3.13$$

Where m and n are numbers.

3.3.2 Subtraction of Vectors

$(-\vec{b})$ to $\vec{a}$ This is similar to what we did during addition of vectors. The difference is that here we shall only be adding negative values to positive quantities. So, subtraction of a vector from vector i.e. can be seen as adding the vector . Thus we can write

a - b = a + (-b) 3.14

$\vec{a}$ You should recall that we touched on this earlier on when we discussed vector addition. So, to subtract from graphically (see Fig. 3.13).

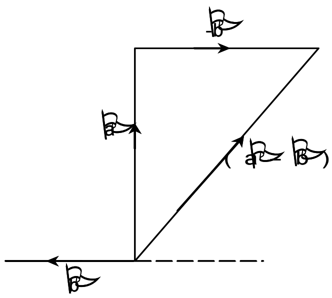


Fig. 3.13

$(-\vec{b})$ to $\vec{a}$ We multiply vector by -1 and add the new vector using either the triangular law or the parallelogram law.

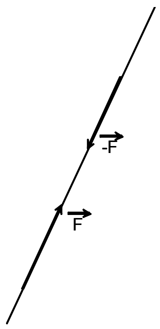


Fig.3.14

3.3.2.1 The Null Vector

$\vec{F} + (-\vec{F}) = \vec{F} - \vec{F}$ Now, we are going to look at another scenario. This is the case where two equal and opposite forces are applied to a point (Fig. 3.14). What do you think is their resultant? From our knowledge of vector addition, we simply add. This gives a vector of zero magnitude. Secondly, we see that we can not define a direction for it. Such a vector is called a NULL VECTOR or a ZERO VECTOR.

So, to define a null vector we say that,

A Null vector is a vector, whose magnitude is zero and whose direction is not defined.

It is normally denoted by O. We also get a null vector or zero vector when we multiply a vector by the scalar zero.

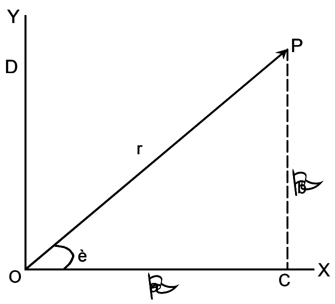


Fig.3.16

3.3.2.2 Unit Vector

We want to explain what we mean by a unit vector. You will now see how simple this section is. You already known what unit means. It simply means unit value. That is 1 or one unit. Unit vector then means a vector whose magnitude is simply one unit.(i.e.1).

$\frac{\hat{a}}{a}$ Now consider the product of vector with a scalar You can see that the magnitude of the vector is 1. This implies that a vector of length or magnitude

1 is called a unit vector. Also, since is a positive number, it follows that the direction

of vector is along vector. Hence, is the unit vector in the direction of. Note that the

unit vector could be denoted by the symbol (Fig. 3.15)
Thus we can write

$$\hat{a} = a . \vec{a} \qquad 3.15$$

A unit vector is used to denote direction in space. So it serves as a handy tool to represent a vector. This means that a vector in any direction can be represented as the product of its magnitude and the unit vector in that direction. By convention, unit vectors are taken to be dimensionless. Let us now go on to define vectors in terms of their components.

3.4 Components of a Vector

3.4.1 Components of a vector in terms of unit vectors

$\vec{OP}$ The vector is defined by its magnitude, r and its direction, . It could also be defined by its two components in the OX and OY directions. What we are saying here is that is a vector acting along a plane and could be resolved into its components. Thus:

$\vec{b}\,P$ is equivalent to a vector in the OX direction plus a vector in the OY direction. i.e.

$$\vec{OP} = \vec{a}$$

(along OX axis) (along OY axis) 3.16

If we take to be unit vector in the OX direction then

$$\vec{a} = a\hat{i} \qquad 3.17$$

Similarly, if we define j to be a unit vector in the OY direction, then

$$\vec{b} = b\hat{j} \qquad 3.18$$

$\vec{OP}$

$$\vec{r} = a\hat{i} + b\hat{j} \qquad 3.19$$

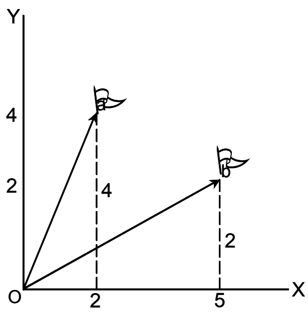


Fig.3.17

Then the vector can be written as

where î and are the unit vectors in the 0X and 0Y directions respectively. The sign (called cappa) denotes a unit vector.

Note: Conventionally, i and j are taken to be the unit vectors along the x and y axis in the cartesian coordinate system.

∧ Since we have defined the unit vectors, we shall in practice omit the sign (cappa) above, i and j, but always remember that they are vectors.

SELF-ASSESSMENT EXERCISE 5

Let

$$\vec{a} = 2i + 4j \; and \; \vec{b} = 5i + 2j$$

$\vec{a} + \vec{b}$, To find draw the two vectors in a chain as shown below, Figure 3.18

$$\vec{a} + \vec{b} = \vec{OP} \qquad 3.20$$
$$= (2+5)i + (4+2)j$$
$$= 7i + 6j \qquad 3.21$$

i.e. we add up the vector components along OX and add up the vector components, along OY.

I would like you to know that we can do this without a diagram like this:

If

$$\vec{P} = 3i + 2j \; and \; \vec{Q} = 4i + 3j$$

Then

And in the same way, if we are subtracting i.e.

$$\vec{Q} - \vec{P} = 4i + 3j - (3i + 2j) \qquad 3.24$$
$$= i + j \qquad 3.25$$

Similarly if

$$\vec{P} = 5i - 2j \; and \; \vec{Q} = 3i + 3j$$
$$and \; \vec{R} = 4i - j$$

Then,

SELF-ASSESSMENT EXERCISE 6

$\vec{P} + \vec{Q} + \vec{R} =$...................... (i)

$\vec{P} - \vec{Q} - \vec{R} =$.................. (ii)

Complete the working above.
Your answers should be
(i) 12i
(ii) -2i - 4j

(i) $\vec{OP} = \vec{r}$ ∴ $\vec{Q} + \vec{R} = 5i - 2j + 3i + 3j + 4i - if$ 3.26

$= (5 + 3 + 4)i + (3 - 2 - 1)j$

$= 12i$ 3.27

(ii) $\vec{P} - \vec{Q} - \vec{R} = (5i - 2j) - (3i + 3j) - (4i - 1j)$ 3.28

$= (5 - 3 - 4)i + (-2 - 3 - 1)j$

$= -2i - 4j$ 3.29

Compare your solution with these

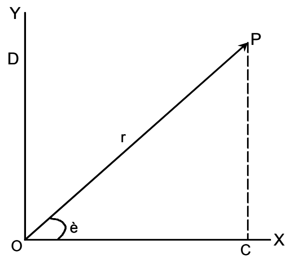


Fig. 3.19

3.4.2 Component of a Vector in Terms of Polar Coordinates

In Polar coordinates the vector as shown in Figure 3.1.9 is resolved along the OX and OY axes thus:

$\vec{p}$ From the end point of vector draw a perpendicular PC and PD on X and Y-axes respectively. Then, OC and OD represent the resolved parts of the vector in magnitude and direction. Hence we have

OC = OP Cos 2 = r Cos 2 ....3.30

OD = OP Sin 2 = r Sin 2 .... 3.31

and $OC^2 + OD^2 = OP^2 = r^2 (Cos^2 2 + Sin^2 2) = r^2$ 3.32

$\vec{OP} = r\ Cos\theta and\ \vec{OD} = r\ Sin\ \theta$ Now, are the components of vector in polar coordinates.

4.0 CONCLUSION

What you have learnt in this unit concerns

a) Definition and representation of vectors
b) How vectors are denoted
c) Composition of vectors
d) How to resolve vectors into their components in two dimensional space
e) How to express vectors in terms of their unit vectors

5.0 SUMMARY

In this unit you have learnt that:-

- Quantities which are completely specified by a number are called scalars with a suitable choice of units.
- Vectors are quantities which are specified by a positive real number called magnitude or modulus and have a direction in space.
- Vectors combine according to the following rules

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

$$\vec{a} + (\vec{b} + \vec{a}) = (\vec{a} + \vec{b}) + \vec{a}$$

$$m(n\vec{a}) = (mn)\vec{a}$$

$$(m+n)\vec{a} = m\vec{a} + n\vec{a}$$

$$m(\vec{a} + \vec{b}) = m\vec{a} + m\vec{b}$$

$\vec{a}$

- Any vector can be expressed as

$$\vec{a} = a\hat{\alpha}$$

$\vec{a}$

- Where is a unit vector in the direction of

- Vectors can be expressed in terms of unit vectors along the X and Y axes of a plane Cartesian coordinate system.

$\vec{\vee}$ Thus the unit vectors i, j point along the X and Y- axes respectively. Then for a vector.

$$\vec{V} = V_x i + V_y j$$

$\vec{V}$ The quantities V_x, V_y are the components of. The magnitude of V is

$$V = \sqrt{V_x^2 + V_y^2}$$

- The NULL vector is the vector of zero magnitude and unspecified direction

6.0 TUTOR-MARKED ASSIGNMENT

1. Let V be the wind velocity of 50km h^{-1} from north-east. Write down the vector representing a wind velocity of
   (i) 75 kmh^{-1} from north-east
   (ii) 100km h^{-1} from south-west in terms of V.

7.0 REFERENCES/FURTHER READING

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UNIT 4 VECTORS IN THREE DIMENSIONS

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  3.2.2 Vector (or Cross) Product
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1.0 INTRODUCTION

The importance of vectors in physics cannot be over emphasised. This is because most physical quantities we come across in physics are vector quantities. These include electric flows, magnetic flux, forces, velocities, etc. Also, we recall that in unit 2 we discovered that every object in the universe is in constant motion. Since it is one kind of force or the other that keeps these objects in motion, and motion could be in one, two or three dimensions and force is a vector. It is important to study vectors in all these dimensions. By so doing we get an understanding of why certain occurrences in nature behave as they do.

In this Unit, therefore, we shall dwell on vector in space, resolution of vectors I three dimensions. You will also learn about vector product.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- write the general equation that gives the magnitude of a vector in space
- resolve a vector in space along three mutually perpendicular axes
- resolve a vector in terms of its Unit vectors along three mutually perpendicular axes.
- calculate the Scalar product of two vectors meeting at a point

- calculate the vector product of two vectors acting at a point.

3.0 MAIN CONTENT

3.1 Vectors in Space

3.1.1 Magnitude of a Vector in Space

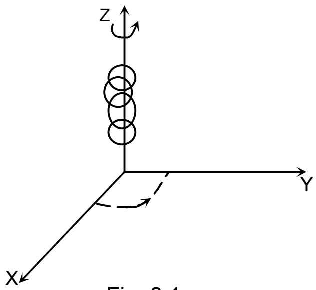


Fig. 3.1

The axes of reference are defined by the right-hand-rule. Ox, OY and OZ form a right-handed set of rotation from OX to OY takes a right-handed corkscrew action along the positive direction of OZ (Figure 3.1)

SELF-ASSESSMENT EXERCISE 1

Where will be the positive direction for a right-handed cork screw action while rotating from OY to OZ?...........................

The Answer is OX

.

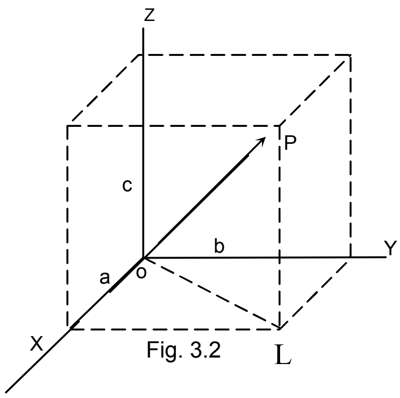


Fig. 3.2

In Figure 3.2,

$\vec{OP}$ is defined by its component

$\vec{a}$ *along OX direction*

$\vec{b}$ *along OY direction*

$\bar{c}$ *along OZ direction*

Let i = Unit vector in OX direction

j = Unit vector in OY direction
k = Unit vector in OZ direction

$$Then\ \vec{OP} = \vec{a}i + \vec{b}j + \vec{c}k \qquad 3.1$$

also

$$OL^2 = a^2 + b^2 \qquad 3.2$$

and

$$OP^2 = OL^2 + C^2 \qquad 3.3$$

$$i.e. \quad OP^2 = a^2 + b^2 + c^2 \qquad 3.4$$

$$So, \quad if\ \bar{r} = ai + bj + ck \qquad 3.5$$

Then

$$r = \sqrt{(a^2 + b^2 + c^2)} \qquad 3.6$$

The value of r here gives the magnitude of the vector $\vec{OP}$ in Figure 3.2. This is also an easy way of finding the magnitude of a vector when it is expressed in terms of its Unit vectors.

SELF/ASSESSMENT EXERCISE 2

Now you can do this one

$$If\ \vec{PQ} = 4i + 3j + 2k, then\ /\ \overline{PQ}\ / = \quad ?$$

$$The\ answer\ is\ \vec{PQ} = |(29) = 5.385$$

This is how to solve it. We are given that

$$|\vec{PQ}| = 4i + 3j + 2k \qquad 3.7$$

$$\therefore |\vec{PQ}| = \sqrt{(4^2 + 3^2 + 2^2} \qquad 3.8$$

$$= \sqrt{(16 + 9 + 4)} \qquad = \sqrt{(29)} \qquad 3.9$$

$$= 5.385\ Answer$$

3.1.2 Resolution of Vectors in the three mutually perpendicular axes

Here we want to resolve a vector in space into its components in a three dimensional rectangular coordinate system. Let the vector $\vec{OP}$ be situated in a 3-dimensional rectangular coordinate system with its starting point O at the origin shown in Figure 3.3

Let OX, OY and OZ represent the axes. Let the coordinates of

$\vec{OP}$ be (X, Y, Z).

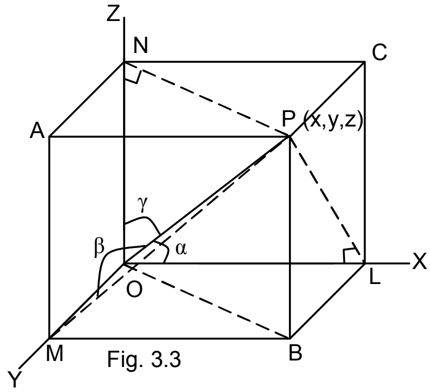


Fig. 3.3

Then, draw the projections of OP and OX, OY and OZ and let these be represented by OL, OM and ON respectively.

If Ɑ and γ are the angles of inclination of, $\vec{OP}$ with OX, OY and OZ axes respectively,
then,

OP Cos A= x 3.10

OP Cos = y 3.11

And

OP Cos γ = z 3.12

$OP^2 (Cos^2 + Cos^2 + Cos^2 \gamma) = x^2 + y^2 + 2^2$ 3.13

But we know that

$OP^2 = OL^2 + OM^2 + ON^2 = x^2 + y^2 + 2^2$ 3.14

$Cos^2 + Cos^2 + Cos^2 \gamma = l^2 + m^2 + n^2 = 1$ 3.15

where l, m, n are called the direction cosines.

$$Also\, op = \frac{x^2}{OP} + \frac{y^2}{OP} + \frac{z^2}{OP} = \frac{x}{OP}.x + \frac{y}{OP}.y + \frac{z}{OP}.z \quad 3.16$$

$$= Cos\alpha\ x + Cos\,\beta y + Cos\, rz \quad 3.17$$

$$= lx + my + nz \quad 3.18$$

Thus the vector OP can be complete resolved in magnitude by the coordinate of its starting point (O, O, O) and end point (X, Y, Z) and in direction by the three direction cosines (l, m, n).

Now considering the case when the vector lies in a plane, say the XOY plane, then Z = 0 and we get that

$$OP = lx + my \qquad 3.19$$

it follows also that for a vector lying in the XOZ plane, then y = 0 and for a vector lying in the YOZ plane, x = 0

3.1.3 Resolution of Vectors in Three Mutually perpendicular axes in terms of the Unit Vectors

The vectors we have considered thus far are two dimensional and Unit vectors in three dimensions. Now, let us generalise for any vector in three dimensional system. This is same as considering a vector in space.

A vector in three dimensions can be specified with Cartesian set of axes x, y and z as we discussed earlier in this Unit. The orientation of the axes is best described using the right -hand rule.

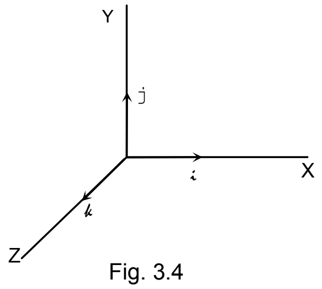
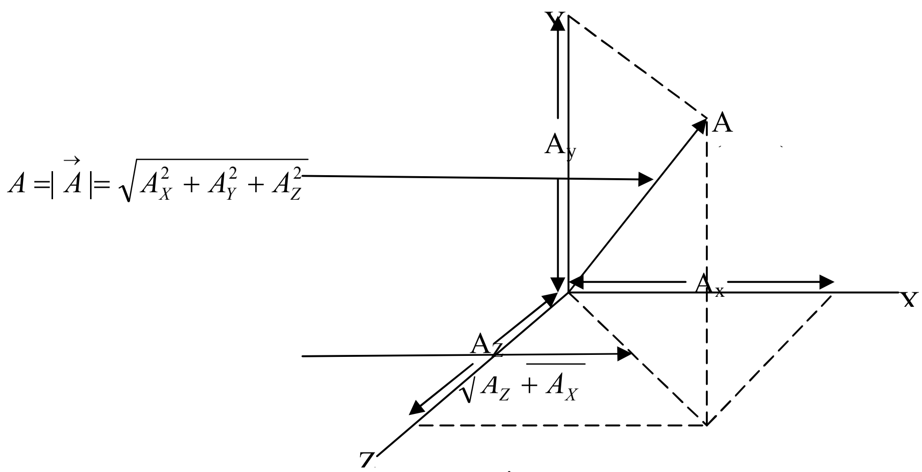


Fig. 3.4

In Figure 3.4 visualize the z axes as pointing out of the plane of the paper and perpendicular to both the x and y axes. The right-hand rule says that if you curt your fingers from the x-axes to the y-axes, your thumb will be pointing towards the positive z-axis. This right hand rule is a well established convention and you will come across it in many areas of physics like in your course in magnetism.



Fig

Figure 3.5 shows how we resolve a vector into its components in the Cartesian coordinate system along the three axes, OX, OY and OZ. The three Unit vectors for the three axes are denoted by I, j and k as shown in Figure 3.4. The Unit vector K points in the Z-direction.

In Figure 3.5 vector $\overline{A}$ with its origin at O is known as the displacement vector for its coordinates at A (x, y, z).

Therefore,
The component of vector $\vec{OA}$ along x axis = $A_x i$
The component of vector $\vec{OA}$ along y-axis = $A_y j$ and
The component of vector $\vec{OA}$ along z axis = $A_z k$

This is the same thing as saying that the projection $\vec{OA}$ along x, y and z axes are A_x, A_y and Az respectively. They are then multiplied by the Unit vectors in the direction of each axis to get the vectors $A_x i$, $A_y j$ and $A_z k$. The sums of these components give the vector $\vec{OA}$ and we write

$$\vec{OA} = A_x i + A_y j + A_z k \qquad 3.20$$

By Pythagoras's theories, we recall that the length or magnitude of

$\vec{OA}$ is

$$\vec{OA} = \sqrt{A_x^2 + A_y^2 + A_z^2} \qquad 3.21$$

SELF-ASSESSMENT EXERCISE 3

Draw a vector $\vec{V}$ that points in the northwesterly direction, making an angle with the northwesterly direction as shown in Figure3.6. If north is chosen as the + y - direction, what is the x component of $\vec{V}$

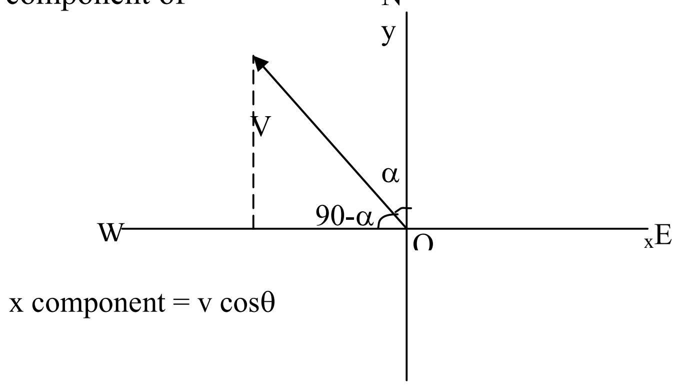


Fig 3.6

x component of
$\vec{V}$ = -V Cos (90 - ?) = - V sin

3.2 Vector Product

3.2.1 Scalar (or Dot) Product

Multiplication of vectors is the same thing as saying product of vectors. There are two kinds of products of vectors.

(1) The Scalar Product
(2) The Vector Product

The Scalar Product

The scalar product of two no-zero vectors $\vec{A}$ and $\vec{B}$ (written as A.B) is a scalar defined as

$$\vec{A}.\vec{B} = AB\,Cos\,\theta \qquad 3.22$$

Where A, B are absolute values or magnitudes of the vectors $\vec{A}$ and $\vec{B}$, and 2 is the angle between $\vec{A}$ and $\vec{B}$ when they are drawn with a common tail. Figure 3.7 shows what we mean.

The scalar product denoted by $\vec{A}.\vec{B}$ is (sometimes called the 'dot product').

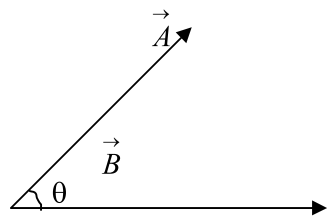


Fig 3.7

$\therefore \vec{A}.\vec{B}$ as given above?= AB Cos

This means $\vec{A}.\vec{B}$ = A x projection of B on A 3.23

Or $\vec{A}.\vec{B}$ = B x projection of A on B 3.24

we note that
In either case, the result is a scalar quantity.

SELF-ASSESSMENT EXERCISE 4

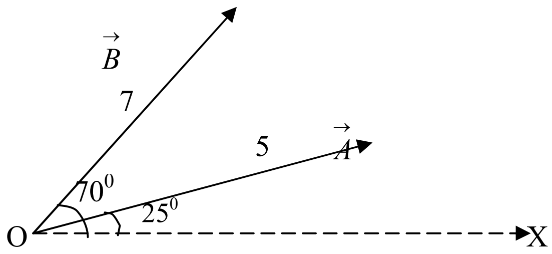


Fig 3.8

What is $\vec{OA}.\vec{OB}$ in Figure 3.8 not that the dot sign means multiplication sign. Try it, before checking on the answer below.
The answer is

$$\vec{OA}.\vec{OB} = \frac{35\sqrt{2}}{2}$$

This is because $\vec{OA}.\vec{OB} = OA.OB\ COS\theta$

$$= 5x7\cos 45^0$$

$$= 35x\frac{1}{\sqrt{2}}$$

$$= 35\frac{\sqrt{2}}{2}$$

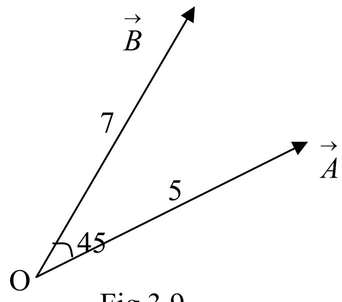


Fig 3.9

Self Assessment Exercise 3.5

Now, what is the dot product of the vectors shown in the diagram below
i.e. The scalar product of

$\vec{a}$ *and* $\vec{b}$ *is.............*

The scalar product of

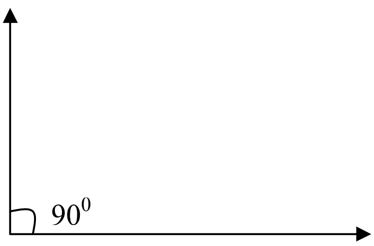


Fig 3.10

$\vec{a}$ *and* $\vec{b}$ $= \vec{a}\,.\vec{b} = 0$ 3.26

This is so because

$\vec{a}\,.\vec{b} = ab\,Cos\,90^0$ 3.27

but

Cos $90^0 = 0$

The scalar product of any two vectors at right angles to each other is always zero.

What happens if the two vectors are in the

(i) Same direction

(ii) opposite direction. For example

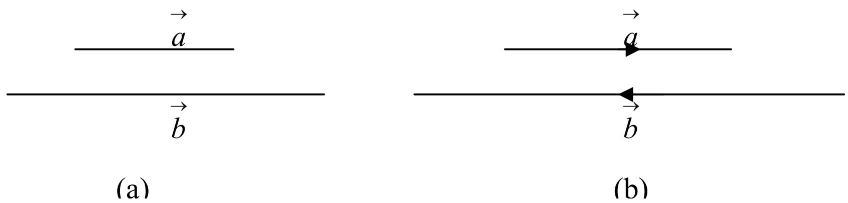


Fig 3.10

In Figure 3.10 a vectors

$\vec{a}$ *and* $\vec{b}$ are in the = 0 same direction, ^{0}.

Then $\vec{a}.\vec{b}$ = ab Cos 0^0 = a.b.1 = ab 3.28

In Fig 3.10b vectors $\vec{a}$ *and* $\vec{b}$ = 180□are in opposite direction, ^{0}. Then

$$\vec{a}.\vec{b} = ab\cos 180^0 = a.b.(-1) = -\,ab$$

Self Assessment Exercise 3.6

When the vectors are expressed in terms their Unit vectors in component form of we have,

$$\vec{A} = a_1 i + b_1 j + c_1 k$$

$$\vec{B} = a_2 i + b_2 j + c_2 k$$

Then

$$\vec{A}.\vec{B} = (a_1 i + b_1 j + c_1 k).(a_2 i + b_2 j + c_2 k) \qquad 3.29$$

$$= a_1a_1\, i.i + a_1b_2\, i.j + a_1c_2 i.k + b_1a_2 j.i + b_1b_2 j.j + b_1c_2 j.k + c_1a_2 k.i + c_1b_2 k.i + c_1c_2 k.k \qquad 3.30$$

Just be careful when expending such brackets above. This will simplify soon, so no need to worry.
Take note that

i.i = 1.1. Cos 0^0 = 1 3.31

Similarly j.j = 1 and k.k = 1 always remember this.

Now i.j = 1 Cos 90^0 = 0 3.32

We see that the following terms will also varnish i.e.
j.k = 0 and k. i = 0 applying these in one expression for

$\vec{A}.\vec{B}$ we have

$$\vec{A}.\vec{B} = a_1a_2 + b_1b_2 + c_1c_2 \qquad 3.33$$

$$Since\ \overline{A}.\overline{B} = a_1a_2 1 + a_1b_2 0 + a_1c_2 0 + b_1a_1 0 + b_1b_2 1 + c_1a_2 0 + c_1b_2 o + c_1c_2 1 \qquad 3.34$$

hence we dropped the terms in zero to assure at our answer above

Properties of dot Product

1. $\vec{a}.\vec{b}$ is a scalar
2. $\vec{a}.\vec{b} = \vec{b}.\vec{a}$ i.e. the dot product is commutative 3.35
3. $\vec{a}.(\vec{b}.\vec{c}) = \vec{a}.\vec{b} + \vec{a}.\vec{c}$ i.e. the dot product is associative over addition 3.36
4. $(m\vec{a}).\vec{b} = m(\vec{a}.\vec{b}) = a.(m\vec{b})$ 3.37
5. If $\vec{a}.\vec{b} = 0, and\ \vec{a}\ and\ \vec{b}$ are not zero, vectors then, a is perpendicular to b 3.38
6. $\left|\vec{a}\right| = \sqrt{a^2} = \sqrt{a.a}$ 3.39
7. $\vec{a}.\vec{a} > 0$ For any non zero vector
8. $\vec{a}.\vec{a} = 0$ only if a = 0 3.40

3.2.2 The Vector (or Cross) Product

The vector of two vectors is also known as the cross product of the two vectors. This is written as $\vec{A}.\vec{B}$ for the cross product of vectors $\vec{A}$ *and* $\vec{B}$ The result of the cross product is another vector. Thus, we define the cross product as

$$\vec{A}\,X\,\vec{B} = (AB\sin\theta)\hat{c} = \vec{C} \qquad 3.41$$

where is the angle between

$\vec{A}$ *and* $\vec{B}$ in Figure 3.11

$(\vec{A}\,x\,\vec{B}) = \vec{C}$

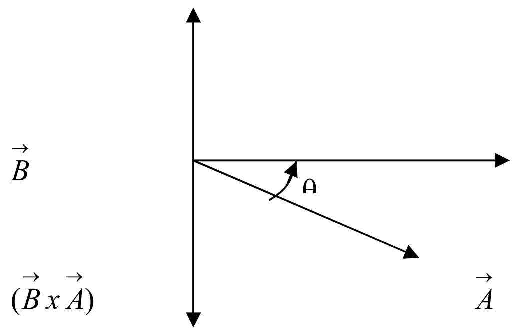
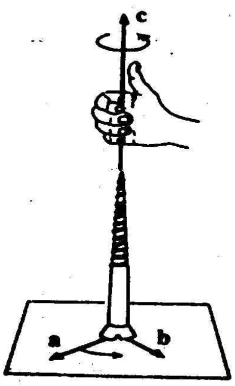


Fig 3.11

The expression $\vec{A}\,x\,\vec{B}$ is pronounced as $\vec{A}$ cross $\vec{B}$.

The magnitude of $\vec{A}$ is□, where □ is A sin, i.e.□the angle smaller than or equal to

$0 \le \theta \le \pi$. Here $\hat{c}$ is a Unit vector perpendicular to $\vec{A}$ and $\vec{B}$ the sense or direction of $\hat{c}$ is given by the right-hand rule: Rotate the fingers of your right hand so that finger tips point along the direction of rotation of $\vec{A}$ into $\vec{B}$ through $\theta\,(\le \pi)$ .the thumb gives the direction of



(Fig. 3.29)

Defined in this way, $\vec{A}, \vec{B}$ and $\vec{C}$ are said to form a right-handed triple or a right-handed triad. Now, think of how you unscrew the cork of a bottle. Unscrewing means turning the cork anti clockwise. The unscrewing motion is like the right hand rule and you notice that the cork moves upward perpendicular to the direction of unscrewing wise motion of the car. Also, unscrewing means anti clockwise motion while screwing means clockwise which will make the cork more vertically downwards in opposite direction to the 1st case.

Note that in the definition of the cross product, the order of $\vec{A}$ and $\vec{B}$ is very important. Thus $\overline{B} x A$ is not the same as $\vec{A} \text{x} \vec{B}$ (Fig.). In fact, you can use the right hand rule to show that

$$\vec{A} x \vec{B} = -\vec{B} x \vec{A} \qquad 3.42$$

We conclude that the vector product is not commutative.

Some properties of the vector product are:

1. $\vec{A} x \vec{B}$ is a vector
2. $\vec{A} x \vec{B} = -\vec{B} x \vec{A}$ **3.43**

3. If $\vec{A}\, and\, \vec{B}$ are non-zero vectors, and $\vec{A} x \vec{B} = 0$ then $\vec{A}$ is parallel to $\vec{B}$ 3.44

4. A x A = 0, for any vector $\vec{A}$ 3.45

   The properties (3) and (4) follow directly from the is zero.□definition because in both cases

   $$\vec{A} x(\vec{B} + \vec{C}) = (\vec{A} x \vec{B}) + (\vec{A} x \vec{C}) \qquad 3.46$$

5. $(\vec{A} + \vec{B}) x \vec{C} = (AxC) + (BxC)$ 3.47

   That is, the vector product is distributive over addition. Notice that the order in which these vectors appear remains the same.

6. $(m\vec{A}) x \vec{B} = M(\vec{A} x \vec{B}) = \vec{A} x(m\vec{B})$ 3.48

SELF-ASSESSMENT EXERCISE 7

If 2 = 0^0, what is $\vec{A}\, \mathbf{x}\, \vec{B}$ = 90 and if ^{0}, what is $\vec{A}\, \mathbf{x}\, \vec{B}$

Solution

If $\vec{A}$ and $\vec{B}$ are given in terms of the Unit vector,

$$then\ \vec{A}\,x\,\vec{B} = (a_1i + b_1j + c_1k)\,x\,(a_2i + b_2j + c_2k)$$
$$= a_1a_2i\,x\,i + a_1b_2ix\,j + a_1c_2i\,x\,k$$
$$+ b_1a_2j\,xi + b_1b_2i\,x\,j + b_1c_2jx\,k$$
$$+ c_1a_2k\,x\,i + b_1b_2\,k\,x\,j + c_1c_2kx\,k \qquad 3.49$$

But i x i = 1. 1 sin 2 = 1. 1 sin 0^0 = 0 3.50

We see that

2 i x i = j x j = k x k = 0 3.51

Also i x j = 1. 1 sin 90^0 1 in direction 0z
i.e. i x j = k i x j = k; j x k = i, k x i = j 3.52

Also, remember that
i x j = - (j x i)
j x k = - (k x j)} since the sense of rotation is reversed 3.53
k x i = - (i x k)

Now applying the result of 3.51 and 3.52 and the expressions, you can simplify the expression for $\vec{A}\,\mathbf{x}\,\vec{B}$. We see that what is left is

$$\vec{A}\,\mathbf{x}\,\vec{B} = (b_2c_2 - b_2c_1)\,i + (a_2c_1 - a_1c_2)\,j + (a_1b_2 - a_2b_1)\,k \qquad 3.54$$

This last expression may remind you of the pattern of expression of determinant.

So we now have that
If

$$\vec{A}\,a_1i + b_1j + c_1k\ and\ \vec{B} = a_2i + b_2j + b_1k \qquad 3.54$$

then in determinant form, it is written as $A\,x\,B = \begin{vmatrix} l & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$

This is the easiest way to write out the vector product of two vectors.

Note: (i) the top raw consists of the Unit vectors in order i, l, k
(ii) the second row consists of the coefficients of $\vec{A}$
(iii) the third row consist of the coefficients of $\vec{B}$

Self Assessment Exercise 3.8

If $\vec{P} = 2_l + 4j + 3k \; and \; \vec{Q} = li + 5j - 2k$

what is $\vec{P} \, x \, \vec{Q}$

Solution*:* First, write down the determinant that represents the vector $\vec{P} \; x \; \vec{Q}$

$$\vec{P} \, x \, \vec{Q} = \begin{vmatrix} i & j & k \\ 2 & 4 & 3 \\ 1 & 5 & -2 \end{vmatrix}$$

Expand the determinant to get

$$\vec{P} \, x \, \vec{Q} = i\begin{vmatrix} 4 & 3 \\ 5 & -2 \end{vmatrix} - j\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} + k\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix}$$

$$= i(-8 - 15) - j(-4 - 3) + k(10 - 4)$$

$$= -23i + 7j + 6k \qquad 3.55$$

Finally, note that the result of the cross product of two vectors is a vector quantity. You should always remember this property of vector product.

4.0 CONCLUSION

What you have learnt in this Unit concerns

a) the determination of the magnitude of a vector in space.
b) how to resolve a vector into its components in three mutually perpendicular axes.
c) the determination of the direction cosines of a vector
d) the resolution of vectors in terms of their Unit vectors
e) the determination of the scalar (dot) product
f) the determination of the vector (cross) product of two vectors

5.0 SUMMARY

In this Unit, you have learnt that

- For vectors $\vec{A} \; and \; \vec{B}$ their magnitude and direction can be expressed in terms of their components and Unit vectors in three-dimensional Cartesian coordinate system as

$\vec{A} = a_1 i + a_2 j + a_3 k$

$|\vec{A}| = A = \sqrt{a_1^2 + a_2^2 + a_3^2}, \tan\theta = \frac{a_2}{a_1}$

$\vec{B} = b_1 i + b_2 j + b_3 k$

$|\vec{B}| = B = \sqrt{b_1^2 + b_2^2 + b_3^2}$

The direction cosines (l, m, n) for $\vec{B}$, say, is

$$l = \cos = \frac{b_1}{B}, \; m = \cos\beta = \frac{b_2}{B} \; and$$

$$n = \cos r = \frac{b_3}{B}$$

Here, a_i and b_i are the components of $\vec{A}$ *and* $\vec{B}$. And i, j, k are the Unit vectors along the positive x, y and z axes. Here also, angle $\vec{A}$ makes with x-axis. A and B are the angles, , And $\vec{B}$ makes with the x, y and z-axes respectively.

- The scalar product of two vectors are defined as

$\vec{A}.\vec{B} = AB\cos\theta$

where 2 in the angle between $\vec{A}$ *and* $\vec{B}$ such that $0 \le \theta \le \pi$

In component form, for

$\vec{A} = a_1 i + b_1 j + c_1 k$ *and* $\vec{B} = a_2 i + b_2 j + c_2 k$

$\vec{A}.\vec{B} = a_1 a_2 + b_1 b_2 + c_1 c_2$

- The vector product of two vectors $\vec{A}$ *and* $\vec{B}$ is defined as

$\vec{A} \; x \; \vec{B} = (AB \sin\theta) \; \hat{c} = \vec{c}$

Where 2 is the angle between

$\vec{A}$ *and* $\vec{B}$ such that

$0 \le \theta \le \pi$. The direction of $\hat{c}$ is obtained by the right hand rule.

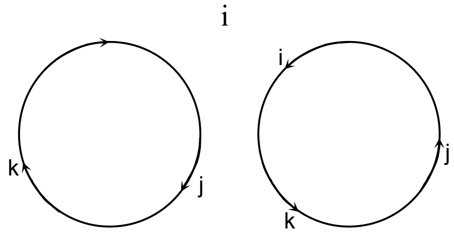
In component form,

$$\vec{A} \, x \, \vec{B} = (b_1 c_2 - b_2 c_1) i + (a_2 c_1 - a_1 c_2) j + (a_1 b_2 - a_2 b_1) k$$

$$= \begin{vmatrix} i & j & k \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

- Rule of thumb for taking cross product of two vectors. You would have observed a cyclic pattern in the cross products.

i x j; j x k; k x i



Going clockwise direction round the circle all vector products are positive i.e. i x j = k and so on.

For anticlockwise direction, the vector products are negative i.e. j x i = - k and so on

6.0 TUTOR-MARKED ASSIGNMENT

1. Find a Unit vector in the yz plane such that it is perpendicular to the vector $\vec{A} = \hat{i} + \hat{j} + \hat{k}$

2. Find the direction cosines [l, m, n] of the vector $\vec{r} = 3i - 2j + 6k$

3. If $\vec{P} = 2i + 4j + 3k$ *and* $\vec{Q} = 1i + 5j - 2k$ what is $\vec{P}\, x\, \vec{Q}$

7.0 REFERENCES/FURTHER READING

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UNIT 5 LINEAR MOTION

CONTENT

1.0 Introduction
2.0 Objectives
3.0 Main Content
3.1 Definition of Motion
3.2 Motion in a Straight Line and Parameters for Describing motion
3.2.1 Displacement
3.2.2 Velocity
3.2.3 Acceleration
4.0 Conclusion
5.0 Summary
6.0 Tutor-Marked Assignment
7.0 References/Further Reading

1.0 INTRODUCTION

The topic of this Unit is what we do every day of our lives and that is motion. A living thing that does not undergo one form of motion or the other is assumed to be dead. So, nothing characterises our daily lives more than motion itself. Understanding motion is one of the key goals of physical laws. That is why we always begin the study of physics and the physical world with mechanics which is the science of motion and it's causes.

No doubt, you have studied motion at the secondary school level years ago. That could be termed as just scratching the subject. In this Unit and the subsequence one's, we shall go into more details on the topic. This Unit will treat rectilinear motion, that is, motion in one dimension (straight line motion) in more details. The topics covered in Units 1 to 4 will help you to understand this unit better-so, relax. The stage has already been set for you. I wish you happy reading. At the back of your mind, as you read, just remember what happens every day: aero planes fly, cars move, pedestrians walk, athletes run etc.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define in scientific terms what motion is.
- define the velocity and acceleration of a particle undergoing rectilinear motion.
- distinguish between average and instantaneous velocity of a particle undergoing rectilinear motion
- state the laws of motion
- solve problems concerning rectilinear motion of objects using the laws of motion

3.0 MAIN CONTENT

3.1 Definition of Motion

Let us begin the study of this Unit by asking the question “what is motion”?. Maybe you are wondering why this question when it is so obvious to everybody as something we do every day. Besides in unit 1, we learnt that everything in the universe is in motion continuously. So what is motion? We say that an object is moving if it changes its location at different times. This means that our study of motion will deal mainly with questions like where and when?

Definition: Motion may be defined as a continuous change of position with time.

During motion, we notice that different points in a body move along different paths. Let for simplicity we shall consider motion of a very small body which we shall refer to as a particle. The position of a particle is specified by its projections onto the three areas of a Cartesian coordinate system. As the particle moves along any path in space, its projections move in straight lines along the three axes. The actual motion can be reconstructed from the motions of these three projections. But firstly, we shall discuss one dimensional motion also known as rectilinear motion and later extend it to two and three dimensional motions.

3.2 Motion in a Straight Line and Parameters for describing Motion

This section describes motion in a straight line and the parameters for describing motion. These are displacement, velocity and acceleration.

3.2.1 Displacement

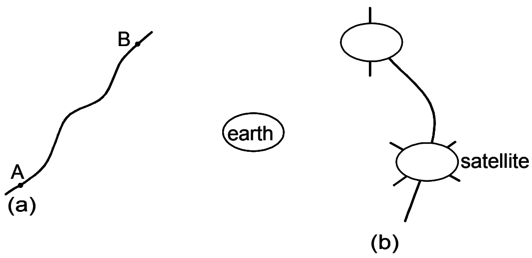

Fig. 3.1

If you run on a winding path from point A to point B (Figure 3.1a) and travel a distance of 240m in 20 seconds, then your average speed is

$$Speed = \frac{240m}{20\sec s} = 12m\ 5^{-1} \qquad 3.1$$

Similarly, a car that takes 2 hours to travel from Lokoja to Abuja along a winding road, a distance of 200km is said to have an average speed given by

$$Speed = \frac{distance}{time} = \frac{200km}{2hrs} \qquad 3.2$$
$$= 100km\ h^{-1}$$

An object changes its position at a uniform rate without reference to its direction. In other words, speed is what we call a scalar quantity. Also if a satellite (Fig. 3.1) revolves round the earth covering a circular path 60,000km in 24 hours its average speed is

$$Average\,speed = \frac{60,000km}{24h}$$
$$= 2500km\ h^{-1}$$

But if the satellite moves through equal distance in equal times, no matter how small the time intervals the satellite is said to have a constant or uniform speed.

In going from one point to another irrespective of the path taken to do the journey the motion is said to be over a distance A to B. For example the cases cited in Fig.3.1 such a journey undertaken during a time interval possess speed. Distance does not have any specified direction, hence it is a scalar quantity.

3.2.2 Velocity

A particle or car travelling between two locations and limited to make the journey in a specified direction, say 30^0 due North in some time interval is said to posses velocity, because

$$velocity = \frac{displacement}{Time} \qquad 3.3$$

Consider a particle moving along the x-axis as in Figure 3.2a above. The curve in Figure 3.2b shows the graph of its displacement with time. At time t, the particle is at point P in Figure 3.2a where its coordinate is x_1. At a later time t_2 whose coordinate is x□it has moved to point $_2$.

The corresponding points on Figure 3.2b are labelled p and q.
$\bar{v}$ The displacement of this particle is then given by the x with magnitude x□vector $_2$ - x_1 along a specified direction the x-axis which is a straight line. The average velocity, of the particle is defined by

$$\vec{v} = \frac{\Delta x}{\Delta t} \qquad 3.4$$

$\vec{\vee} t$ where $\Delta x = x_2 - x_1$, is the displacement and Δt is the time interval between when Δt the particle is at point P and when it is at point Q. Note that average velocity here is a vector quantity because is a vector quantity since is a scalar quantity. The

direction of is the same as the direction of the displacement vector. The magnitude of the average velocity is

$$\left|\overline{\vee}\right| = \vee = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} \qquad 3.5a$$

In figure 3.2b, the average velocity is represented by the slope of the chord pq given by the ratio of

$x_2 - x_1, or \Delta x \ to \ t_2 - t_1 \ or \ \Delta t$

$i.e.$

$$Slope = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t} = \overline{\vee} \qquad 3.5b$$

This is exactly the same as we got from figure 2a. Rewriting equation 3.5a we get that

$$x_2 - x_1 = \overline{\vee}(t_2 - t_1) \qquad 3.6$$

If we take the time the particle starts its journey to be time t = 0, then the corresponding position is taken as x_0 (i.e. initial position). After a later time t, the particle is taken to be at position x then equation 3.6 becomes

$$x - x_0 = \vec{\vee} t \qquad 3.7$$

Now, if the particle is at the origin when t = 0, then $x_0 = 0$ and equation 3.7 reduces to

$$x = \vec{\vee} t \qquad 3.8$$

Instantaneous velocity

Δt The velocity of a particle at someone instant of time, or at some one point of its path, is called its instantaneous velocity. We have seen that average velocity is associated with the entire displacement and the entire time interval. When the point Q is taken to be closer and closer to point p, the average velocity could be computed for very small time intervals- until a limiting time interval is reached. This limiting time interval we refer to as an instant of time. Hence we define instantaneous velocity as

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \qquad 3.9$$

Δx Instantaneous velocity is also a vector quantity whose direction is the limiting direction of the displacement vector. By convention, a positive velocity indicates that it is towards the right along the x-axis of the coordinate system.

Note that as point Q approaches point p in Figure 3.2a, point q approaches point p in Figure 3.2b, in the limiting case the slope of the chord pq equals the slope of the tangent to the curve at point p. (Take note that the diagrams have not been drawn to scale).

The instantaneous velocity at any point of a coordinate-time graph therefore equals the slope of the tangent to the graph at that point.

As a rule of thumb, if the tangent slopes upwards to the right , its slope is positive, the velocity is positive, and the motion is to the right. But if the tangent slopes downwards to the right, the velocity is negative. At a point where the tangent is horizontal, its slope is zero and its velocity is zero. If distance is given in meters and time in seconds, velocity is expressed in meters per second (m s^{-1}). Other common units of velocity are:

Feet per second (ft s^{-1}) centimeters per second(cm s^{-1}) miles per hour (mi h^{-1}) and knot (1 knot = 1 nautical mile per hour).

Self-Assessment Exercise 3.1

Suppose the motion of the particle in Figure 3.2 is described as the equation x = a + bt^2, where a = 20cm and b = 4cm s^2,

(a) Find the displacement of the particle in the time interval between t, = 2s and t_2 = 5s

(b) Find the average velocity in this time interval

(c) Find the instantaneous velocity at time t_1 = 2s.

Solution

For (a) at time t_1 = 2s the position is

x_1 = 20cm + (4cm s^2) (2s)

= 36cm

at time t_2 = 5s;

x_2 = 20cm + (4cm s^2)(5s) = 120cm

The displacement is therefore

x_2 - x_1, = (120 - 36)cm

= 84cm

For (b): The average velocity in this time interval is

$$\vec{v} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{84cm}{3s}\ 28cm\ s^{-1}$$

Δt x = 20cm + (4cm s^2) (2s + $)^2$

Δt = 36 cm + (16cm s^{-1}) + (4cm s^{-2}) $()^2$

Δt The displacement during the interval is

Δt^2 = 36cm + (16cm s^{-1}) (4cm s^{-2}) - 36cm

Δt^2 = (16cm s^{-1}) + (4cm s^2)

Δt The average velocity during is

$$\bar{v} = \frac{\Delta x}{\Delta t} = 16cm\ s^{-1} + (4cm\ s^{-1})\,\Delta t$$

Δt For the instantaneous velocity at t = 2s, we let approach zero in the expression for

$\bar{v}$

$$\therefore \bar{v} = 16cm\ s^{-1}$$

This corresponds to the slope of the tangent at point P in Figure 3.2b.

3.2.3 Acceleration: Average and Instantaneous Acceleration

When a body accelerates in motion, it means that its velocity changes continuously as the motion proceeds.

Figure 3.3 shows a particle, P moving along the x-axis. The vector v_1 is its instantaneous velocity at point P and the vector v_2 represents its instantaneous velocity at point Q. Its instantaneous velocities between points P and Q are plotted against time in Figure 3.3b as shown above. Points p and q corresponds to points P and Q in part (a). Thus, the average acceleration of the particle is given as the ratio of change in velocity to the elapsed time i.e

$$\vec{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t} \qquad 3.10$$

$\vec{v}_2 - \vec{v}_1\ \vec{}_2$ where t_1, and t_2 are the times corresponding to the velocities. Note that since are vectors, the quantity is a vector difference and must be found by the method of vector subtraction you learnt in units 3 and 4. But in rectilinear motion both vectors lie in the same straight line. So in this case, the magnitude of the vector difference equals the difference in the magnitudes of the vectors. In Figure 3.3b the magnitude of the average acceleration is represented by the slope of the chord pq.

Δt The instantaneous acceleration, of a body i.e. its acceleration at some one instant of time or at some one point of its path is defined in the same way as for instantaneous velocity. Hence it is defined as the limiting value of the average acceleration when the second position of the particle is taken much closer to the first position as tends to zero.

i.e.

$$\vec{a} = \lim_{\Delta t \to o} \left(\frac{\Delta v}{\Delta t} \right) \qquad 3.11$$

Note that the direction of the instantaneous acceleration is the limiting direction of the vector change in velocity. Instantaneous acceleration plays an important role in physics and is more frequently used than average acceleration. Subsequently, the term acceleration will be used to mean instantaneous acceleration. Acceleration, of course, is a vector quality and the definition just given above applies to whether the path of motion of the particle is straight or curved. In Figure 3.3b the instantaneous acceleration is equal to the slope of the tangent to the curve at any point say, p of a velocity-time graph. If velocity is expressed in metres per second, then acceleration is expressed in metres per square second ($m\ s^{-2}$). Other common units of acceleration are feet per square pound ($ft\ s^{-2}$) and centimeters per square second ($cm\ s^{-1}$).

Note that when a body is slowing down its motion, we say it is decelerating.

SELF-ASSESSMENT EXERCISE 2

Given that the velocity of a particle is $V = m + nt^2$ where $m = 10cm\ s^{-1}$ and $n = 2cm\ s^{-1}$

(a) find the change in velocity of the particle in the time internal between $t_2 = 2s$ and $t_2 = 5s$

(b) find the average acceleration in this time interval.

(c) find the instantaneous acceleration at time $t_1 = 2s$

Solution:

Given $v = m + nt^2$

for (a) : At time $t_1 = 2s$ (m = 10cm 5^{-1}, n = 2cm

$v_1 = 10cm\ s^{-1} + (2cm\ s^{-3})(2s)^2$

$= 18cm\ s^{-1-}$

At time $t_2 = 5s$

$V_2 = 10cm\ s^{-1} + (2cm\ s^{-3})(5s)^2$

$= 60cm\ s^{-1}$

The average velocity is therefore

$v_2 - v_1 = (60 - 18)cm\ s^{-1}$

$= 42cm\ s^{-1}$

for (b)

$$\vec{a} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{42cm\ s^{-1}}{3s} = 14cm\ s^{-2}$$

This corresponds to the slope of the chord pq in Figure 3.3b

For (c) at time t = 2s + Δt

$v = 10cm\ s^{-1} + (2cm\ s^{-3})(2s + \Delta t)^2$

$= 18cm\ s^{-1} + (8cm\ s^{-2})\ \Delta t\ (2cm\ s^{-3})\Delta t^2$

t is□The change in velocity during

-18cm s^{-1}

= (8cm s^{-2}t□) + (2cm s^{-3}t)□)(²

t is□Hence, the average acceleration during

v = 18cm s□$^{-1}$ + (8cm s^{-2}t +□) (2cm s^{-3}t)□)(²

$\vec{a} = \Delta\vec{v} = 8cm\,s^{-2}\ \Delta t + (2cm\ s^{-3})\ (\Delta t)^2$

$\vec{a}\ t \to 0$ at time t = 2 as is

$\vec{a} = 8cm\ s^{-2}$.

This corresponds to the slope of the tangent at the point p in Figure 3.3b

3.2.4 Rectilinear Motion with Constant Acceleration

Rectilinear motion with constant acceleration means that the velocity of the particle changes at the same rate throughout the motion. The velocity time graph is then a straight line.

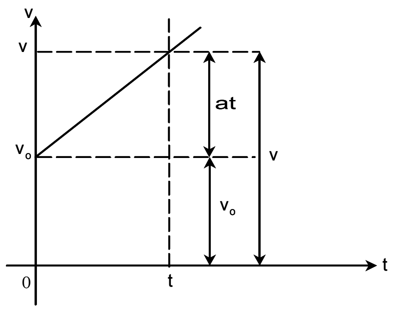

Fig. 3.4

Fig. 3.4: Velocity time graph for readilinear motion with constant acceleration

Since the slope of the chord between any two points on the line are the same, the average velocity is the same as the instantaneous acceleration in this case. Hence eg 3.10 can be replaced by

$$\vec{a} = \frac{\vec{V_2} - \vec{V_1}}{t_2 - t_1} \qquad 3.12$$

Now let $t_1 = 0$ and t_2 = any arbitrary time. Let v be the velocity at time t = 0 and v the velocity at time t.

Then eqn. 3.12 becomes,

$$a = \frac{v - v_0}{t - 0}$$

or simply,

$$v = v_0 + at \qquad 3.13$$

Hence, we say that acceleration is the constant rate of change of velocity or the change in velocity per unit time.

Note that for motion with constant acceleration, we have that average velocity between time t = 0

and time, t is

$$\bar{v} = \frac{v_0 + v}{2} \qquad 3.14$$

When the acceleration is not constant the velocity time graph is curved as in Figure 3.3.

Recall that by definition

$$\vec{v} = \frac{x_2 - x_1}{t_2 - t_1}$$

for time t = 0 and a later time t

Let x_0 be the position at t = 0 (initial position) and let x be the position at time t (final position). Then, the proceeding equation becomes

$$x - x_0 = \vec{v}\, t \qquad 3.15$$

3

substituting the expression for v in equation 3.14 gives

$$x - x_0 = \left(\frac{v_0 + v}{2}\right)t \qquad 3.16$$

Then using equation 3.13 and 3.16 to eliminate v and t and substituting for v in equation 3.13. We get

$$x - x_0 = (\frac{v_0 + v_0 + at}{2})$$

or

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \qquad 3.17$$

Now solving eqn. 3.13 for t and putting the result in eqn. 3.16 we have

$$x - x_0 = (\frac{v_0 + v}{2})(\frac{v - v_0}{a}) = (\frac{v^2 - v^2_0}{2a})$$

or

$$v^2 = v^2_0 + 2a(x - x_0) \qquad 3.18$$

Note that the following equations 3.13, 3.16, 3.17 and 3.18 are called the equations of motion with constant acceleration. You are required to know these equations of motion by root so that you can easily apply them in solving problems in physics.

SELF-ASSESSMENT EXERCISE

A boy rolls a ball along a flat straight platform. The ball possesses an initial velocity of $2m5^{-1}$ when the boy release it and it shown down with constant negative acceleration of -0 2m 5^{-2}. How far does the ball roll before stopping, and how long does it take to stop?

Solution

Choose a coordinate system with x = at the point where the ball leaves the boy's hand, and start the stop watch (clock) at t = 0 when the ball leaves his hand. The aim is along the direction of the balls motion. The initial conditions are then

$x_0 = 0m$
$t_0 = 0s$
$v_0 = 2m5^{-1}$

The acceleration is negative along t x-direction and has constant value a = - 2m 5^{-1}

Now, we know the initial and final velocities (zero) as well as the acceleration. We do not know the final position of the ball or the time elapsed. So, how world you solve this problem? We look at the equations of motion and find out by process of elimination which are to apply here to help us arrive at the answer. We see that we need eqns. (3.18) and 3.13

Hence,

$$x = x_0 + \frac{V^2 - V_0^2}{2a}$$ Substituting our values we get

$$x = \frac{(0m) + (0m/s)^2 - (2m\,5^{-1})^2}{2(-0.2m\,5^{-2})}$$

$= 10m$ *Answer*

Also for the second part

$$t = \frac{v - v_0}{a}$$

i.e. $$t = \frac{(m5^{-1}) - (2m\,5^{-1})}{-0.2m\,5^{-2}}$$

$= 10s$ *Answer*

Now, having concluded this section, let us move on to the discussion of motion in more than one dimension for which you will find your knowledge of resolution of vectors treated in units 3 and 4 very.

4.0 CONCLUSION

a) In the unit you have learnt that
b) Motion involves change in the position of an object with time.
c) The language used to describe motion is distance displacement, velocity and acceleration.
d) To define velocity and acceleration of a moving particle
e) You have also learnt how to compute the displacement, velocity and acceleration of a
f) body in motion alone dimensional axis-the x-axis.
g) To state the laws of motion
h) To solve problem concerning rectilinear motion of particles using the laws of motion.

5.0 SUMMARY

What you have learnt in this unit are:
In discussing distance and speed a body changes position at a uniform rate without reference to its direction i.e. $\frac{\text{distance}}{\text{time taken}} = \text{speed}$

Speed is a scalar quantity
In discussion displacement and velocity a body changes position at a uniform rate with reference to a specified direction e.g. Motion due east or 45^0 west of North.

$$velocity\,\vec{v} = \frac{displacement}{time\,taken} = \frac{\Delta \vec{x}}{\Delta t}$$

Velocity is vector quantity
That instantaneous velocity is the velocity of a particle at any one instant of time. It is also the limiting value as the time interval two positions (initial + final positions) of a particle tends to zero i.e.

$$\frac{\rightarrow}{\vee} = \frac{\lim}{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t}$$

That when the velocity of a particle changes with time it results in the acceleration or deceleration of the particle depending or whether the motion is increasing in speed or decreasing i.e.

$$\frac{\rightarrow}{a} = \frac{\overrightarrow{V_2} - \overleftarrow{V_1}}{t_2 - t_1} = \frac{\Delta \frac{\rightarrow}{V}}{\Delta t}$$

$$\frac{\rightarrow}{a} = \frac{\lim}{\Delta t \rightarrow 0} = \frac{\Delta \frac{\rightarrow}{\vee}}{\Delta t}$$

That instantaneous acceleration is the acceleration of a particle at someone instant of time i.e. . It is also given by the slope of the tangent to the curve at any point of a velocity-time graph.

That constant acceleration results from when the velocity of the particle changes at the same rate throughout it motion.
That the equations of motion are given by

1. $\vee = \vee_0 + at$
2. $x - x_0 = (\frac{\vee_0 + \vee}{2})t$
3. $x - x_0 = \vee_0 t + \frac{1}{2} at^2$
4 $\vee^2 = \vee_0^2 + 2a(x - x_0)$

$\vee_0$ where x is the displacement of the particle, is the velocity at time t = 0 i.e. $t_{0;}$ V is the velocity at a later time t ; a is the acceleration of the particle.

6.0 TUTOR-MARKED ASSIGNMENT

1. A runner bursts out of the starting blocks 0.1s after the gum signals the start of a race. She runs at constant acceleration for the next 1.9s % the race. If she has gone 8.0m after 2.0s, what are her acceleration and velocity at this time?
2. Table 1 Times for 100m Race

Distance (m)	Time(s)
0	0
5	1.50
10	2.00
15	2.50
20	3.10
25	3.60
30	4.10
35	4.60
40	5.00
45	5.50
50	6.00
55	6.50
60	7.00
65	7.50
70	7.70
75	8.20
80	8.70
85	9.10
90	9.60
95	10.00
100	10.50

For table 1 above, using graphical techniques determine the velocity at times t = 2s.

3. Suppose that a runner on a straight track covers a distance of 1mile in exactly 4 minutes. What was his average velocity in (a) mi h^{-1} ? (b)ft 5^{-1} ? (c) cm 5^{-1} ?
The Answers are
(a) 1.5m h^{-1} ; (b) 22 ft 5^{-1} ; (c) 672 cm5^{-1}.

4. A body starts from zero and attains a velocity of 20m 5^{-1} in 10s. It continues with this velocity for the next 20s until it is brought to rest after another 10s. Sketch the -Time graph for the motion and find the acceleration and the distance covered during the motion.
Ans: acceleration = 2m 5^{-2}
Retardation = - 2m 5^{2}
distance covered = 600m

7.0 REFERENCES/FURTHER READING

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Indira Gandi National Open University School of Sciences. PHE 01, Elementary Mechanics.