UNIT 1: GRAPHS

TABLE OF CONTENTS

1.0 INTRODUCTION

As a physics student, you will be involved in observing some physical phenomena and measuring physical quantities such as length, mass, time, temperature and current of electricity. These are called fundamental physical quantities.

You should also note that such fundamental quantities are used to obtain derived quantities such as force, velocity, pressure, density, etc.

Through such measurements we are able to learn more about nature. We are able to measure some constants about nature for example acceleration due to gravity, the resistance of a wire, and specific heat capacity of a substance. We determine these constants by identifying how two variables are related through the use of graphs.

Thus in your practical sessions in the laboratory, you will be developing practical skills in measuring physical quantities and the show the relationship between the two physical quantities to be measured through the use of graphs.

In this unit therefore, you will be introduced to the various types of graphs and how to use such graphs to obtain the physical constants required.

2.0 OBJECTIVES

At the end of this unit you should be able to:

- identify the importance of graphs in the study of physics;
- identify the two variables as physical quantities to be measured,
- show how the two variables are related either linearly or non-linearly,
- identify linear relationship from non-linear relationship;
- translate non-linear relationships to linear relationships,
- use graphs to determine physical constants through the use of slopes (gradients) or intercepts.

How to Study this Unit

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

3.0 GRAPHS

In physics, we are always interested in knowing how two variables are related to each other.

There are two types of variables

- independent variable
- dependent variable

For example, if the value of quantity P depends on the value of quantity Q, then, Q is the independent variable while P is the dependent variable.

For example, the change in position of an object with respect to time; time (t) is regarded as the independent variable while the change in position (x) is regarded as the dependent variable.

If you therefore measure time (t) of a moving object and its corresponding change in position, it is possible to show the relationship between x and t. That is it is possible to show how x relates with t by means of a graph.

A graph, therefore gives a vivid picture of how two physical quantities are related. The following are the advantages of graphs:

- if t is plotted against x the relationship between the two variables is shown I in a pictorial form, the corresponding values of x and t other than those actually observed or measured can be read from the graph, it is possible to verify a known or expected relationship between x and t and to determine the numerical values of constants occurring in it, in many cases, the relationship between x and t is of a simple form and is not known or is suspected before hand, a graph enables us to discover this form of relationship.

3.1 TYPES OF GRAPHS

The following are the types of graphs you will come across.

3.1.1 LINEAR GRAPH THROUGH THE ORIGIN

Consider the equation

$$y = mx$$

where, x is the independent variable y is the dependent variable
m is a fixed number

When x=0 then y=0
When x = I then y = m
When x = 2 then y = 2m
When x = 3 then y = 3m and so on.

If we therefore plot the graph of y on the vertical axis and x on the horizontal axis we would obtain a graph as shown in fig. 3.1.

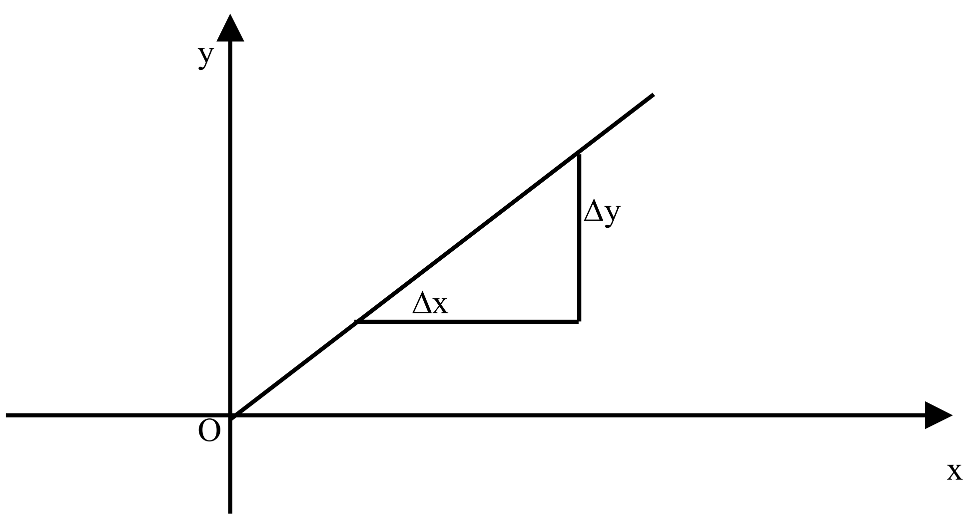

Fig. 3.1

We would have a straight line graph passing through the origin.
You will observe that the ratio of y = m = a constant that is y is directly proportional to x.

The gradient or the slope of the graph is obtained from this ratio:
Increase in y - Increase in y = m = Δy
Increase in x Δx

Thus a straight line through the origin shows that the quantity y is directly proportional to x. The constant of proportionality, m is given by the slope or the gradient of the graph.

3.1.2 LINEAR GRAPH NOT PASSING THROUGH THE ORIGIN

Consider another equation $y = mx + b$
where, x = independent variable
y = dependent variable m and b are constants
When $x = 0$ then $y = b$ When $x = 1$ then $y = m + b$ When $x=2$ then $y=2m+b$
When $x = 3$ then $y = 3m$ +band soon.
Again if we plot the graph of y on the vertical axis against x on the horizontal axis we also obtain a straight line. It will be noted that this time the graph does not pass through the origin as shown in fig. 3.2.

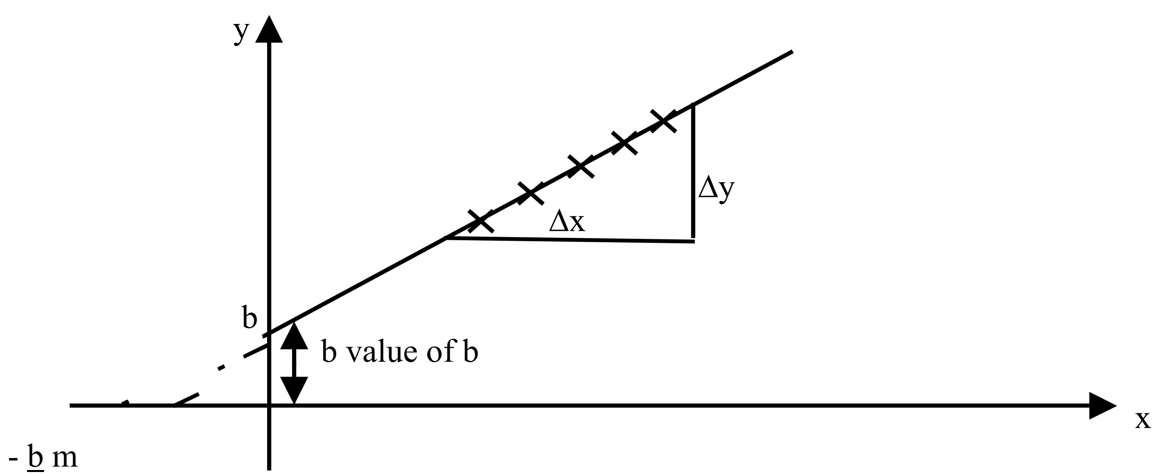

Fig. 3.2

You will also observe that when $x = 0$, $y = b$ gives the intercept of the graph on the y-axis.
When $y = 0$ then $x = -\frac{b}{m}$ gives the intercept of the graph on the x-axis.
The relationship between y and x is also linear. It is however to be noted that this is not a direct relationship.

The slope or the gradient of the graph is given by

$\frac{\text{Increaseiny}}{\text{Increase in x}} = \frac{\Delta y}{\Delta x}$ = while b is the intercept on the axes

If the line slopes downwards as shown in fig. 3.3, then the graph is said to have a negative slope.

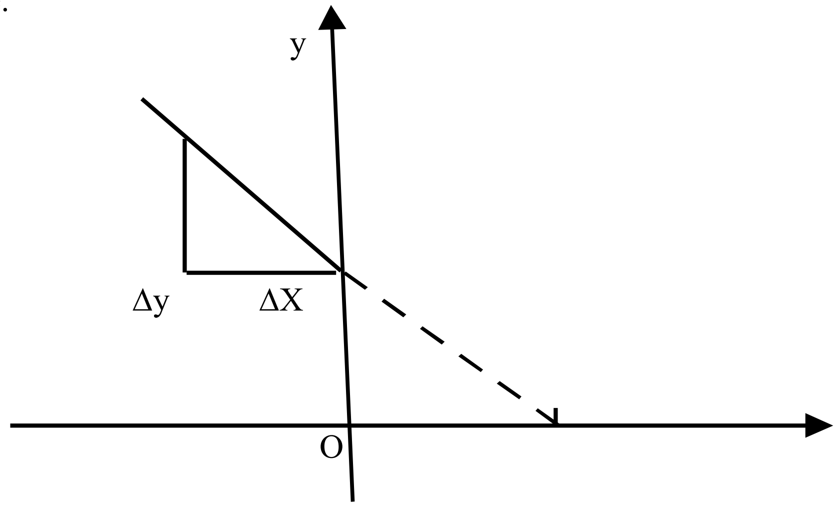

Fig 3.3

Then y = -mx + b .
In conclusion, therefore, whenever the graph of y against x is a straight line, this shows that the relationship between y and x is linear and can be expressed as y=mx+b

If the graph passes through the origin when b = 0 then y is directly proportional to x.

The slope or gradient = $\frac{\text{Increase iny}}{\text{Increase in x}}$ = m

3.1.3 REDUCING NON-LINEAR EQUATION TO LINEAR EQUATION

Consider the expression that relates the period of oscillation of a simple pendulum (T) with the length (l) of the pendulum.

$$T = 2\pi \, g \sqrt{\frac{l}{g}}$$

If T is plotted against $\sqrt{l}$ there will be no linear relationship between T and $\sqrt{l}$ But on squaring both sides of the equation we would obtain this expression

$$T^2 = 4\pi^2 \frac{l}{g}$$

$$\therefore T^2 = \frac{4\pi^2}{g} l$$

Plotting the graph of T^2 on the vertical axis and ℓ on the horizontal axis will give us a straight line passing through the origin as shown in fig. 3.4.

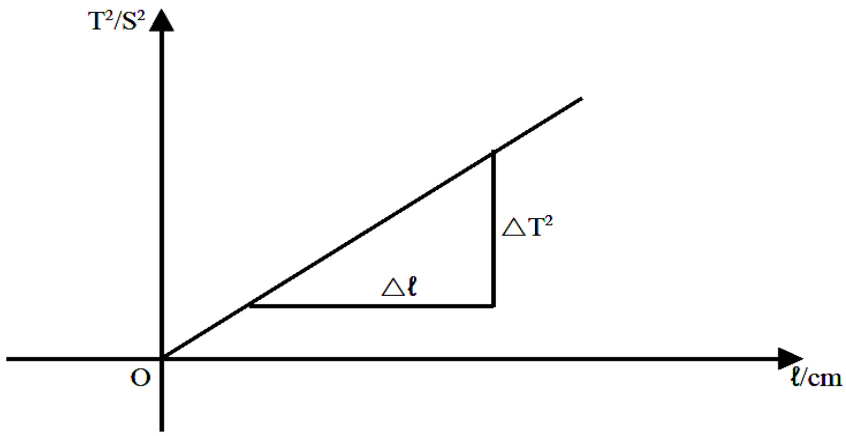

Fig. 3.4

The slope or the gradient of the graph is given as

$$\frac{\Delta T^2}{\Delta \ell} = \frac{\text{Increase in } T^2}{\text{Increase in } \ell} = m = \frac{4\pi^2}{g}$$

The value of g, acceleration due to gravity may then be obtained, by substitution of values of known variables in the relation.

Thus we have reduced a non-linear relationship to a linear one. Another example of this reduction may be found in Boyle's law. Boyle's law shows the relationship between the pressure exerted on a given mass of gas (P) and its volume V provided the temperature of the gas is kept constant.

PV =K

If the values of P are plotted against the corresponding values of V, we do not obtain a linear relationship as shown in fig. 3.5 (i).

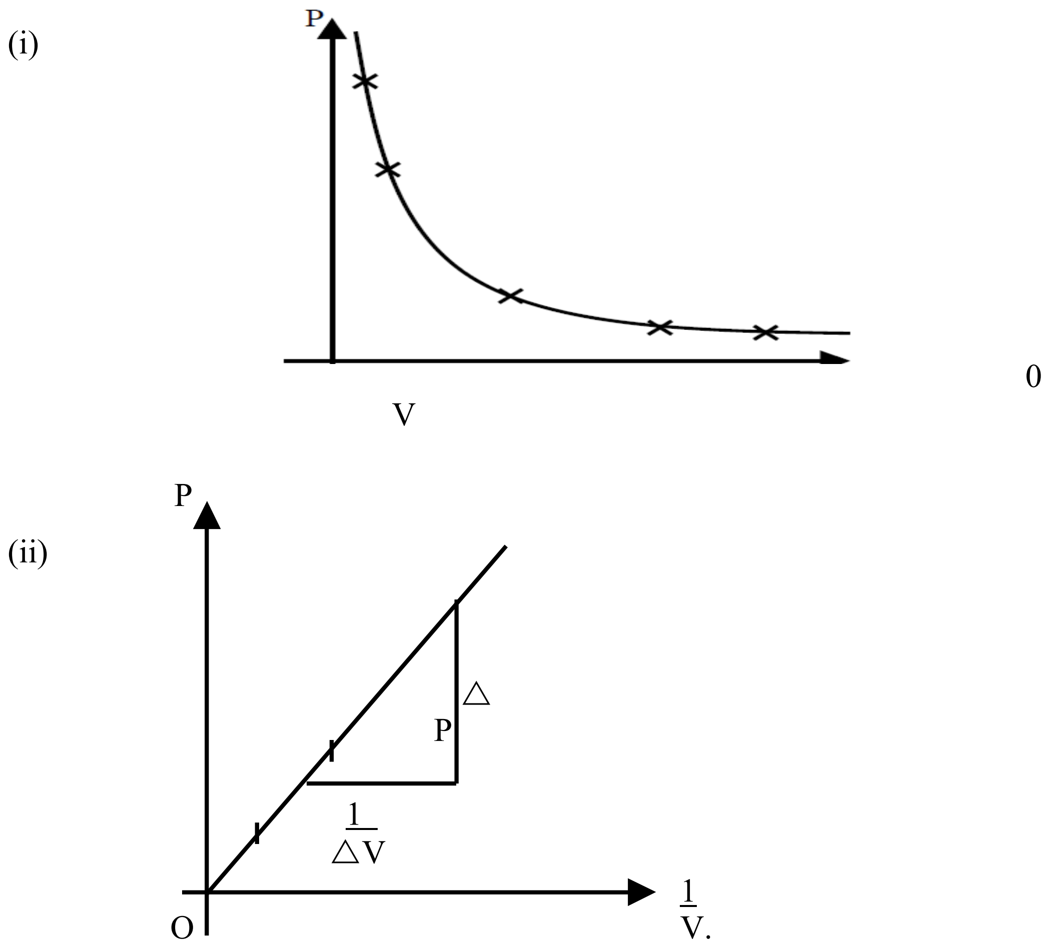

Fig. 3.5

By rearranging the expression we would obtain this

$P = k\ \frac{1}{V}$

When P is therefore plotted against $\frac{1}{V}$ the reciprocal of V, then we obtain a straight line passing through the origin as shown in fig. 3.5 (ii). The gradient or the slope m of the graph is expressed as

$$\frac{\text{Increase in pressure}}{\text{Increase in the reciprocal of volume}} = \frac{\Delta P}{1/\Delta V}$$

$= m$
$= K$ a constant

The oscillation of a weighted spiral spring is another example.
There is a relationship between the period of oscillation T, the loaded mass M and the effective mass of the spiral spring m.

$$T = 2\pi\sqrt{\frac{M+m}{k}}$$

Where, k is the spring constant.

We then reduce the expression to a linear one by squaring both sides of the equation. T^2

$$T^2 = 4\pi^2 \frac{(M+m)}{k}$$

$$\therefore T^2 = \frac{4\pi^2}{k} M + \frac{4\pi^2}{k} m \text{ compare this equation with}$$

$$y = ax + b \text{ where } y \cong T^2,\ \frac{4\pi^2}{k} M \cong ax \text{ and } \frac{4\pi^2}{k} m = b$$

By plotting the values of T^2 against the corresponding values of M, we would obtain a straight line graph which does not pass through the origin as shown in fig. 3.6.

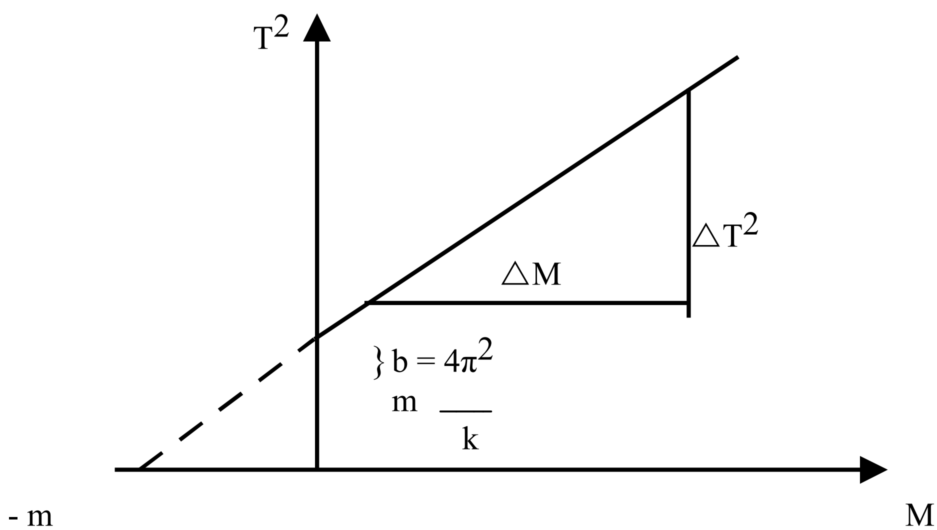

Fig. 3.6

The gradient or the slope of the graph is expressed as

$$\frac{\text{Increase in } T^2}{\text{Increase in } M} = \frac{\triangle T^2}{\triangle M} = \frac{4\pi^2}{k}$$

The intercept on the vertical axis is obtained when M = 0.

:. $T^2 = \frac{4\pi^2}{k} m = b$

The intercept on the horizontal axis is obtained when $T^2 = 0$.

:. M = - m

Which gives us the effective mass of the spiral spring. Thus, through the knowledge of m, k, could be determined.

3.1.4 REDUCING TO A LINEAR EQUATION FROM UNKNOWN RELATIONSHIP

Suppose two physical quantities P and Q are related as

$P = kQ^n$

Where the values of k and n are not known we can reduce the expression to a linear one by taking the logarithm of both sides of the equation.

log P = log k + n log Q or log P = n log

Q + log k

compare it with

y = nx + b where n log Q nx and log k b.

Therefore if we plot log P on the vertical axis against log Q on the horizontal axis, we shall obtain a straight line graph as shown in fig. 3.7.

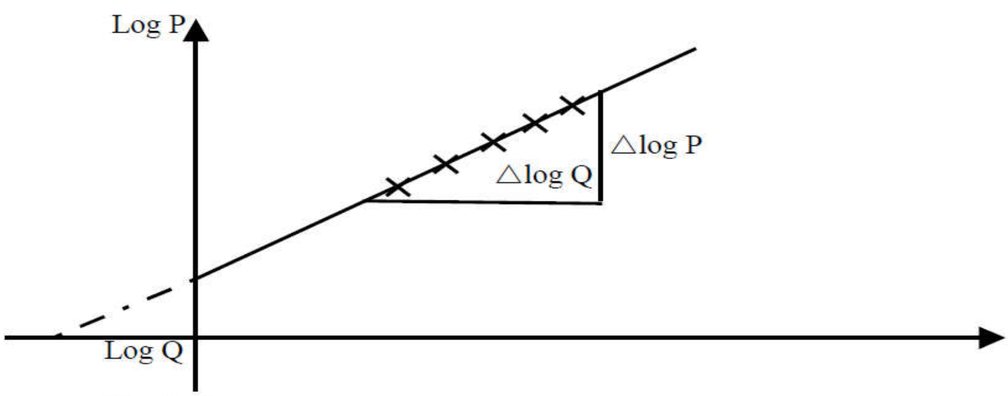

Fig. .3.7

The slope or the gradient of the graph is expressed as

<u>Increase n logP</u> △ log P = n _______

Increase in log Q △ log Q

When log Q = 0 gives the intercept of the graph on the vertical axis log P = log k

Thus the value of k is obtained.
You need to be careful when working with logarithm.

- If the value of P or Q is greater than l, log P is taken as positive.

For example, l o g 9 = 0.9542
log 90 = 1.9542
log 900= 2.9542 etc

- If the value of P or Q is less than 1 then you needs to sensitize yourself on what value of log P or log Q you have to use. For example,

log 0.5 = 1.6990 = -1 + 0.6990 = -0.3010 log 0.006 =
3.7781 = -3 + 0.7781 = -2.22

If the intercept is -2.73
-2.73=-3+x
x=3-2.73=0.27
-2.73 = 3 + 0.2700

The antilog of 0.2700 = 1.86
Antilog of -2.73 = 1.86 x 10^{-3}

3.2 DETERMINATION OF GRADIENT OF NON-LINEAR GRAPH AT A POINT

From the equations
y = mx
y = mx +b
the highest power to which x is raised is 1 hence when y is plotted against x, we obtain a straight line graph.

But for equations such as $y = ax^2$
y = ax + bx $y = ax^2 + c$
they produce non-linear graphs because when you plot the values of y against x, we do not produce a straight line graph. You will observe that the graphs produced are usually parabolic because they are quadratic functions. The equations are described as being quadratic because the highest power to which x is raised is 2. Fig. 3 (i) – (vi) are

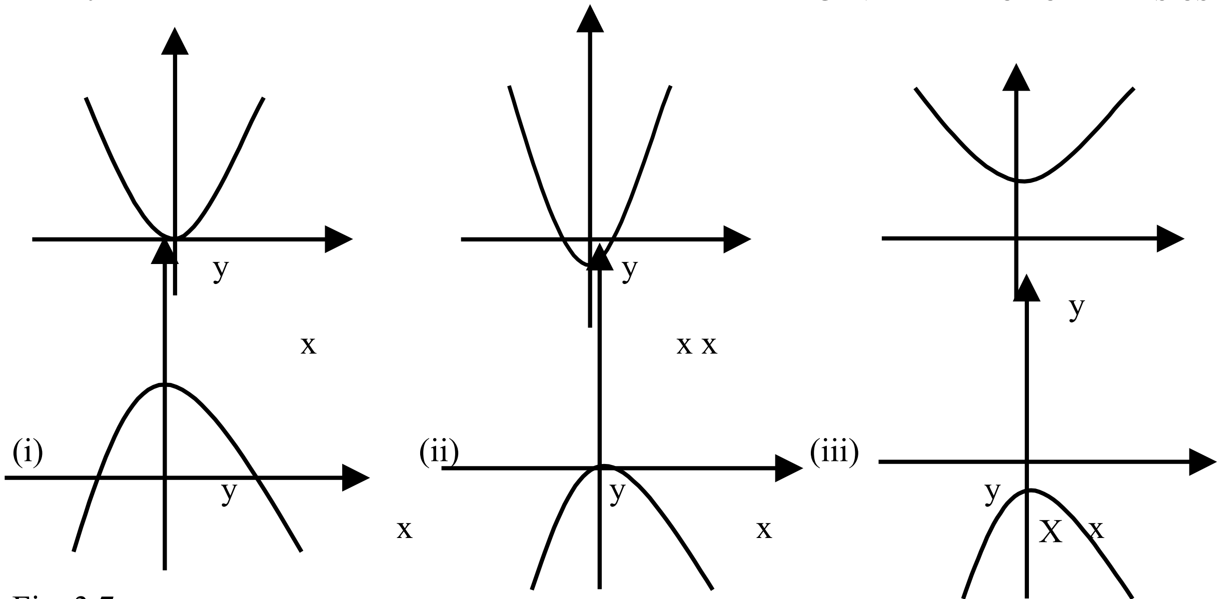

Fig. 3.7

Some typical graphs of such quadratic functions for the values of – x to + x. You will observe that the slopes of the graphs rise and fall with a turning point usually described as the minimum or maximum points of values of y with respect to x.

The slopes or gradients vary from one point of x to the other. At the minimum or maximum point, the gradient at that point is zero.

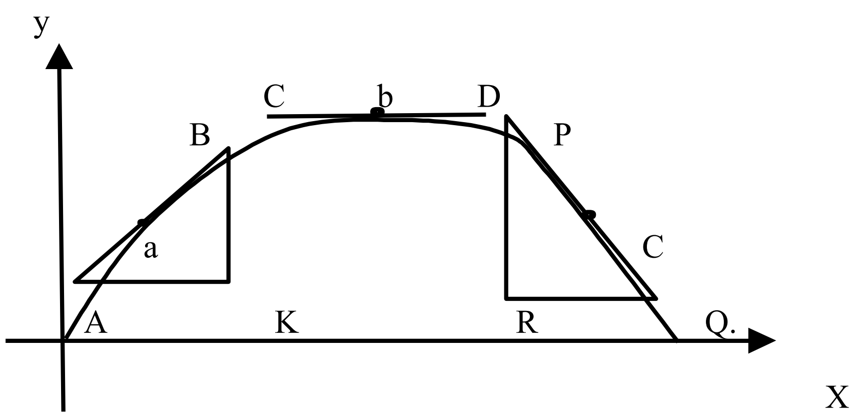

Figure 3:8

The straight lines on the graph at points a, b, and c or the graph in fig. 3.8 define the slopes of the graph at such points. The lines AB, CD and PQ describe the tangents at points a, b and c.

AB gives a positive slope or gradient while PQ produces a negative slope or gradient. The line CD is parallel to the horizontal axis to produce no slope or gradient.

Because the slope or gradient progresses from positive values through zero at b to negative values then point b gives maximum value of y. Otherwise as in fig. 3.9 it is a minimum value.

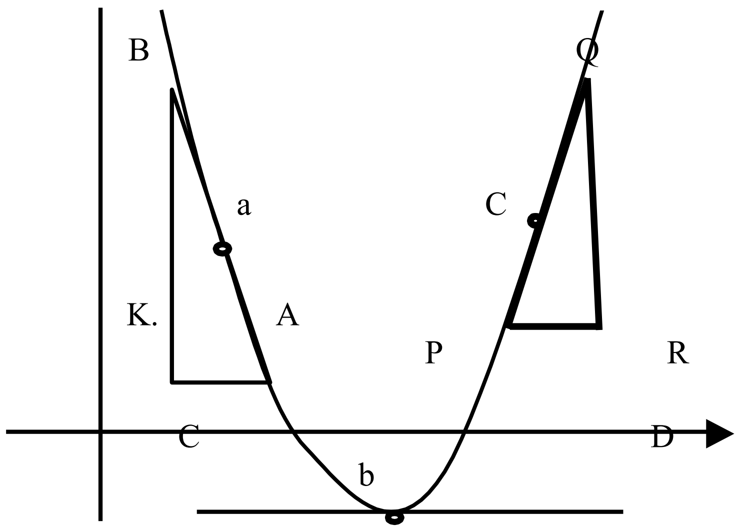

Fig. 3.9

To obtain the gradient at a, we then produce a right-angled triangle on the tangent AB such that AB forms the hypotenuse. Hence the gradient at point a is given as

$$\frac{BK}{AK}$$

which must be interpreted to scale.

Displacement-time, velocity-time and Newton's cooling curves are examples of nonlinear graphs, which enable us to measure the rate of change of displacement, velocity and other quantities such as temperature with time.

Example

The table below shows the temperature of a cooling calorimeter at different times. Use the data to determine the rate of fall of temperature at 2 min.

Time in minutes	0	1	2	3	4	5	6
Temperature in °C	45.0	33.0	26.1	21.7	19.1	17.4	16.5

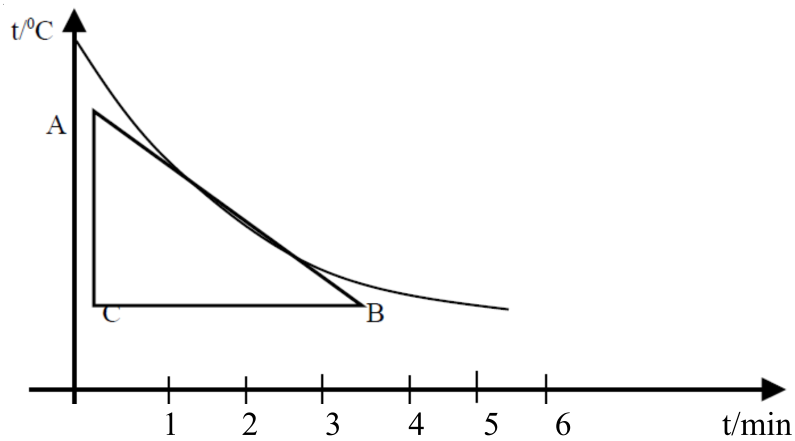

Fig 3.10

The gradient or slope is obtained from the tangent drawn on the graph at time t = 2 min. The gradient is defined as $\frac{AC}{BC}$ = negative which is about 5.6 °C per minute.

The negative sign shows that the temperature is falling.

ACTIVITY 1
Discuss the advantages to be gained by a graphical representation of a set of physical measurements.

4.0 CONCLUSION

Graphs are pictorial ways of showing how two physical measurements are related. Such physical measurements are regarded as variables. If the graph produced is a straight line, then the relationship is described as being linear. Then the equation that relates the two variables such as y and x are expressed as

(1) $y = mx$ or

(2) $y = mx + c$

where, x = independent variable y = dependent variable
m = the slope or gradient of the slope c = intercept on the
y-axis when x = 0

The first equation tells us that the line passes through the origin while the second equation does not.

It is possible to reduce a linear graph from unknown relationship by taking the logarithm of the equation expressing the unknown relationship.

5.0 SUMMARY

At the end of this unit you have learnt,

- the importance of graphs in physics practicals
- that a straight line graph shows that there is a linear relationship between the two variables in question
- non-linear relationship can be reduced to a linear one by squaring the equation expressing the relationship between the two variables such as

$$T = 2\pi \sqrt{\frac{l}{g}}$$

the slope of the graph is determined by the ratio of

$$\frac{\text{IncreaseoftheDependentVariable}}{\text{Increase of the Independent Variable}}$$

- how to determine the slope of a non-linear graph at a given point
- non-linear graphs are usually in form of quadratic equations

6.0 TUTOR MARKED ASSIGNMENTS

EXERCISE 1

Plot the displacement-time graph for the following motion of an object and determine the velocity after 3 seconds of its motion.

Time/s	0	1	2	3	4	5
Displacement /m	0.0	4.0	12.0	36.0	64.0	100.0

EXERCISE 2

Plot the velocity-time graph for the following motion of an object and find the acceleration at t = 2 seconds.

Time in seconds	0	1	2	3	4
Velocity in cm/s	15	29.5	36	38	35

EXERCISE 3

The following values were obtained for the period of vertical oscillation for a spiral spring carrying different loads.

Load M/g	25	50	75	100	125
Period T/s	0.96	1.14	1.28	1.41	1.54

Suppose the formula relating T and M is

$$T = 2\pi \sqrt{\frac{M+m}{k}}$$

where m = effective mass of the spring and k = spring constant
Determine the values of m and k.

7.0 REFERENCES AND OTHER RESOURCES

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UNIT 2 MEASUREMENT

TABLE OF CONTENTS

1.0 INTRODUCTION

In carrying out scientific investigations as in physics practical, we know that scientists use devices to measure and thereby quantify physical quantities. But even the best of devices yield inexact measurements. We express these measurements as approximate numbers. We distinguish between numbers such as 3.2 cm and 3.20 cm. These are the results of measurements using different devices. While doing computations with these numbers special care is required.

You may have wondered why the ratio of two measurements such as 12 is expressed as 2.7 not as 2.68 or 2.675.The numbers of digits used in a measurement have some significance regarding the quality of measuring instruments. In this unit we will aim at the meaning and usage of approximate numbers. We will also learn about the techniques of computations with these numbers. These techniques are of basic importance in calculating the results of experiments that we will do later. The mastery of these techniques is therefore, essential at this stage. In the next unit we will study errors, which arise due to defects in measuring instruments, fluctuations in the quantity to be measured and several other reasons. We will also learn how these errors are propagated and how the final results of an experiment are expressed.

2.0 OBJECTIVES

At the end of this unit you should be able to,

- observe that all measurements are inexact and are expressed in numbers resulting from approximations or approximate numbers
- distinguish between precision and accuracy
- express a measurement in scientific notation
- add, subtract, multiply and divide approximate numbers

How to Study this Unit

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.

Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

3.1 ERRORS: EXPRESSING THE RESULTS OF MEASUREMENTS

We are familiar with at least two reasons why all measurements are inexact. Firstly, the measuring instrument itself, such as the zero error, causes error. Secondly, error can be due to limitations of human judgment and perception, such as in aligning the end of a rod to be measured with the zero of the centimeter scale. To better appreciate the inexact nature of measurement let us reflect on the process of measurement of length. Let us obtain a `perfect' centimeter scale, which has clear and equal marking of millimeters. We desire to measure the length of three arrows A, B and C (Fig. 3.1).

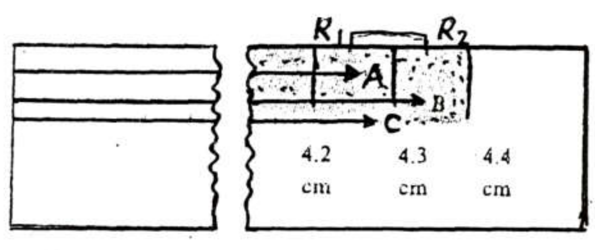

Fig .3.1

Let us suppose that we are able to perfectly align the tails of the arrows with zero marking on the scale. Of course, this is impossible to achieve in practice, but let us assume it to gain an insight into the process of measurement.

In order to measure the length of these arrows we look at the arrowheads. The head of arrow A is closer to the 4.3 cm mark than to the 4.2 cm mark. We will report the length of arrow A as 4.3 cm to the nearest millimeter. Let us now measure the length of arrow B. The heads of arrow B is closer to 4.3 cm mark than to the 4.4 cm mark. Therefore, we will also report its length as 4.3 cm to the nearest millimeter or simply 4.3 cm. Similarly the length of arrow C would be reported as 4.3 cm. Thus the lengths of all arrows whose tails are aligned with zero marking, and whose heads lie in the range Rl and R2, would be reported as 4.3 cm. We can conclude that a measurement which is reported as 4.3 cm (which is in the middle of RIR2) might possibly be in error by 0.05 cm (or one-half of the unit of measure which is 0.1 cm) or less. Thus in the measurement 4.3 cm the last digit is in error. We will, therefore, report measurements in such a manner that only the last digit will have error.

3.1.1 POSSIBLE ERROR AND PRECISION

We have seen that the maximum possible error, barring any mistake in measuring, in a measurement is 1/2 of the unit of measurement. The possible error is thus due to inherent imprecision in measuring devices. The measurements having less possible error are more precise. Since possible error is proportional to the unit of measure the instruments having smaller units of measure will give more precise measurement. A measurement reported to one hundredth of a centimetre, such as 5.32 cm is more precise than a measurement reported to one tenth of a centimetre, such as 5.3 cm.

ACTIVITY 1

Consider the following pairs of measurement. Indicate which measurement in each pair is more precise.

a. 17.9 cm or 19.87 cm

b. 16.5 s or 3.21 s

c. 20.56 °C or 32.22 °C

3.1.2 RELATIVE ERROR AND ACCURACY

So far we have considered measurement of nearly equal lengths with emphasis on precision. Let us now consider measurement of much different lengths. Suppose, two measurements yield 3.2 cm and 98.6 cm using the same metre stick. The possible error in both of these measurements is equal to 0.05 cm but the 98.6 cm is much bigger than measurement 3.2 cm. Would you say that the 98.6 cm is more accurate? How would you compare the accuracy of measurement such as 7.4 s and 98 s? In order to compare such measurements we define relative error as the ratio of possible error to the total measurement. In the table 1.0 below we have computed the relative error in some measurement. (The exact method of expressing the relative error will be discussed in section 1.5)

Table 1.0

Measurement	Unit of measure	Possible error	Relative error
3.2 cm 98.6 cm	0.1 cm 0.1 cm	0.05 cm 0.05 cm	0.02 0.0005
7.4 s 98 s	0.1 s 1 s	0.05 s 0.5 s	10.007 0.005

* Relative error is the ratio of possible error to the total measurement

Let us compare measurements 3.2 cm and 98.6 cm. Both have equal unit of measure and are therefore equally precise. But the measurement 98.6 cm has less relative error (0.0005 compared to 0.02) and is therefore more accurate. Comparison of measurements 7.4s and 98s is more revealing. The measurement 7.4 s is more precise than the measurement 98 s (possible errors 0.05 s and 0.5 s respectively) but less accurate (relative error 0.007 as compared to 0.005). You will therefore appreciate that a smaller measurement needs to be more precise for the same accuracy. This is why when measuring the dimensions of a room, metre is used as unit of measure while in measuring inter-city distances the unit kilometre is used for the same accuracy.

ACTIVITY 2

Consider the following pairs of measurements. Indicate which measurement in each pair is more accurate.

a. 40.0 cm or 8.0 cm b. 0.85 m or 0.05 m

3.2 SCIENTIFIC NOTATION

In the system of measurement that we use (SI-system) a measurement is expressed in decimal numerals. While measuring inter atomic distances, we use very large,` numbers. On the other hand, while measuring interstellar distances we use very large numbers. In scientific notation these numbers are written as a number between one and ten multiplied by an integral power of ten. For example, the diameter of the sun is l, 390, 000, 000 metres and the diameter of hydrogen atom is only 0.000000000106 metres. In scientific notation we write the diameter of the sun as 1.39 x 10^9 m and the diameter of the hydrogen atom as 1.06 x 10^{-10} m.

ACTIVITY 3

The mass of a water molecule is 0.000 000 000 000 000 000 000 03 g. Express this in scientific notation.

You have probably guessed that writing numbers in scientific notation will make computations easier. This is because we can apply the laws of exponents readily.

3.3 SIGNIFICANT DIGITS

We have seen in section 3.1.1 that a measurement reported as 5.32 cm is more precise than 5.3 cm. The number of digits in these measurements are three and two, respectively. This suggests that the number of digits used in reporting a measurement have some significance. All non-zero digits are significant.

However, in measurements such as 0.05 m or 0.005 m, none of the zeros is significant. The zeros to the left of the decimal are merely flags pointing to the decimal. The other zeros are placed to help locate the decimal point. Let us investigate this by calculating the possible error and relative errors as in the table 2.0 below.

Table 2.0

Measurement	Unit of measurement	Possible Error	Relative error
0.5m	0.1 m	0.05m	0.1
0.05 m	0.01 m	0.005 m	0.1
0.005 m	0.001 m	0.0005 m	0.1
0.00005 m	0.00001 m	0.000005 m	0.1

We can see from this table that the unit of measure and the possible error in all the cases are different. But the relative error is the same. Therefore, we can assert that these zeros are not significant because they do not affect the relative error. We can thus conclude that a digit is significant if and only if it affects the relative error.

ACTIVITY 4

Complete the following table,

S/NO	Measurement	Possible error	Relative error
1	0.2m	0.05m	$\frac{0.05m}{0.2m} = 0.25$
2	0.20m		
3	0.2000m		
4	25m		
5	250m		
6	25000m		
7	102m		
8	1002m		

(a)What can you conclude regarding the significance of `trailing' zeros in the first three measurements?

(b)What can you conclude about zeros in the fifth and sixth measurements?

(c)What can you conclude regarding the significance of zeros between non-zero digits in the seventh and eight measurements?

ACTIVITY 5
From the above discussion justify that a measurement possessing greater number of significant digits has greater relative accuracy.

ACTIVITY 6
Comment on the following:
"The distance to the sun from the roof of a house (height 20 m) is 150 million kilometres. Therefore the distance to the sun from ground is 15 million kilometres plus 20m"

3.4 COMPUTATIONS WITH APPROXIMATE NUMBERS

In section 3.1, we have seen that the reported measurements have errors in the last digit. For example, a measurement reported as 3.2, has error in the digit 2, which is indicated by placing a bar (-) over this digit. In computing values of physical quantities from observed experimental data we have to do computations. We will now establish some rules for expressing the results of basic operations with approximate numbers.

3.4.1 MULTIPLICATION AND DIVISION

Let us consider multiplication first. We want to multiply 1.23 by 2.3. At each step of the computational process we will put a bar (-) over a significant digit, which arises from computation with a digit containing error as below:

$$
\begin{array}{r}
1.2\bar{3} \\
\times\, 2.\bar{3} \\
\hline
\bar{0}.\bar{3}\bar{6}9 \\
2.4\bar{6} \\
\hline
2.829
\end{array}
$$

We see that the product contains three digits, which contain errors. Since we report the result in a number having only one digit containing error, we should round off the product to 2.8. Thus the product has two significant digits.

This is also equal to the number of significant digits contained in a factor having the least number of significant digits, namely 2.3. Therefore we formulate the following rule:
RULE: The product (or quotient) of two measurements should be rounded off to contain as many significant digits as the measurement having fewer numbers of significant digits.

ACTIVITY 7
Divide 2.1 by 1.54. Round off the result according to the above rule.

ACTIVITY 8
Divide 9.5362 by 3.2

3.4.2 ADDITION AND SUBTRACTION

Let us study the process of addition given below: 2.135

$$
\begin{array}{r}
2.13\bar{5} \\
\text{x } 2.5\bar{3} \\
\text{x } 1.0\bar{2} \\
\hline
5.6\bar{8}\bar{5}
\end{array}
$$

The sum has two error-containing digits. We, therefore, round off the sum to 5.69 so that it contains only one digit containing error. Rounding off is necessary because the sum cannot be more precise than individual measurements. We note that the sum 5.69 has the same unit of measure as the least precise addend. Thus we formulate the following rule:

Rule: While adding (or subtracting) approximate numbers, round off the sum (or difference) to the same unit of measure as the least precise measurement.

ACTIVITY 9
Subtract 2.11 from 2.1546

ACTIVITY. 10.
Compute the sum of 2.1546 m, 2.11 m and 2.125 m.

4.0 CONCLUSION

In this unit, you have learnt that errors do occur in physical measurements. All measurements are not exact. They are expressed in numbers resulting from approximations.

Precision of a measurement is a function of possible error only while accuracy is related to relative error that occurs in a measurement.

Scientific measurements are usually expressed by using the scientific notations. There are rules for multiplication, division, addition and subtraction of measurements.

5.0 SUMMARY

At the end of this unit you have learnt, exact measurement is impossible. The result of every measurement is expressed in numbers resulting from approximation such that only the last digit contains error. In scientific notation a measurement is expressed as a decimal number between one and ten multiplied by powers of ten. Possible error is only one-half the unit of measurement. Precision is a function of possible error only. Relative error is the ratio of possible error to total measurement. Accuracy is related to relative error. A digit is significant if and only if it affects the relative error rule for multiplication (or division) The product (or quotient) of two measurements should be rounded off to contain as many significant digits as the measurement having fewer numbers of significant digits.

- rule for addition (or subtraction)

While adding (or subtracting) approximate numbers, round off the sum (or difference) to the same unit of measure as the least precise measurement.

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UNIT 3 ERROR ANALYSIS

TABLE OF CONTENTS

1.0 INTRODUCTION

In the last unit we studied about errors in measurements due to imprecision of measuring devices. The results of measurements were expressed as approximate numbers. We also learn about performing basic operations of addition, subtraction, multiplication and division of approximate numbers and expressing results using correct number of significant digits. We assumed that the measuring instruments as well as the observers were perfect. However, as you are aware, there can be defects in measuring instruments and also humans are not perfect. If the environment is not perfectly controlled its changes will affect the object to be measured thereby introducing errors in measurements. In this unit we will familiarise ourselves with these and other sources of errors. We will also learn how to estimate and possibly eliminate or account for such errors. In most of the physics experiments our objective is to determine relationship between physical quantities. Therefore, we will estimate the errors in the measurement of various physical quantities and make efforts to determine valid relationships as mentioned above. In the next couple of experimental write-ups we will apply our knowledge of errors and its propagation to actual measurements and deduce relationships. We will first concentrate on the measurements of fundamental quantities such as mass, length and time, and then do experiments involving two or more of these quantities.

2.0 OBJECTIVES

After studying this unit you should be able to

- distinguish between random errors and systematic errors
- eliminate to some extent the systematic errors
- compute errors in the measurement of various physical quantities
- analyse data by calculation and by plotting graphs to functional relationship
- interpret the slope of a graph and to determine the value of certain physical quantities from the slope of a straight-line graph.

How to Study this Unit

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

3.1 TYPES OF ERRORS

Every measuring instrument has a limitation in that it cannot measure physical quantities smaller than a certain value known as the least count of instrument. For example, a meter scale can measure only up to 1 mm (smallest division of the scale). A venire caliper can generally measure up to 0.1 mm whereas a spherometer and screw gauge can measure lengths up to 0.01 mm. Similarly a thermometer usually has the least count of half a degree. In addition to these limitations, which are inherent in a measuring device, there are other sources of error. These arise due to changes in environment, faults in observational, malfunctioning of measuring devices etc. errors in any measurement can be classified into two broad headings namely - Systematic errors

and Random errors.
Let us now study the causes of such errors, and see how they are eliminated or minimized.

3.1.1 SYSTEMATIC ERRORS

The systematic errors, also called determinant errors, are due to causes which can be identified. Therefore, these errors, in principle, can be eliminated. Errors of this type result in measured values which are consistently too high or consistently too low. Let us discuss these errors one by one.

(i) Zero Error

In the case of vernier callipers, for example, when the jaws are in contact, the zero of the vernier may not coincide with the zero of the main scale. The magnitude and sign of the `zero error' can be determined for the scale readings. We can easily eliminate this error from the measurement by subtracting or adding the zero error.

(ii) Back lash Error

While measuring a physical quantity there may be an error due to wear and tear in the instruments like screw gauge or spherometer due to defective fittings. Such an error is called back lash error and can be minimised in a particular set of measurements by rotating the screw head in only one direction.

(iii) End Correction

Sometimes the zero marking of the metre scale may be worn out. Unless we are careful, this will lead to incorrect measurements. We must therefore compensate for this by shifting our reference point.

(iv) Errors due to changes in the Instrument parameters

Usually, in experiments involving electrical quantities, the value of the electrical quantities change during the course of the experiment due to heating or other causes. For example, the value of the resistance of a wire will increase because of current passing through it. This will lead to errors, which are generally difficult to calculate and compensate for. To some extent this can be avoided by not allowing current to flow through the circuit while observations are not being taken.

(v) Defective Calibration

Occasionally instruments may not be properly calibrated leading to errors in the results of measurement. This type of error is not easily detected and compensated for. This is a manufacturer's defect and if possible the instrument should be calibrated against a standard.

(vi) Faulty Observation

This could be due to causes like parallax in reading a metre scale. These errors are eliminable by using proper techniques.

3.1.3 RANDOM ERRORS

You must have noticed that many times repeated measurements of the same quantity do not yield the same value. The readings obtained show a scatter of values. Some of those values are high while others are low. This function is due to random errors whose possible sources are:

(i) Observational

These arise due to errors in judgment of an observer when reading a scale to the smallest division.

(ii) Environmental

These arise due to causes like unpredictable fluctuations inline voltage, variation in temperature etc. They could also be due to mechanical vibrations and wear and tear of the systems. There could be a random spread of readings due to friction say, wear and tear of mechanical parts of a system.

ACTIVITY 1

Which of the figures 1(a) or (b) show random errors only?

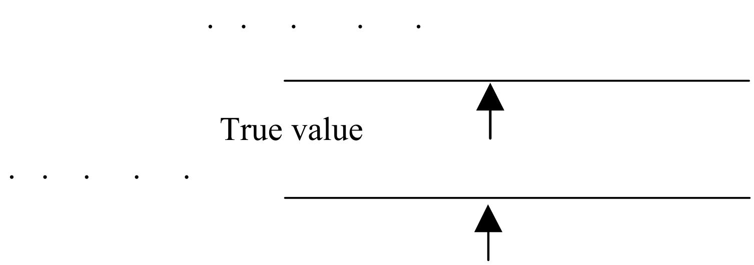

Fig. 1 Set of measurements. Each point indicates the result of a measurement

3.2 DETERMINING SIZE OF ERROR

When we measure a quantity it is important to take several readings. It may be preferable that readings are taken by independent observer. This has the advantage that bias of a single observer is eliminated. The value obtained will indicate whether the data is scale limited or random. An error analysis can be made to determine the size of error from these readings. A typical set of values of a measurement is given below in table l. The quantity to be measured as a `true' value is independent of our measuring process. But the imperfection of our measuring process prevents us from obtaining that value every time. Which one of the values listed in table I would be `true' value? It is impossible to tell that from the measurements because of this spread. Under the circumstances the average A value can be quoted. To get the average value we simply add up all the measurements and divide the sum by the total measurements. As you can see from the table l, the average is 3.68. Also notice that most of the data in table I deviates from the average d we first take the difference of each data from the average to get individual deviations d. These deviations are then added and their sum is divided by the number of observations to obtain d. As you can see from table 1 the average deviation in this case is 0.009.

Table 1

S/No	Data	Deviation (d)
1.	3.69	0.01
2.	3.67	0.01
3.	3.68	0.0
4.	3.69	0.01
5.	3.68	0.0
6.	3.69	0.01
7.	3.66	0.02
8.	3.67	0.01
	A = 3.68	$\bar{d}$ = 0.009

As you are aware, repeated measurements of the same quantity yield results with better precision. A measure of this precision index S whose definition (without proof) is

$$S = \frac{\bar{d}}{\sqrt{n}}$$

where d is the average derivation and n is the number of observations. The precision index S is a measure of uncertainty of average. Using the data of table l, the precision index is

$$S = \frac{d}{\sqrt{n}} = \frac{0.009}{\sqrt{8}} = 0.003$$

Thus the final result can be expressed as A ± S. In this case the result of random data analysis gives 3.68 ± 0.003. We can see that this error is much less than the possible error, which is ± 0.005. Thus in such cases we will consider the possible error only.

ACTIVITY 2
The measurement of the length of a table yields the following data.
11=135.0cm,/2=136.5cm,13=134.0 cm, /4=134.5 cm Calculate (a) the average value and (b) precision index. How does the precision index compare with possible error? How will you express the final result?

3.3 PROPAGATION OF ERROR

We have so far learnt how to determine the error in the measurement of a quantity, which can be measured directly. In actual practice, however, we determine values of a quantity from the measurements of two or more independent quantities. In such cases the error in the value of the quantity to be determined will depend on the errors in other independent quantities. In other words, the error will `propagate'. The actual analysis of propagation of error is beyond the scope of this course. We shall therefore, quote some rules which can be used in our laboratory.

3.3.1 ERROR PROPAGATION IN ADDITION AND SUBTRACTION

What will be the error, in quantity E, defined by $E = x + y + z$?
Let us take the differential of this quantity, we get $\delta E = \delta x + \delta y + \delta z$
if the error is small compared to the measurement we can replace the differential by `delta' to get $\delta E = \delta x + \delta y + \delta z$
which is simply the sum of errors in x, y and z. It, therefore, is the maximum error in E. Statical analysis shows that a better approximation is

$$\delta E = \sqrt{(\delta x)^2 + (\delta y)^2 + (\delta z)^2}$$

We only consider the magnitude of errors in the above calculation. Therefore, the error in the quantity $(x + y - z)$ will also be the same.

SOLVED EXAMPLE: Let the measured value of two lengths be
$L_1 + \delta L_1 = 1.746 \pm 0.010$ m
$L_2 + \delta L_2 = 1.507 \pm 0.010$ m
The error in the quantity $L = L_1 + L_2$ will be $\delta L =$

$$\sqrt{(0.010\ m)^2 + (0.010\ m)^2} = 0.014\ m$$

3.3.2 ERROR PROPAGATION IN MULTIPLICATION AND DIVISION

If a quantity $E = A \times B$ and the result of measurement of A & B is $A \pm \delta A$ and $B \pm \delta B$ then what will be the error δE in E? Here if we take differentials we get
$dE = B\,dA + A\,dB$

Dividing by E = AB and changing differentials by `deltas' we get

$\frac{\delta E}{E} = \frac{\delta A}{A} + \frac{\delta B}{B}$

ACTIVITY 3
Take logarithm of E = AB and then differentiate to show that

$$\frac{\delta E}{E} = \frac{\delta A}{A} + \frac{\delta B}{B}$$

Which is generally known as the logarithmic error?

RULE 1: When independent measurements are multiplied or divided the fractional error in error in the result is the square root of the sum of squares of fractional errors in individual quantities.

SOLVED EXAMPLE: In an experiment we calculate velocity from measurement of distance and time. If the distance is $S \pm \delta S = 0.63 \pm 0.02$ m

$\frac{\delta S}{S} = 0.03$

and time is $T \pm \delta T = 1.71 \pm 0.10$ $\frac{\delta T}{T} = 0.06$

Then the velocity (V) $V = \frac{S}{T} = 0.368\ MS^{-1}$

The fractional error in V is given by

$$\frac{\delta V}{V} = \sqrt{\left(\frac{\delta S}{S}\right)^2 + \left(\frac{\delta T}{T}\right)^2} = 0.07$$

$\delta V = 0.368\ ms^{-1} \times 0.07 = 0.02\ ms^{-1}$

Thus the final result becomes

$V \pm \delta V = 0.37 \pm 0.02\ ms^{-1}$

3.3.3 ERROR PROPAGATION IN OTHER MATHEMATICAL PERATIONS

Errors in exponential quantity: Let us first consider a special case where a quantity appears with an exponent. For example, $S = A^2 = A \times A$. Here the two numbers multiplied together are identical and hence not independent. The rule mentioned above does not apply. Detailed analysis shows that logarithmic error gives a good estimate. Taking the logarithm of the above equation we get,

$\log S = 2 \log A$

on differentiation and changing differentials to `deltas' we get $\frac{\delta S}{S} = 2 \frac{\delta A}{A}$

Therefore, the fractional error in A^2 would be twice the error in A, the fractional error in A^3 will be 3 times the fractional error in A, and the fractional error in $\sqrt{A}$ will be 1/2 the fractional error in A.

RULE: The fractional error in the quantity An is given by n times the fractional error in A.

EXAMPLE: Suppose two measurements of mass are $M_1 \pm \delta M_1 = 0.743 \pm 0.005$ kg and $M_2 \pm \delta M_2 = 0.384 \pm 0.005$ kg.

Determine the value of $M = 2M_1 + 5M_2$ along with δM. What will be the error in $(M_1 + M_2)^2$ and $(M_1 - M_2)^{-3}$.

HINT: The error in $2M_1$ is $2\,\delta M_1$ and in $5M_2$ is $5\,\delta M_2$. Thus

error in $2M_1 + 5M_2$ is $\delta M = \sqrt{(2\delta M_1)^2 + (5\delta M_2)^2}$

Error in $(M_1 + M_2)^2 = 2 \sqrt{(\delta M_1)^2 + (\delta M_2)^2}$

Error in $(M_1 - M_2)^{-3} = 3 \sqrt{(\delta M_1)^2 + (\delta M_2)^2}$

Similarly in other mathematical operations and deducing results from graphs (about this you will learn in the next subsection) the flowing rule is used.

RULE: The error in the result is found by determining how much change occurs in the result when the maximum error occurs in the data.

EXAMPLE: Let us compute the error in the sine of 30° ± 0.5°. Using the logarithmic tables we get,

sin 30° = 0.5, sin 30.5° = 0.508, sin 29.5° = 0.492

The difference between sin 30° and sin 30.5° is 0.008, and the difference between sin 30° and sin 29.5° is also 0.008. Thus the error in sin 30° would be ± 0.008.

ACTIVITY 4
Determine the error of sine of 90°, when the error in the angle is 0.5°. Compare your result with that of the example above.

3.3.4 ERROR PROPAGATION IN GRAPHING

Very often we can better visualise the functional relationship between two physical quantities by plotting a graph between them. This is another useful way of handling experimental data because the values of some quantities can be obtained from the slope. While plotting a graph we will use the following guidelines:

1. A brief title may be given at the top

2. Label the axes with the names of the physical quantities being presented along with units. It is customary to plot the independent variable (the quantity which is varied during the experiment at one's will) on the x-axis and the dependent variable on the y-axis (the dependent variable is the one that varies as a result of change in the independent variable). We would write to the name of the variable represented on each axis along with units in which they are measured.

3. We should choose the range of the scales on the axis so that the points are suitably spread out on the graph paper and not cramped into one corner. Check for the minimum and maximum values of the data that has to be plotted. We may then round off these two numbers to slightly less than the minimum and slightly more than the maximum. Their difference may be divided by the number of divisions on the graph paper. For example, if we are to plot 5.2 and 17.7 it would be convenient to allow the scale to run from 5 to 20 rather than from 0 to18. Each set of data points is indicated by a point within a circle on the graph paper and the error is shown by using bars above and below this point as shown in fig. 2. The graphed data show that velocity V is the linear function of time T.

We recall that the general equation of a straight line is $y = mx + c$ where m is the slope of line and c the vertical intercept in the value of y when $x = 0$. From the graph we can thus write $V = aT + V_0$. By comparing the above equation we can conclude that the slope of the graph gives the acceleration and the intercept gives the velocity V_o at $T = 0$. From the graph $V_o = 0.32\ ms^{-1}$. To determine the slope we consider two points on the straight line, which are well separated.

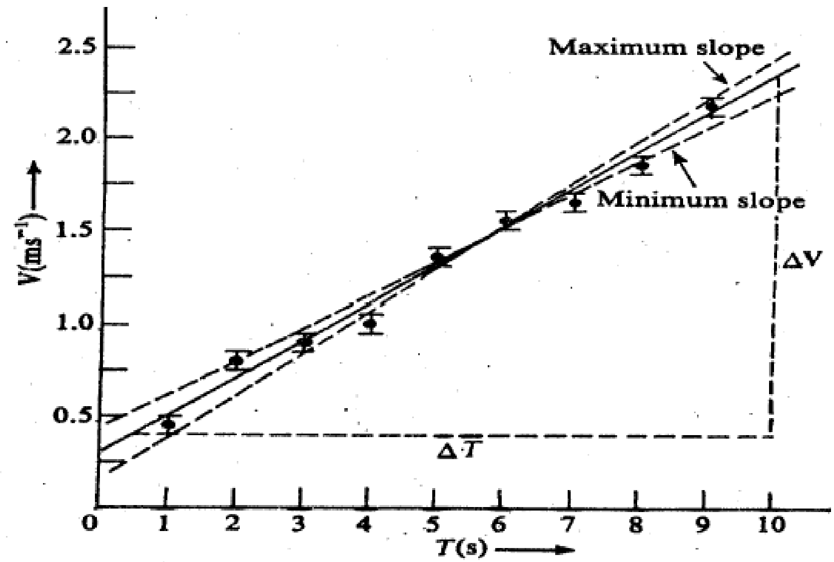

Fig. 2: Graph between velocity and time

Then,

$$a = \text{Slope} = \frac{V}{T} = \frac{2.35 - 0.40\ (ms^{-1})}{10.0 - 0.5\ (s)} = 0.20\ ms^{-2}$$

In the above example, we have plotted the variable V, which is a linear function of T in a linear graph paper. In some experiments we may get data where the relationship between the measured variable is not linear. Suppose a man gets a salary of ₦ 20, 000.00 on the 1^{st} of every month and he decides that each day he will spend half the money he has with him on that day. Then the amount of money, which the man will have over a period of first seven days of any month, will be given as in table 2.

TABLE 2

Day of any month	Money left with the man (M) ₦ K x 10^2
1st	200.00
2nd	100.00
3rd	50.00
4th	25.00
5th	12.50
6th	6.25
7th	3.12
8th	1.56

Let us plot these data on a linear graph paper. The paper will be of the type shown in fig. 3. Look at the graph carefully. You will find that seven of the ten experimental points are clustered together near the bottom right-hand corner of the graph. The shape of the curve we have drawn also involves a bit of guesswork. Therefore, we have to find some method so that these data can be plotted in a better way.

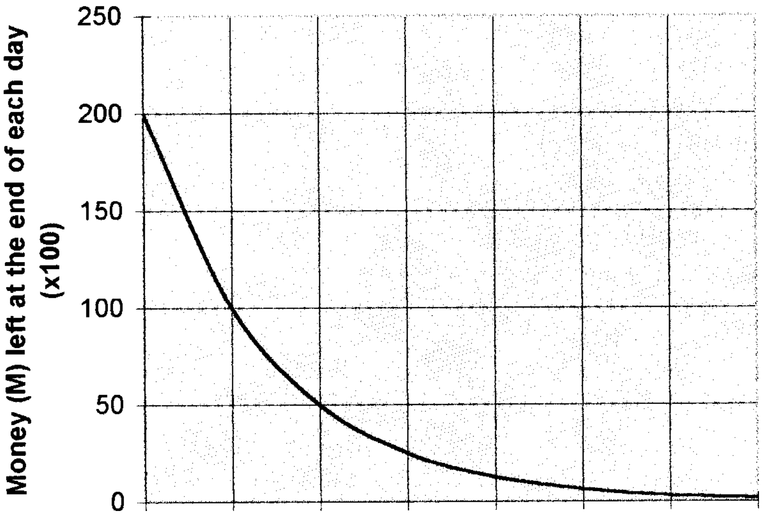

Fig. 3 Graphical representation of Table 2

Try to recollect what you used to do in school when you used to come across data like this, which range over a few orders of magnitude or having big gaps between the points. We will tell you, in such cases you used to take the logarithm of the data and then plot those data in a linear graph paper. When you did this, you must have found that the result was a straight line. So let us take the logarithm of the data of table 2 and tabulate them as shown in table 3.

Day of any month	Money left in the man (m) N K x 10^2	Log N
1st	200. 00	4.301ø
2nd	100. 00	4.000 ø
3rd	50. 00	3. 699 ø
4th	25. 00	3. 398
5th	12. 00	3.097
6th	6. 00	2.706
7th	3. 12	2.494
8th	1. 00	2.193

Day of any month	Log M
1^{st}	3.301
2^{nd}	2.000
3^{rd}	1.699
4^{th}	1.397
5^{th}	1.097
6^{th}	0.796
7^{th}	0.494
8^{th}	0.193

Now plot log M against days as shown in fig. 4. You obtain a graph in which points are more clearly spaced evenly and hence you can more easily draw a straight line through the points.

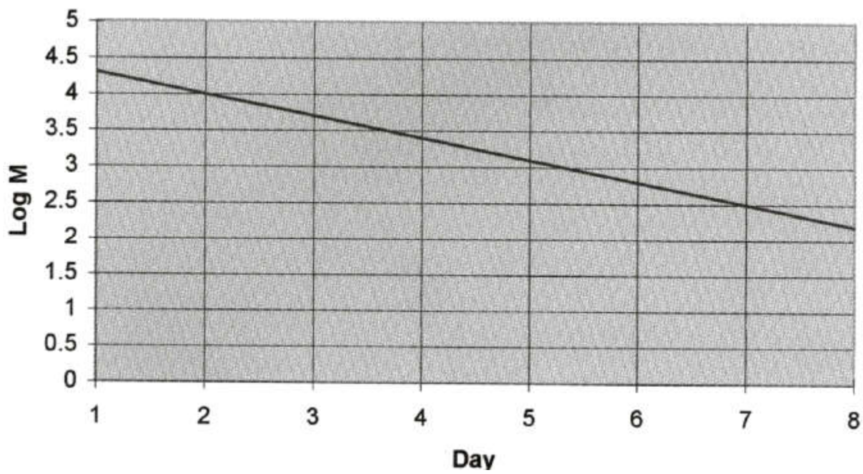

Fig. 4 Graphical representation of Table 3

You might have realised that working out the log values for each data is tedious and it also introduces another step, which may introduce error between the data and the graph. Therefore, to plot such a data we use a graph paper called semi-logarithmic or log-linear graph paper where the lines on one axis have been drawn in a logarithmic fashion. On a semi-log paper (see the graph paper of fig. 5) the horizontal scale is an ordinary one, in which the large division are divided into tenths and each division has the same size. The vertical scale is a logarithmic scale (is automatically takes logarithms of data plotted), in which each power of ten or decade (also called frequency) corresponds to the same length of scale. In each decade, the divisions become progressively compressed towards the upper end. Now to the semi-log graph paper we plot the data of table 1. We obtain a straight line as shown in fig. 5. If you compare fig. 4 & 5 you will see that the points plotted on semi-log paper are distributed on a linear graph paper. A question may strike your mind that how to calculate the slope of the straight line of fog. 5? Also what is the equation of the straight line? Let log M be represented by y and by t then we have a straight line graph of y against t. Let the question be represented as

$y = b + kt$

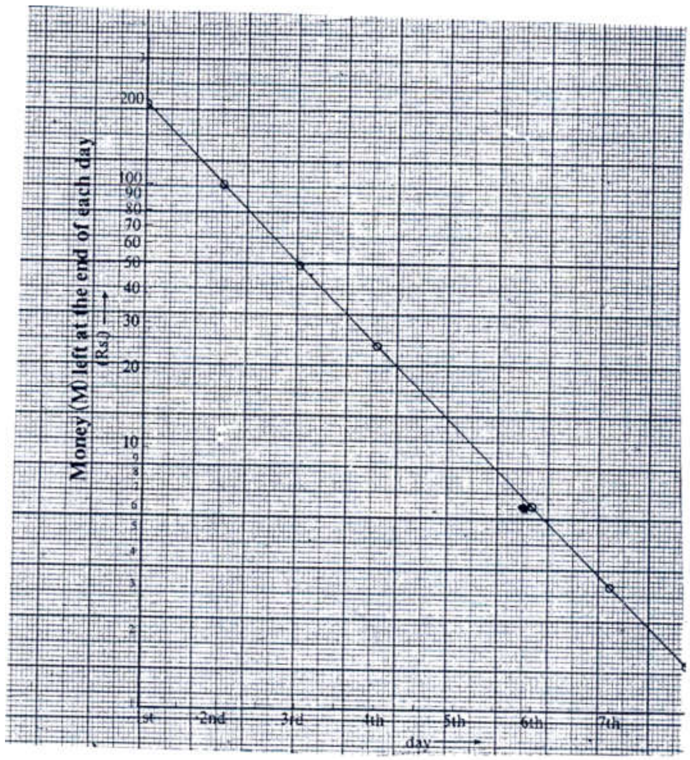

Fig. 5 Representation of Table 3 graphically on semi-log paper

Where b is the intercept of the line on the y-axis and k the slope of the line. We can find the values of b and k from the graph as follows. When t = 0, M = 200 then log M =log 200=2.30=y

2.30=b+0 or b=2.30

...y=2.30+kt

When t = 7th day, M = 1.56 and log M = log 1.56 = 0.193 = y

Putting these values in equation (1) we get

0.193=2.30+7k

.'. The slope $k = -\frac{2.1}{7} = -0.3$

and the equation of the straight line is

$y = 2.3-0.3t$ (2)

From the graph of fig. 5 or in other words from equation (2) can you find the equation of the curve plotted in fig. 3?

Let the value of M at t = 0 be denoted as M_O then equation (2) becomes log M =

$\log M_O + kt$

or $\log M - \log M_o = kt$

or $\text{Log} \frac{M}{M_O} = kt$

or $\frac{M}{M_O} = 10\ kt$

or $M = M_O 10^{kt}$

or $M = 200 \times 10{-}0.3t$ (3)

This is the equation of curve plotted in fig. 3. It tells us that the money is decreasing logarithmically (also called exponentially) with each day.

In science, you will come across many logarithmic or exponential relations of the form of equation (3). In such cases it would be convenient to plot the data on semilogarithmic graph paper because the graph will be convenient to plot the data on semilogarithmic graph because the graph will be a straight line if the relationship is logarithmic. Also the slope of the line (which may give you the value of any physical constant) can be read simply and directly from the graph.

Sometimes we find that we wish to plot a graph where both variables range over several powers of ten. For example, you know that according to Keplar's law, the semi-major axis of the orbit of a planet (R) is related to its period (time for one revolution around the sun) T by the following power-law relation:

$R^3=kT^2$ (4)

Where k is another constant.

If you consider the experimental data that shows how the quantity T depends on quantity R you will observe that R varies by two orders of magnitude and T varies by three orders of magnitude. In other words the experiment data follows equation (4). For a moment, suppose you do not know the exact relationship between the variables T and R.

Then you can suppose that

$$R=kT^{n} \qquad (5)$$

where n is another constant.

Using the conventional method to find the value of n, you will take logarithm of equation (5) as follows:

$$\log R = \log k + n \log T$$

Now you will plot log R vs. log T on a linear graph paper. The slope of straight line obtained will give the value of exponent n. But again, as mentioned above, taking logarithm of each experimental date is rather tedious so it would be convenient to plot both the variables T and R on a logarithmic scale where the lines on both axes are drawn in a logarithmic fashion. A log- log graph is shown in fig. 6. The points lie upon a straight line. The slope of the straight line will give the exponent (n) of the power law relation and hence the exact relationship between R and T will be found out.

To determine the error in the value of the slope of the straight line drawn in any graph paper (linear or semi-log or log - log) we draw two dashed lines representing the greatest and the least possible slopes which reasonably fit the data as shown in fig. 2. Thus the error in the slope is defined as

$$\text{Error in slope} = \frac{\text{maximumslope-minimumslope}}{2}$$

Thus from the graph we get the error in the slope as

$$\delta a = \frac{0.23 - 0.19 \text{ ms}^{-2}}{2} = 0.02 \text{ ms}^{-2}$$

Thus the experimental value of acceleration from the graph is

$$a \pm \delta a=0.20 \pm 0.02 \text{ ms}^{-2}.$$

Similarly the error in intercept = (intercept of minimum slope line - intercept of maximum slope line)/2

$$\delta VO = \frac{(0.45-0.17)}{2} \text{ ms}^{-1} = 0.14 \text{ ms}^{-1}{}_{s}$$

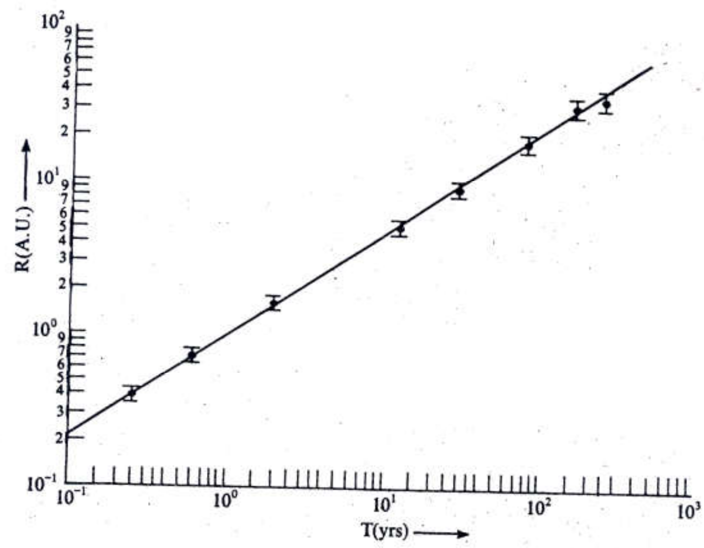

Fig 6

3.4 USES OF π

It appears that most of our students are under the impression that the value of π is equal to $\frac{22}{7}$ exactly. Unfortunately many books writers also have contributed to perpetuate and establish this false idea by setting numerical problems with data cooked up that using π = $\frac{22}{7}$, the factor always happily cancels out and the simplification becomes very easy. However, in the real world the values of actual physical quantities are not such as to facilitate cancellation with 7. Also, we may as well acknowledge that the value of π cannot be expressed exactly in terms of any whole number.

The value of $\pi = \frac{22}{7}$ is one of the many approximations that can be used. In fact, a better approximation is

$$\frac{355}{113} = 3.1415928.$$

Compare this with the calculator value π = 3.141592654 and 22 = 3.14286. It may be noted that the value of $\frac{22}{7}$ deviates from the more accurate value from the calculator in the third decimal place; if we round it off to 5-digit accuracy, π = 3.1415 (from calculator), whereas the approximations $\frac{355}{113}$ = 3.1416 and $\frac{22}{7}$ = 3.1429

For practical purposes at undergraduate level, the most convenient and comparatively more accurate thing to do will be to remember

$\pi = 3.142$; $\log \pi = 0.4972$;

$\pi^2 = 9.870$; $\log \pi^2 = 0.9943$

Wherever the value of π is to be used in our calculations, the above values prove fruitful.

4.0 CONCLUSION

In this unit you have been able to distinguish between random errors and systematic errors when measuring physical quantities and know Systematic errors to some extent, can be eliminated. Errors in the measurement of various physical quantities can also be computed. When graphs are used to show relationship between certain physical quantities, they can also be used to determine the errors in the calculation of slopes.

5.0 SUMMARY

In this unit you have learnt the following:

- the difference between random errors and systematic errors,
- systematic errors, to some extent can be eliminated;
- errors in the measurement of various physical quantities can be computed;
- data collected can be used to show relationship between two physical quantities through graphs;
- from the graphs plotted, the errors in the determination of the slopes of such graphs may also be calculated;
- the use of π in calculations .

6.0 REFERENCES AND OTHER RESOURCES

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UNIT 4 EXPERIMENTS

Objectives

After going through unit 4 you are expected to have gained and master the skills of:

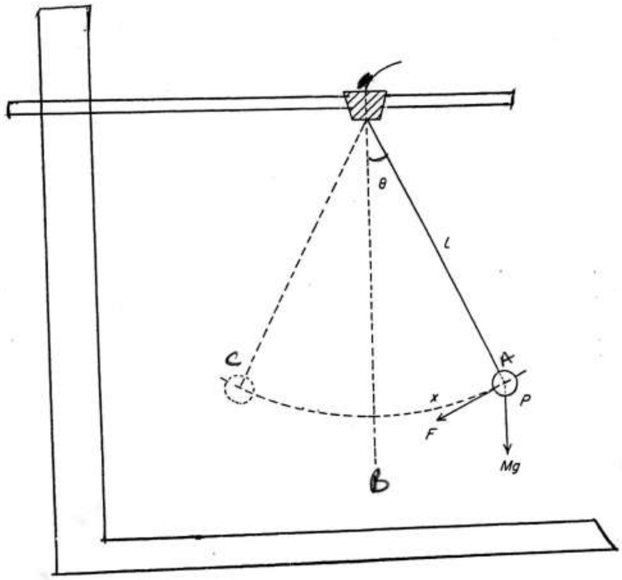
- Choosing and reducing scales
- Follow practical instructions carefully to take records
- Plot a neat graph making use of at least ¾ of the page
- Report practical conducted following an accepted procedure

How to Study this Unit
In this last unit quite unlike units 1 to 3, students will be involved in doing and reporting while the facilitator does the marking and necessary feedback. Have your hard cover graphical notebook and use it to report experiments 1 to 10. Ensure that the facilitator signs records taken per practical before leaving the laboratory.
Where you are expected to take records, report and submit within a given time before leaving laboratory, adhere to such instruction strictly. You can call to ask question while conducting practical each time.
***All the Experiments in this Unit are Tutor Marked Assignment**

EXPERIMENT 1
Determination of acceleration due to gravity by means of the Simple Pendulum.

AIM: To determine the value of acceleration due to gravity by using the Simple Pendulum.

APPARATUS
- Small pendulum bob made up of lead or brass
- Cotton thread
- Split cork
- Meter rule
- Stop watch
- Retort stand and clamp



THEORETICAL BACKGROUND

The motion of a simple pendulum is described as being a simple harmonic motion. A simple harmonic motion (SHM) is defined thus:

When an object moves to and fro in such a way that its acceleration is directly proportional to its displacement and is always directed to its equilibrium position, then, that object is said to perform a simple harmonic motion.

The simple pendulum has its equilibrium position at point B if the mass of the pendulum bob is m and length *l*.

If it is displaced to point A such that the angle of displacement ϴ is small and released, the bob will oscillate i.e. move to and fro.

The restoring force $F = mg \sin \Theta$

Where, m = mass of bob and g = acceleration due to gravity. From the second law of motion,

$F = ma - mg \sin \Theta$

$\therefore a = -g \sin \Theta$

where, a = acceleration

The negative sign indicates that the acceleration does not increase perpetually, it increases and decreases as it moves to and fro.

From $a = -g \sin \Theta$

When ϴ is very small then $\sin \Theta = \Theta$ rad

$\therefore$ a = -g θ

Using the sector PAB,

$\therefore \frac{\theta}{2\pi} = \frac{x}{2\pi}$

$\therefore \theta = \frac{x}{l}$

$\therefore a = -g \frac{x}{l}$

$\therefore a = -g \frac{x}{l} x$

where, g and P are constants.
$\therefore a \propto x$
where, x is the displacement.

Hence, the acceleration a is proportional to the displacement x. Consequently, the motion is simple harmonic.

Compare $a = -\omega^2 x$
Where, $-\omega^2$ = angular velocity

$\therefore -\omega^2 x = g \frac{x}{l}$

$\therefore \omega = \sqrt{\frac{g}{l}}$

The period $T = \frac{2\pi}{\omega}$

$\therefore T = 2\pi \sqrt{\frac{l}{g}}$

By squaring both sides of the equation,

$$T^2 = 4\pi^2 \frac{\ell}{g}$$

:. $T^2 = \frac{4\pi^2}{g} l$

by plotting the graph of T^2 versus ℓ , we shall then obtain the slope m = $\frac{4\pi^2}{g}$

where g can then be determined from g = $\frac{4\pi^2}{m}$

To obtain T, we shall then find the time for 20 oscillations. Thus T = $\frac{t}{20}$ s.

PROCEDURE

(1) Set the simple pendulum as shown in the diagram above.

(2)Hang the pendulum bob on one end of the cotton thread and clamp the other end firmly between the two split corks

(3) Allow the pendulum to dangle freely on the working bench

(4) Use the metre rule to measure 95 cm length of the thread

(5) Displace the pendulum through a small displacement, release and allow to swing

(6) Time 20 complete swings (A --> B -> C --> B --> A) is a complete swing or 1 oscillation as t1

(7) Repeat for another 20 swings for the same length a time t2

(8) Find the mean time for t1 and t2

(9) Repeat the above procedure of timing for other values of ℓ such that ℓ = 85 cm, 75 cm, 65 cm, 55 cm 45 cm.

(10) Tabulate your observations as shown below and complete the table for the period T and T^2.

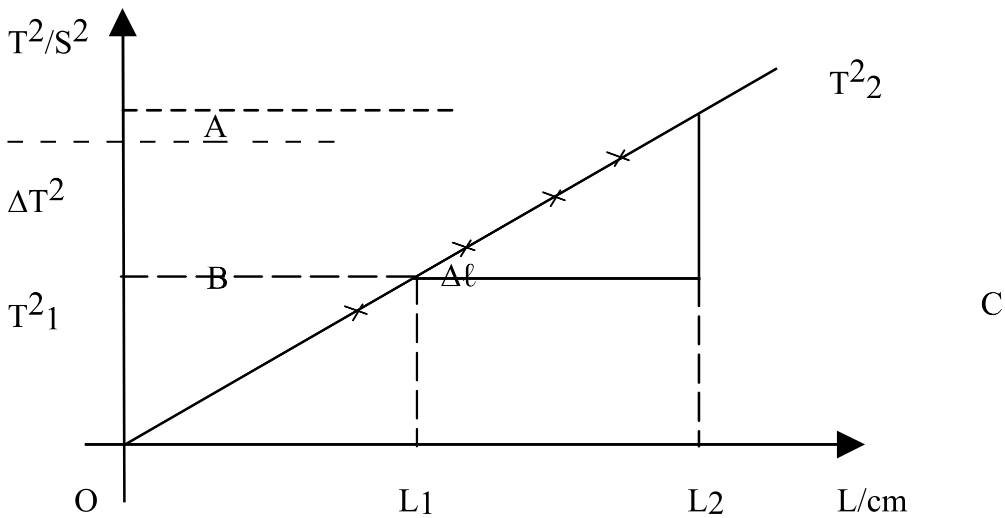
TABLE OF OBSERVATIONS

ℓ/cm	Time for 20 oscillations			Period	T^2/s^2
	t1/s	t2/s	Mean t/s	T/s= (t) 20	
95					
85					
75					
65					
45					

GRAPH

Plot the graph of T^2 on the vertical axis and P on the horizontal axis. Determine the slope m of the graph as

$$m = \frac{\Delta T^2}{\Delta \ell}$$



Slope or gradient m = <u>AC</u> BC

$$= \frac{\Delta T^2}{\Delta l}$$

$$= \frac{T_2^2}{L_2} - \frac{T_1^2}{L_1}$$

From $\Delta T^2 = \frac{4\pi^2}{g} \Delta l$

$$\frac{\Delta T^2}{\Delta L} = \frac{4\pi^2}{g} \Delta l$$

$$g = \frac{4\pi^2}{m} = \frac{4\pi^2}{\Delta T^2} \Delta l$$

Estimate the error in g.

CONCLUSION

Acceleration due to gravity = $\frac{m}{s^{-2}}$ or mS^{-2}

EXPERIMENT 2

Acceleration due to gravity by means of the simple pendulum from inaccessible height.

AIM:

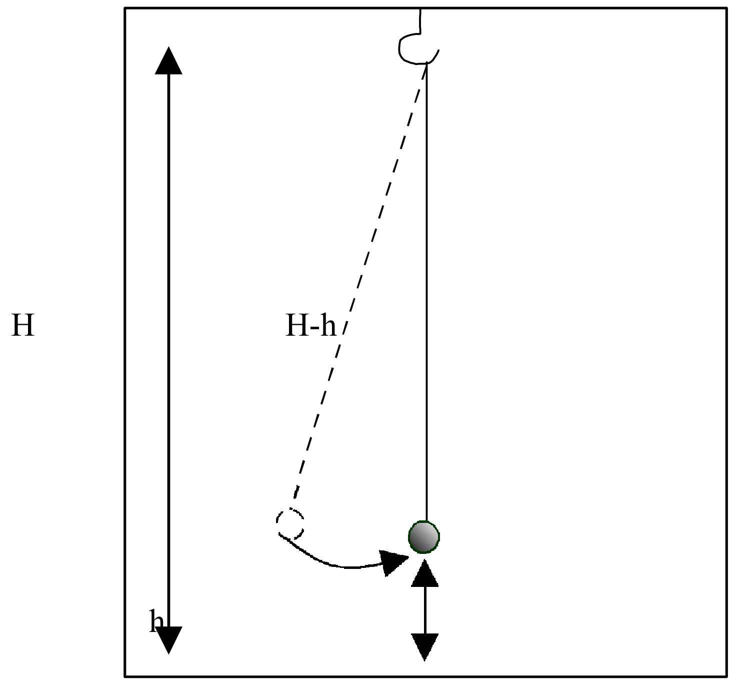
(i) To determine the value of acceleration due to gravity by using a simple pendulum hanging from an inaccessible height

(ii) To determine the height of the room experimentally

APPARATUS

- Small pendulum bob made up of lead or brass
- Cotton thread of about 3 m long
- A hook on the ceiling of the room or laboratory
- Meter rule
- Stop watch
- A pair of scissors or blade

DIAGRAM



THEORETICAL BACKGROUND

We know that the period of oscillation (T) of a simple pendulum of length (*l*) is expressed as

$$T = 2\pi\sqrt{\frac{l}{g}}$$

If we therefore have a simple pendulum hanging from a ceiling, the only measurement available to us is the height of the pendulum bob from the floor of the room or the laboratory.

If the height of the room or laboratory is H and the height of the bob above the floor is h, therefore the length of the simple pendulum (*l*) measured from the ceiling H is

$$l = H - h$$

We can now write the period of oscillation T as

$T = 2\pi \sqrt{\frac{H-h}{g}}$

Squaring both sides of the equation will produce

$T^2 = \frac{4\pi^2}{g}(H - h)$

$T^2 = \frac{4\pi^2}{g} H - \frac{4\pi^2}{g} h$

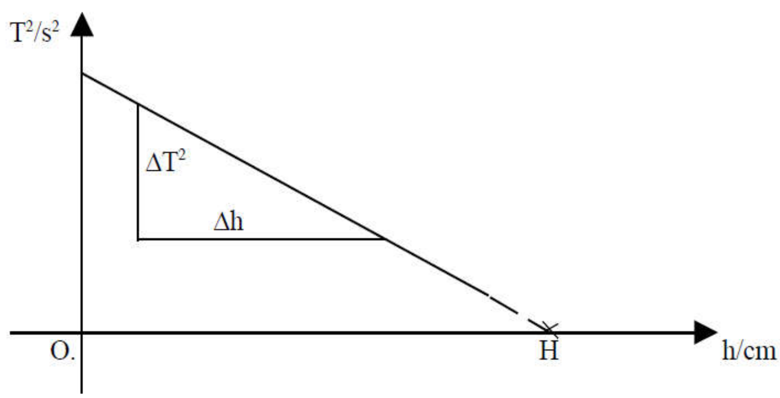
If we therefore plot the values of T^2 against h we will obtain a graph similar to a linear graph represented by the equation
y = c - mx
where, c = the intercept when x = 0
-m = slope of the graph

Similarly from

$T^2 = \frac{4\pi^2}{g} H - \frac{4\pi^2}{g} h$

the slope of the graph m = $-\frac{4\pi^2}{g}$ -

The intercept of the graph on the x-axis i.e. when T^2 = 0 gives us the height H of the room. T^2/s^2



Consequently two values of g and H can be determined experimentally.

PROCEDURE

(1) Hang the simple pendulum from the ceiling using the hook such that the bob is about 30 cm from the floor.

(2) Measure accurately and record the height h from the floor of the room to the pendulum bob

(3) Displace the pendulum bob through a small angle Ө and allow it to oscillate freely.

(4) Determine the time t for 20 oscillations and record as t_1

(5) Repeat steps (2) - (4) to determine time t_2 for 20 oscillations for the same height h above the ground

(6) Determine the mean time t

(7) Cut off 5 cm of the thread, retie the bob as closely as possible to the end of the thread to obtain a new height h above the ground and obtain the time t_1 and t_2 for the new height

(8) Determine the new mean time t for the 20 oscillations

(9) Repeat steps (7) and (8) three times to obtain three new heights h and t_1, t_2 and t

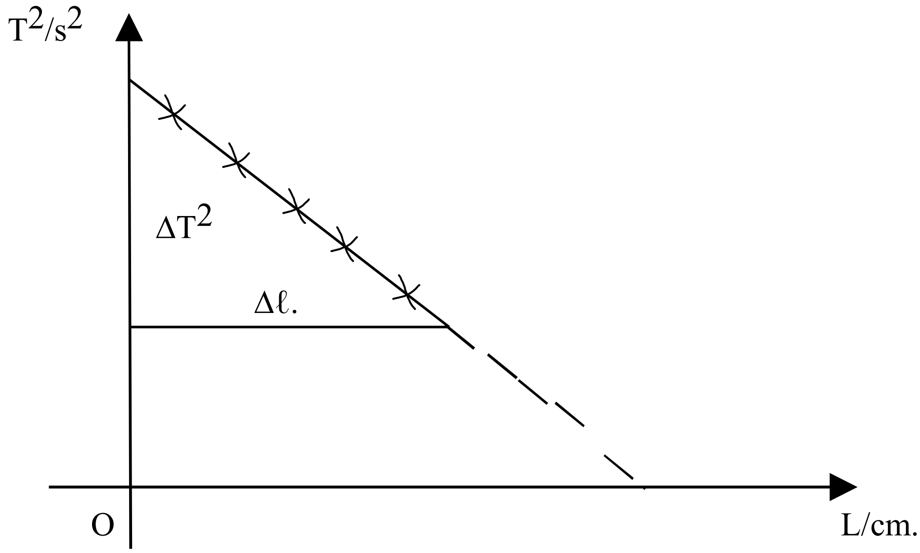
(10) Tabulate your observations as shown below

TABLE OF OBSERVATIONS

	Time for 20 oscillations		Meant t	$\frac{t}{20} = T$ T/s	T^2/s^2
	t_1	T_2			

GRAPH

Plot the graph of T^2 on the vertical axis and h on the horizontal axis. You should obtain a graph similar to the one shown below,



(i) Determine the lope m of the graph as $m = \frac{\Delta T^2}{\Delta h}$

Calculate the value of g from $m = -\frac{4\pi^2}{m}$

(ii) Determine the intercept of the graph on the x-axis when $T^2 = 0$
Estimate the error in (i) g
(ii) H

CONCLUSION

1. Acceleration due to gravity
2. Height of the room

Now with the assistance of your friend, determine the height H of the room by using a tape.
Compare your answers.

EXPERIMENT 3

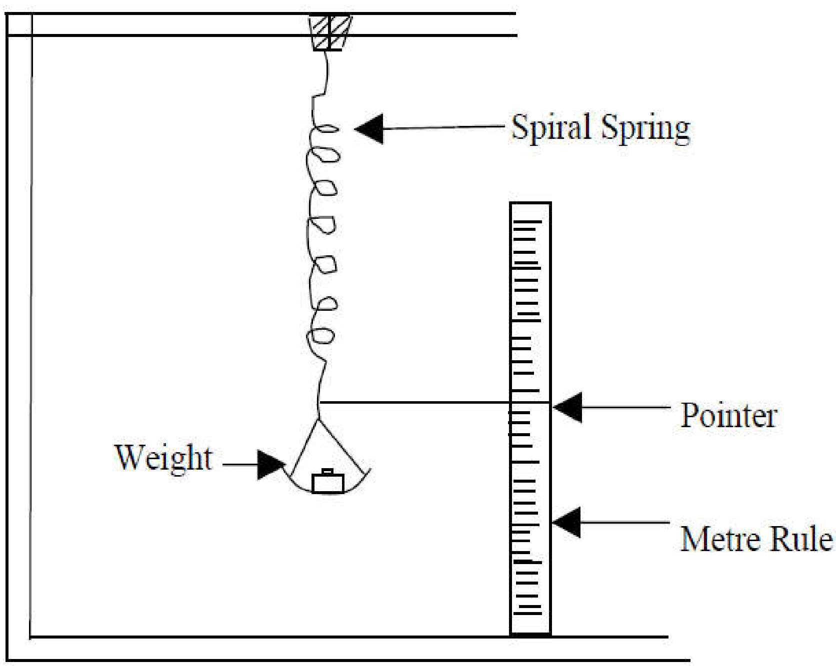
The determination of the force constant of a spiral spring

AIM: (1) To verify Hooke's law
(ii) To determine the force constant of a spiral spring

APPARATUS

- Spiral spring
- Slotted weights
- Retort stand and clamp
- Metre rule
- A pointer

DIAGRAM



THEORETICAL BACKGROUND

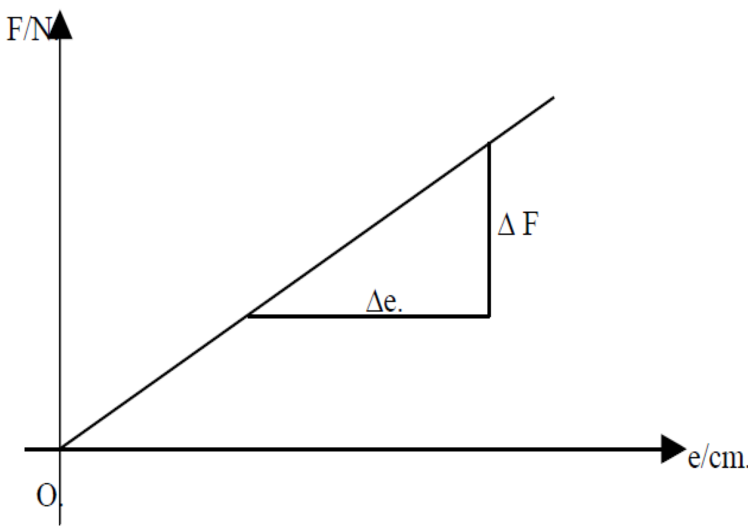
The spiral spring will behave as an elastic material if subjected to a force. Provided its elastic limit is not exceeded you will expect that Hooke's law will be obeyed.

You will recall that Hooke's law states that, the force on an elastic material is directly proportional to the extension produced by the force provided the elastic limit is not exceeded.

i.e. $F = - ke$

where k is the force constant of the spiral spring.

The negative sign shows that the original shape of the spring can be restored. If the graph of the force F is plotted against the extension (e) we shall obtain a linear graph like this



Where the slope $m = \frac{\Delta F}{\Delta e} = k$ the force constant provided the elastic limit is not exceeded.

A linear graph shows that Hooke's law is obeyed while the slope obtained gives the force constant (k)

PROCEDURE

(1) Suspend the spiral spring vertically on the retort stand firmly

(2) Attach to the lower end the first of the slotted weights

(3) Clamp the metre rule vertically along the side of the spiral spring so that a small pointer (a pin) attached to the bottom of the weights moves along lightly against the metre rule

(4) Record the reading of the pointer as e, and the weight wl attached to the spiral spring

(5) Increase the force by successive increments of 10 or 20 g and record the pointer reading each time

(6) When you have taken about five of such readings start unloading the weights and again record the pointer readings

(7) Tabulate your readings as in the section for observations

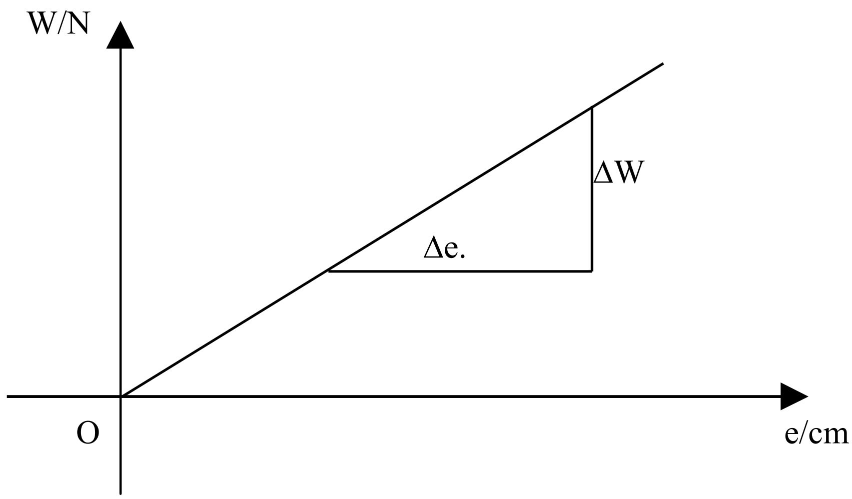
TABLE OF OBSERVATIONS

Mass	Weight /N	Pointer Reading		Mean Pointer Reading
		Weight Increasing	Weight Unloading	
	W1	e1	e1	$\bar{e}1$
	W2	e2	e2	$\bar{e}2$
	W3	e3	e3	$\bar{e}3$
	:	:	:	:

Additional Load/g	Pointer position (cm)			Extension (cm)	Extension for
	Loading	Unloading	Mean		
10				a	
20				b	b - a
30				c	c – a
40				d	d – a
50				e	e – a
60				f	f – a
70				g	g – a
80				h	h - a

GRAPH

Plot the graph of weight in Newtons on the y-axis against the extension on the x-axis.



A straight-line graph shows a linear relationship between the force and the extension. This shows that $W \propto e$.
That is Hooke's law is obeyed. $W = ke$
The slope $m = \frac{\Delta W}{\Delta e} = k$, the force constant

CONCLUSION

(I) $F \propto e$

(II) $k = \frac{\Delta W}{\Delta e}$ = …………….

EXPERIMENT 4

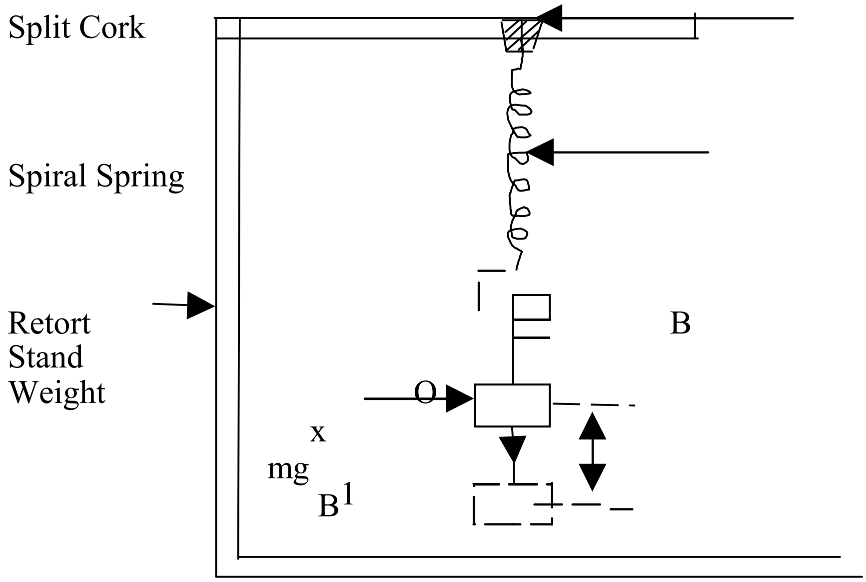
Determination of effective mass of a spiral spring and the spring constant

AIM: To determine the effective mass of a spiral spring by oscillation method

APPARATUS

- A spiral spring
- A set of slotted weights
- A stop watch
- Retort stand and clamp
- Split cork

DIAGRAM



THEORETICAL BACKGROUND

Let M be the mass of the slotted weight. The weight on the spiral spring is then Mg. According to Hooke's law, this force produces an extension e.

.'.$Mg=ke$

where k = spring constant

$F=Mg = ke$

If the mass is pushed down through a displacement x due to a small increased force Fδ.

$F + \delta F = k (e + x)$

with $F = ke$

$F + \delta F - F= k (e +x) - ke$.

$\delta F = ke + kx - ke$

$\delta\delta F = kx = Ma$

where, a = acceleration imparted to mass M

$Ma = kx$

.'. $a = - \frac{k}{M} x$

i.e. a = x

Therefore the motion is a simple harmonic motion because the acceleration is proportional to the displacement x and always directed to the equilibrium point.

Compare $a = -\frac{k}{M}x$ with $a = -\omega^2 x$

$$\omega^2 = \frac{k}{M}x$$

$$\therefore \omega = \sqrt{\frac{k}{M}}$$

The period of oscillation $T = \frac{2\pi}{\omega}$

$$\therefore T = 2\pi\sqrt{\frac{m}{k}}$$

This equation does not include the effective mass of the spiral spring in. So we add m to M.

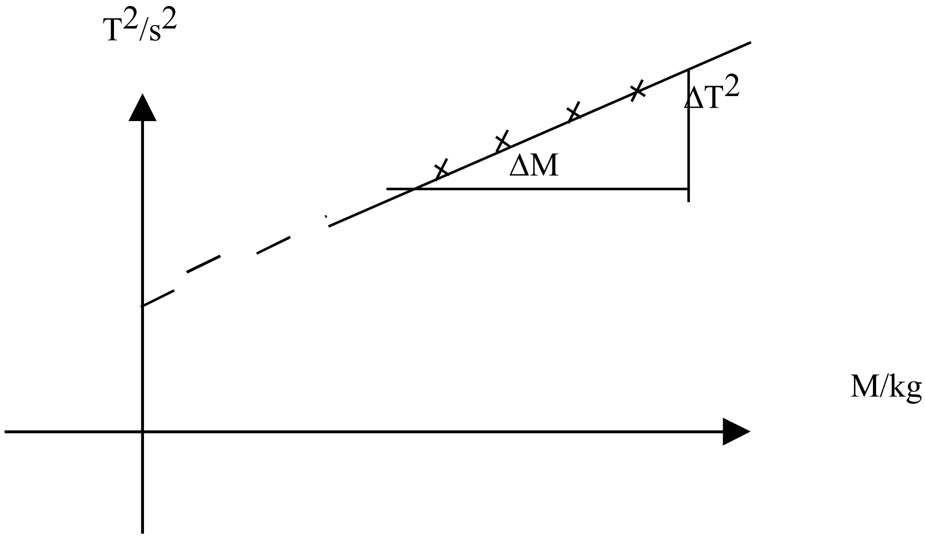
$$\therefore T = 2\pi\sqrt{\frac{M+m}{k}}$$

Squaring both sides of the equation.

$$\therefore T^2 = 4\pi^2 \frac{M+m}{k}$$

$$\therefore T^2 = \frac{M}{k} + 4\pi^2 \frac{M}{k}$$

If we plot the values of T^2 against the corresponding values of M, a graph shown below is obtained.



The slope $\frac{\Delta T^2}{\Delta M}$ = $\frac{4\pi^2}{k}$ = m

The value of k can be obtained
The intercept on the x-axis gives us the value of m when $T^2 = 0$.

$0 = \frac{4\pi^2}{k} M + \frac{4\pi^2}{k} m$

$\therefore M = - m$

Hence the effective mass of the spiral spring is the intercept of the graph on the x axis.

PROCEDURE

(1) Set the spiral spring on the retort stand rigidly by using the split cork. (2) Mark

the equilibrium position of the slotted weight and use it as a reference point

(3) Give the hanger and weight a small displacement downwards to set the spiral spring oscillating

(4) Determine the time t for 20 complete oscillations using 10 g mass of the slotted weight

(5) Repeat this timing

(6) Increase the load by another 10 g weight and again take the time for 20 complete oscillations twice

(7) Continue to increase the load on the hanger for another three sets of weights and then determine the corresponding time for 20 complete oscillations.

(8) Tabulate your readings under the observations below

TABLE OF OBSERVATIONS

Mass M	Time for oscillations		Mean time t	Period $T = \frac{t}{20}$ / s	T^2/s^2
	Loading	Unloading			

GRAPH

Plot the graph of T^2 on the y-axis against the corresponding values of M on the x-axis. A straight line graph shows the linear relationship between T^2 and M as shown below:

Calculate the value of k from slope = $\frac{4\pi^2}{k}$

Find the intercept of the graph on the x-axis. What is the effective mass of the spiral spring?

Estimate your error in (1) the value of k
(ii) the value of m

CONCLUSION

The spring constant k =.............................

The effective mass of the spiral spring =.............................

EXPERIMENT 5

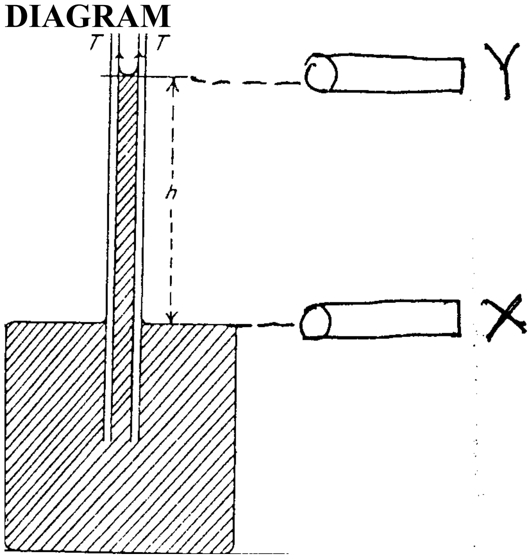
Determination of surface tension of water by rise in a capillary tube

AIM: To determine the surface tension of water by the rise of water in a capillary tube

APPARATUS

A set of glass capillary tubes
Travelling microscope
A beaker
Distilled water
Retort stand with clamp
Clearing fluids such as nitric acid and caustic soda solution

DIAGRAM



THEORETICAL BACKGROUND

The weight of the water column of height h supported in the capillary tube due to surface tension y is

$\pi^2 hpg$

where, r = radius of the capillary tube

h = height of the water column in the capillary tube p = the density of water

g = acceleration due to gravity

This weight is being supported by the upward surface tension acting around the circumference of the meniscus of the water in the tube is given as 2πry.

$$2\pi ry = \pi^2 hpg$$

$$y = \frac{4\pi^2 hpg}{2\pi r}$$

$$= \frac{rhpg}{2} Nm^{-1}$$

With very fine capillary tubes the meniscus surface may be considered to be hemispherical and the weight of the liquid above the lowest point of the meniscus is given as

$$\pi r^3 - \frac{2\pi^3}{3} \Big) pg = \frac{1}{3}\pi^3 pg$$

This weight has to be added to the main weight of the water column.

$$:.\ 2\pi ry = 1\pi^2 \ hpg + \frac{1}{3}\pi^3 pg$$

$$y = \frac{\pi^2 hpg + \frac{1}{3}\pi^3}{2\pi} pg$$

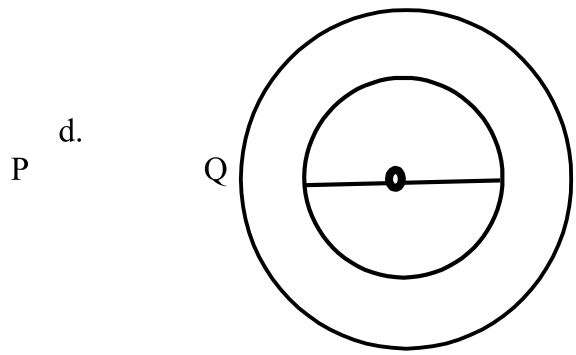
$$= \frac{\pi^2 pg}{2\pi r} \ h + \frac{r}{3} \quad [\ \ ]$$

$$y = \frac{1}{2} rpg \ \ h + \frac{r}{3} \quad (\ )$$

We can then determine various values of r and their corresponding values of h to obtain 7 at the room temperature.

PROCEDURE

(1) Obtain about five different capillary tubes of different radii
(2) Clean them thoroughly by first washing with caustic soda solution and nitric acid and then rinse them with pure water
(3) Fill the beaker with water to the brim
(4) Insert the capillary tube in the water contained in the beaker
(5) Observe the capillary tube in the water contained in the beaker
(6) Observe the capillary rise of water in the tube
(7) Use the travelling microscope to determine the height (h) of the capillary rise of water in the tube by recording X and Y positions of the travelling microscope. Thus h =- Y-X
(8) Remove the tube from the water and clamp it horizontally to show the circular section of the tube



(9) Measure the diameter PQ of the tube to determine the average radius of the tube
(10) Repeat the above procedure with other given capillary tubes
(11) Tabulate your readings as shown under observations
(12) Measure and record the temperature of the room

TABLE OF OBSERVATIONS

Tube	Point X	Point Y	h (Y-X)	P	Q	d (Q-P)	r d (-) 2	y
1								
2								
3								
4								
5								

CALCULATIONS

For each tube calculate the value of y and find the average. What would you consider as your sources of error?

EXPERIMENT 6

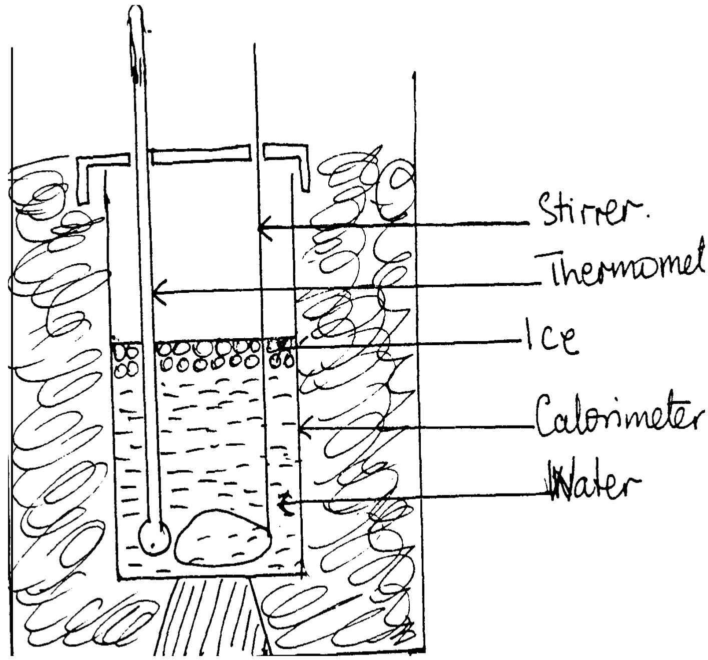
Determination of Specific Latent Heat of Fusion of ice

AIM: To determine the specific Latent Heat of fusion of ice

APPARATUS

- Lagged calorimeter
- Thermometer (0 - 50 °C in tenths)
- Pieces of ice block
- Water
- Beaker
- Beam balance
- Blotting paper

DIAGRAM



THEORETICAL BACKGROUND

When ice block melts, it changes its state from solid to liquid at its melting point which is 0 °C. The amount of heat in joules required to change 1 kg mass of ice (solid) to liquid at its melting point defines the specific latent heat of fusion of ice (Lf).
Thus if we have m kg mass of ice the total amount of heat Q1 absorbed by the ice is mLf joules.

If the temperature of this liquid water at 0 °C now changes to 0 °C then the amount of heat in joules required to change its temperature from 0 °C to Θ °C is given by Q2, where Q2 = mCΔΘ.

Where, m = mass of ice converted to liquid
C = specific heat capacity of water = 4200 $\frac{J}{kg^0C}$

AΘ = change in temperature = (0 - 0) °C = Θ °C

.'. Q2 = MCΘ

The question is "Where does the heat absorbed by the ice come from?"
The heat comes from the water in the calorimeter in which it melts. Let m1 be the mass of the water in the calorimeter, then the heat lost by this water $Q_3 = m_1C\Delta\Theta_1$.

Where, m_1 = mass of water in the calorimeter
C = specific heat capacity of water
$\Delta\Theta_1$ = change in temperature of the water from Θ_1 - 0 where, 0_1 =
original temperature of water in the calorimeter
0 = final temperature of the mixture
$Q_3 = m_1C(\Theta_1 - \Theta)$

But calorimeter and stirrer also lost some heat to the ice. Thus the quantity of heat lost by the calorimeter and stirrer is Q_4 where.

$Q_4 = M_2C_2\Delta\Theta_2$

But $\Delta\Theta_2 = \Delta\Theta_1$

$\therefore Q_4 = m_2C_2\Delta\Theta_1$

Where, m_2 = mass of calorimeter and stirrer
C_2 = specific heat capacity of calorimeter and stirrer which can be made of copper or aluminium
$\Delta\Theta_1 = (\Theta_1 - \Theta)$
$\therefore Q_4 = m_2C_2(\Theta_1 - \Theta)$

We then apply the principle of conservation of heat energy, Heat
gained by ice = heat lost by water.
Provided no heat goes out or enters the system.

Consequently, $Q_1 + Q_2 = Q_3 + Q_4$

i.e. $mL_f + mC\Delta\Theta = m_1C\Delta\Theta_1 + m_2C\Delta\Theta_1$

$mL_f + MC\Theta = m_1C(\Theta_1 - \Theta) + m_2C_2(\Theta_1 - \Theta)$

$$\therefore mL_f = \frac{m_1C(\Theta_1-\Theta)+m_2C_2(\Theta_1-\Theta)-mC\Theta}{m}$$

Hence, L_f can be determined.

PROCEDURE

(1) Weigh the calorimeter and stirrer empty and dry = m_1

(2) Use the thermometer to measure the room temperature $=\Theta_1$

(3) Warm some water say to 3 5 °C

(4) Fill the calorimeter halfway with the warm water and weigh it containing the warm water and stirrer

(5) When the temperature is about 10 °C above the room temperature Θ_1. Record the temperature at t_1 °C

(6) Dry a few small pieces of ice between the sheets of blotting paper and quickly add them to the water in the calorimeter

(7) Stir the content until all the ice has melted

(8) Continue to add the dry ice until the temperature of the calorimeter and contents is about as far below room temperature as it was originally above

(9) Record the new temperature as t_2 °C

(10) Weigh the calorimeter + water + the melt ice (m_3)

TABLE OF OBSERVATIONS

Mass of calorimeter empty = m_1 = ... kg

Mass of calorimeter + warm water = m_2 = ... kg Temperature of

calorimeter + warm water = t_1………… ^{0}C

Temperature of calorimeter + water + melted ice = m_3 = ... kg

CALCULATIONS

Mass of melted ice = $(m_3 - m_2)$
Mass of warm water = $(M_2 - M_1)$
Heat gained by ice to melt it = $Q_1 = (m_3 - m_2) L_f$
Heat gained by melted ice to change its temperature to t_2 °C

$$= Q_2 = (m_3 - m_2)\, C\, (t_2 - 0)\, ^oC$$
$$= (m_3 - m_2)\, C t_2$$

Heat lost by warm water = $Q_3 = (m_2 - m_1)\, C\, (t_1 - t_2)$
$Q_3=(m_2-m_1)C(t_1-t_2)$
Heat lost by calorimeter and stirrer = $Q_4 = m_1\, C_1\, (t_1 - t_2)$
where, C_1 = specific heat capacity of calorimeter and stirrer
Using the conservation of heat energy

Heat lost = Heat gained
$\therefore Q_1 + Q_2 = Q_3 - Q_4$
$(m_3 - m_2)L_f + (m_3 - m_2)Ct_2 - (m_2 - m_1)C(t_1 + t_2) + m_1C_1(t_1 - t_2)$

$. : (m3 - m2)Lf - (m2 - m1)C(t1 - t1) + m1C1(t1 - t2(m3 - m2)Ct2$

$$.'. Lf = \frac{(m2-m1)C(t1-t2)+m1Cl(t1-t2)-(m3-m2)Ct2}{(m3 - m2)}$$

CONCLUSION

The specific latent heat of fusion of ice = ...$\frac{J}{kg}$

PRECAUTIONS

Mention two precautions taken in performing this experiment.

UNIT 4G: EXPERIMENT 7

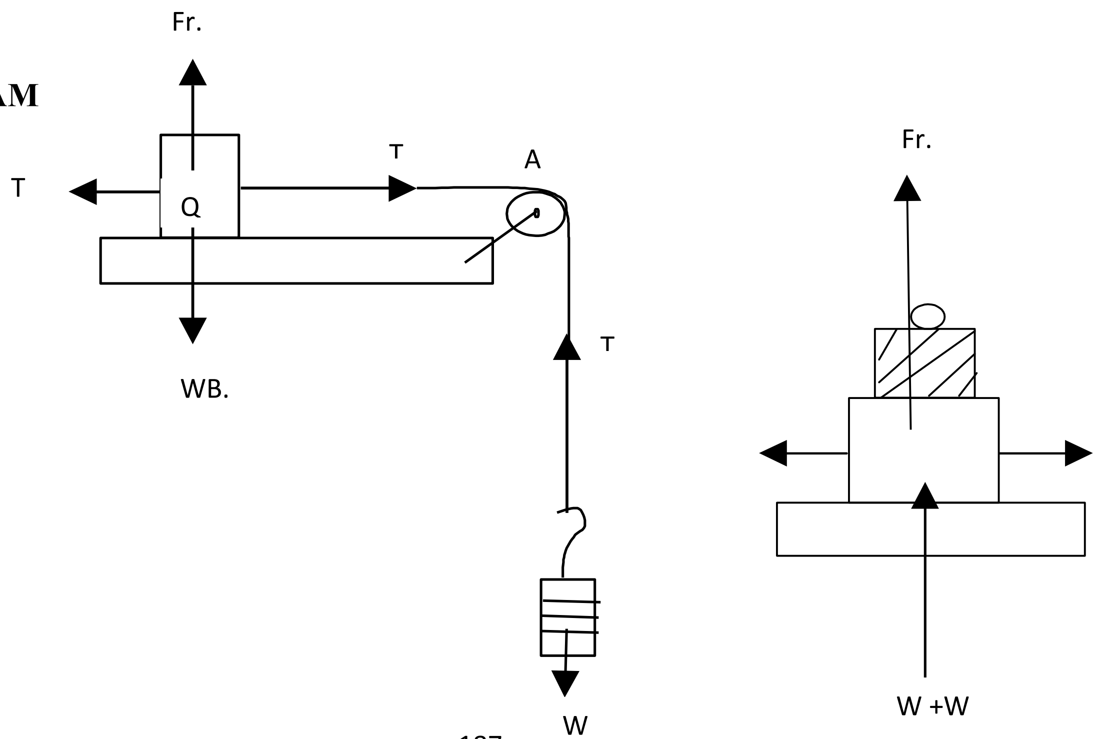
Determination of the coefficient of limiting static friction between two surfaces.

AIM: To determine the coefficient of limiting static friction between two surfaces.

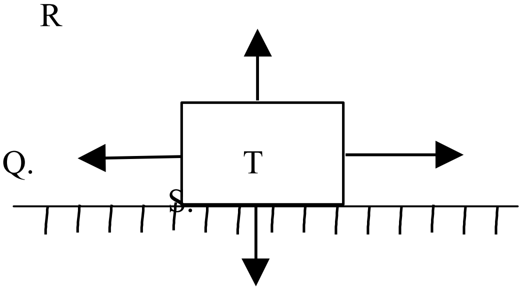
APPARATUS

- A long smooth piece of wood P
- A block of wood Q having one smooth surface
- A single pulley A
- Two sets of known weights
- Light cord
- Spring balance
- Pieces of chalk

DIAGRAM



THEORETICAL BACKGROUND



When an object Q is on a surface S, the vertical forces acting on it are the weight W and the normal reaction R acting in opposite directions to each other. The resultant of these two forces is zero because there is no relative motion downwards or upwards from the surface S.

However if a horizontal force T is brought to play on Q, there is a limiting value of T that will make the body Q to move. This limiting value is equal to the frictional force Fr that will oppose the motion.
The coefficient of friction p is an associated property of the two surfaces in contact and it is defined as

$$\mu = \frac{FrictionalForce(Fr)}{Normal\ Reaction\ (R)}$$

But the normal reaction R is equal to the weight of the object (W), which is equal to (mg) i.e. the product of the mass of Q and the acceleration due to gravity (g)

$$\therefore \mu = \frac{Fr}{w} = \frac{Fr}{mg}$$

$\therefore Fr = \mu W_B$ where, W_B = weight of wood

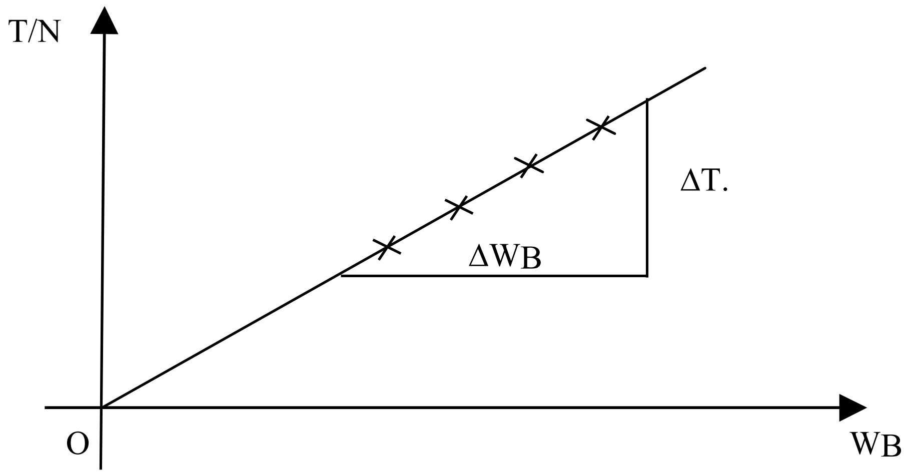
$Fr = \mu m_B g$ where, m_B = mass of the wood

But the frictional force (Fr) is equal to the limiting value of the tension in the string (T) that is equal to W.

$\therefore T = \mu W_B$

By adding more weights on top of the wood Q, we can vary the normal reaction of the wood on the horizontal surface.

By plotting the values of T against the corresponding values of W_B, the coefficient of static friction may be determined.



The slope or gradient of the graph gives the coefficient of static friction between

the two given surfaces. $\mu = \frac{\Delta T}{\Delta W_B}$

PROCEDURE

(1) Set the apparatus as shown in the diagram
(2) Ensure the long board is horizontal by using the spirit-level
(3) Rub the surfaces of P and Q with the chalk to ensure smooth surfaces of the wood
(4) Determine the weight of Q by using the spring balance
(5) Record this weight as W_B
(6) Add the load W to the string until Q just begins to move
(7) Record the weight W that is used to make the wood Q move
(8) Place a known weight w on top of Q in order to increase the normal reaction
(9) Record the new normal reaction as $W_B + w$
(10) Determine the new load W that will make the wood move again
(11) Record the new load W
(12) Repeat this procedure three more times by increasing w and recording the corresponding values of W that will make the wood just move

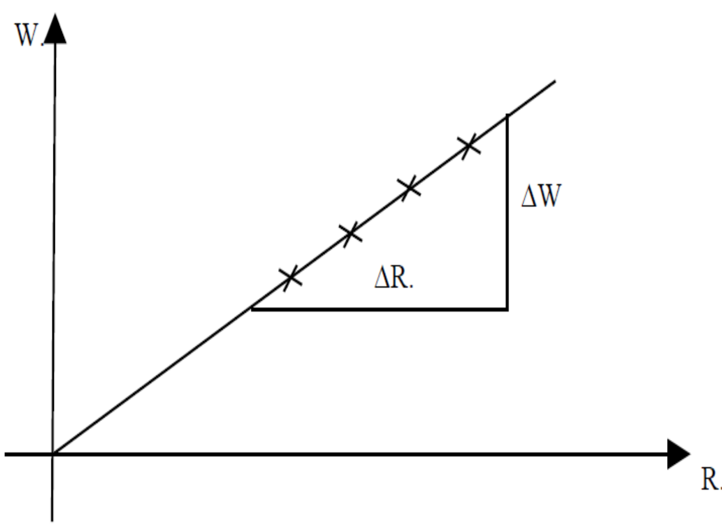
TABLE OF OBSERVATIONS

W_B = …………………N

	Normal Reaction (R)	W = Fr	$\frac{Fr}{R}$ =
1	W_B	$W_1 + Fr_1$	
2	$W_B + W_1$	$W_2 + Fr_2$	
3	$W_B + W_2$	$W_3 + Fr_3$	
4	$W_B + W_3$	$W_4 + Fr_4$	
5	$W_B + W_4$	$W_5 + Fr_5$	

GRAPH

Plot the graph of W on the vertical axis against the values of the normal reaction R on the horizontal axis.



Determine the slope as $m = \frac{\Delta W}{\Delta R} = \mu$

ΔR

Complete the table of observations by calculating the ratios of $\underline{Fr} = R$

Compare your answers with that obtained from the graph.

CONCLUSION

The coefficient of limiting friction, between the two surfaces = ...

EXPERIMENT 8

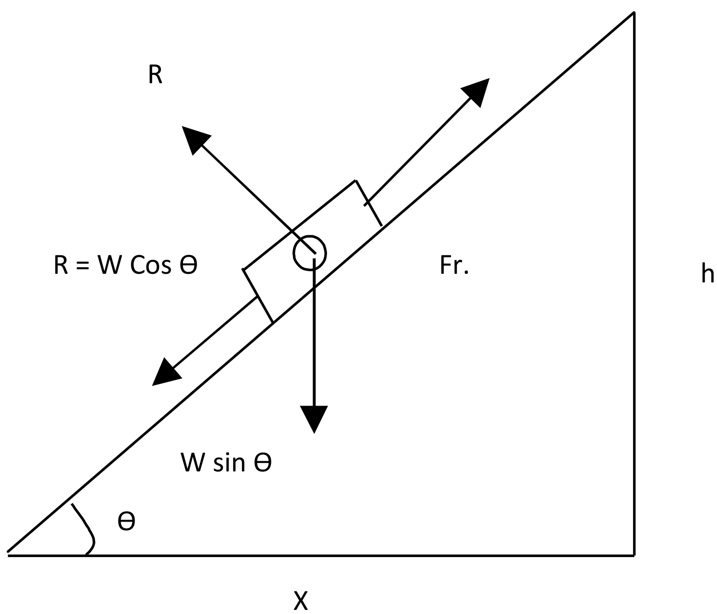
Determination of the coefficient of static friction on two surfaces using an inclined plane

AIM: To determine the coefficient of static friction between two surfaces by using the inclined plane.

APPARATUS

- An inclined wooden plane
- A wooden block
- A set of weights
- A protractor

DIAGRAM



THEORETICAL BACKGROUND

Another way to determine the coefficient of static friction is the use of an inclined plane. The inclined plane is made up of two wooden boards hinged at one end.

On top of the upper board is placed a wooden block. By gradually raising the upper board an angle Θ is made between the upper and lower boards.

The wooden board is then made just to glide down the upper board through the resolved weight (WSinΘ) of the wooden block. The angle Θ, which enables the wooden block just to glide, is then noted.

The limiting frictional force (Fr) is equal to WSinΘ. The normal reaction R on the inclined plane is WCosΘ. Hence, the coefficient of friction p,is defined as

$$\mu = \frac{W Sin\Theta}{W Cos\ \Theta}$$

$$\therefore \mu = \frac{Sin\Theta}{Cos\ \Theta}$$

$$Sin\ \Theta = \quad Cos\ \Theta$$

$$\therefore \mu = Tan\ \Theta$$

Attaching different weights on the wooden block can vary the normal reaction of the wooden block. Corresponding angles of inclination ϴ, just to make the wooden block move can then be determined.

Thus by plotting the graph of Sinϴ on vertical axis against Cosϴ on the horizontal axis, we can then determine the slope m, which is equal to the coefficient of static friction.

$$m = \frac{\Delta Sin\Theta}{\Delta Cos\ \Theta} = Tan\ \Theta = \mu$$

PROCEDURE

(1) Weigh the wooden block Q and record its weight as WB.

(2) Place the wooden block Q on the upper wooden board of the inclined plane

(3) Raise the upper board gradually to determine the angle Θ when the wooden block just starts to move.

(4) Record the angle ϴ by using the protractor. Note that in the absence of a protractor, you can use the height h and distance x to determine angle Θ

Where, $Tan\ \Theta = \frac{h}{x}$

$:.\ \Theta = Tan^{-1}\frac{h}{x}$

(5) Attach a known weight w on the wooden block Q and place it again on the inclined plane

(6) Raise the board of the inclined plane to determine the new angle A just to make the wooden block slide on the plane

(7) Record the new angle ϴ

(8) Repeat this procedure three times by adding more known weights on the wooden block Q and then determe the corresponding angle Θ to make it just slide.

(9) Tabulate your observations

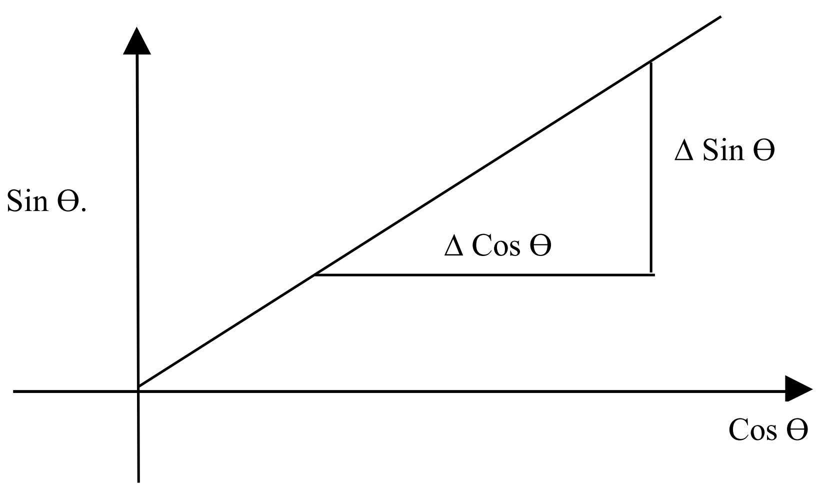
TABLE OF OBSERVATIONS

Load	Angle of inclination	Sin Θ	Cos Θ	$\frac{Sin\Theta}{Cos\ \Theta}$ =
W_B	Θ_1			
$W_B + W_1$	Θ_2			
$W_B + W_2$	Θ_3			
$W_B + W_3$	Θ_4			
$W_B + W_4$	Θ_5			

Complete the table of observations by using the four-figure table to determine SinΘ and Cos Θ.

GRAPH

Plot the graph of Sin Θ on the y-axis against Cos Θ on the x-axis. Determine the slope/gradient of the graph.



CALCULATIONS

Calculate also the ratio of $\frac{Sin\Theta}{Cos\Theta}$ for each set of reading

Compare your results with that obtained from the graph.

CONCLUSION

The coefficient of limiting friction, p, between the two surfaces = ...

EXPERIMENT 9

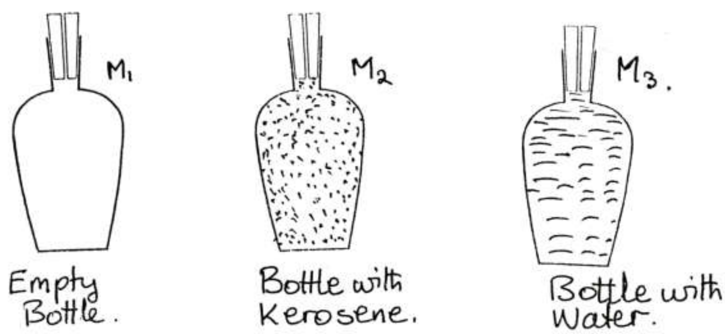
Determination of relative density of kerosene using the specific gravity bottle

AIM: To determine the relative density of kerosene or any other liquid by using the specific gravity bottle.

APPARATUS

- The specific gravity bottle
- Kerosene
- Methylated spirit
- Balance and weights
- Beaker of water

DIAGRAM



THEORETICAL BACKGROUND

The relative density of a liquid is defined as

$$Relative\ Densty = \frac{\text{Density of the liquid}}{\text{Density of water}}$$

$$= \frac{\text{mass of the liquid}}{\text{volume of the liqud}}$$

$$= \frac{\text{mass of the water}}{\text{volume of the water}}$$

$= \frac{m_1}{v_1} \div \frac{m_2}{v_2}$

$= \frac{m_1}{m_2} \times \frac{v_2}{v_1}$

Where, m_l = mass of liquid m_2 = mass of water
V_l = volume of liquid
V_2 = volume of water

If $V_1=V_2$

Relative Density $= \frac{m_1}{m_2}$

That is the relative density of the liquid is defined as the ratio of the mass of the liquid over the mass of an equal volume of water.
The volumes of the liquid and the water are made to be equal by using the specific gravity bottle, which has a constant volume V. The special stopper made for the bottle helps to achieve this. The stopper has a narrow opening to ensure that the volume of the liquid contained in the bottle is constant.

If the mass of the bottle when empty	=	m_l
And the mass of bottle + kerosene	=	m_2
And the mass of bottle + water	=	m_3
:.mass of kerosene	=	$(m_2 - m_l)$
and mass of equal volume of water	=	$(km_3 - m_l)$

. . Relative density of kerosene = $\frac{m_2-m_l}{m_3 - m_1}$

PROCEDURE

(1) Clean the bottle and stopper with Methylated spirit and dry it by using a hot air drier

(2) Weigh the bottle and stopper empty by using the balance and record this mass as m_1.

(3) Fill the bottle with kerosene whose relative density is to be determined

(4) Weigh the bottle, stopper and the kerosene using the balance

(5) Record this as mass m_2

(6) Pour out the kerosene and clean it again with the Methylated spirit and dry

(7) Fill the bottle with water and use the blotting paper to wipe the excess water on the bottle

(8) Then reweigh the bottle, stopper and water

(9) Record the mass of bottle, stopper and water as m3

NOTE

- In this experiment always weigh the bottle with the stopper
- Observe that no air bubbles are left in the liquid near the base of the stopper. If there is any, remove the stopper add more liquid and then reinsert the stopper carefully

•Be careful not to warm the bottle by the heat of the hand. This may make the liquid expand and that will not provide the desired constant volume. Handle the bottle with care.

CALCULATIONS

If the mass of empty bottle and stopper = m1
The mass of bottle + kerosene = m2
And the mass of bottle + water = m3
.'. mass of kerosene = m2 – m1,
and the mass of equal volume of water = m3 - ml

:. Relative density of kerosene = $\frac{m_2}{m_3} - \frac{m_1}{m_1}$

If p = Density of kerosene
pw = Density of water
pr = Relative density of kerosene

By definition $Pr = \frac{p}{p_w}$

:. $P = Pr\,Pw$

$$P = \frac{m_2}{m_3} - \left(\frac{m_1}{m_1} P_w\right)$$

Where, P_W, = 1000kg/m^3 or 1g/cm^3
Therefore, the density of kerosene can be determined.

CONCLUSION

The relative density of kerosene == ……………………….
The density of kerosene ……………………..

EXPERIMENT 10

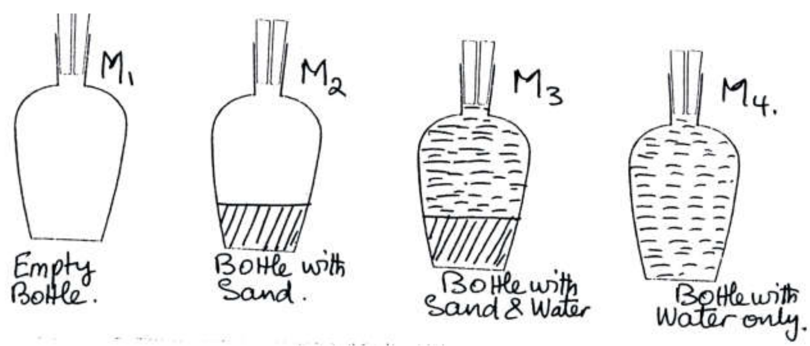
Determination of the relative density of a granular substance not soluble in water using the specific gravity bottle.

AIM: To determine the relative density of granular sand using the specific gravity bottle.

APPARATUS

- Specific gravity bottle
- Granulated sand
- Water
- Balance
- Set of weights

DIAGRAM



THEORETICAL BACKGROUND

The relative density (R.D.) of a substance is defined as

$$\text{R.D.} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

$$= \frac{\text{mass of substance}}{\text{volume of substance}}$$

$$= \frac{\text{mass of water}}{\text{volume of water}}$$

Let pr represent the relative density of the substance
P_S = density of the substance
P_W = density of water m_S = mass of substance
V_S = volume of substance
V_W = volume of water

.'. By definition

$$Pr = \frac{PS}{PW}$$

$$\therefore Ps = Pr\ P_W$$

Thus if pr and p_W are known p_S can be determined. But what is p_r?
If

$$Pr = \frac{p_s}{p_w}$$

$$= \frac{m_s}{v_s}$$

$$Pr = \frac{m_w}{v_w}$$

$$= \frac{m_s}{v_s} \div \frac{m_w}{v_w}$$

$$= \frac{m_s}{v_s} \times \frac{v_w}{m_w}$$

$$= \frac{m_s}{M_m} \times \frac{v_w}{v_s}$$

If V, ie if the volume of water is equal to the volume of the substance.

$$Pr = \frac{m_s}{m_w}$$

$$= \frac{\text{massofthesubstance}}{\text{mass of an equal volume of water to that of the substance}}$$

How do we make $V_W = V_S$?

We do so by using the specific gravity bottle.
For the granular sand we determine the following:

1. Mass of the empty bottle = m1

2. Mass of the bottle + some sand = m_2
   .-.Mass of sand = $m_2 - m_1$
3. Mass of bottle + sand + water = m_3
4. Mass of bottle + water only = m_4
   .-.Mass of equal volume of water = $(m_4 - m_1) - (m_3 - m_2)$
   .'. pr of sand = m

$$Pr\ of\ sand = \frac{m_2 - m_1}{(m_4 - m_1) - (m_3 - m_2)}$$

PROCEDURE

(1) The specific gravity bottle is cleaned and dried
(2) It is then weighed empty as (m_1)
(3) The specific gravity bottle is now filled with one-third full of sand
(4) The bottle with sand is then weighed as (m_2)
(5) Fill the bottle containing the sand with water and shake very well to ensure that it does not harbour any air bubbles
(6) Weigh the bottle containing the sand and water as (m_3)
(7) Remove the content of the bottle, clean it and now fill it with only water
(8) Weigh the bottle with water only as m_4

CALCULATIONS

The mass of the sand = $m_2 - m_1$
Mass of the water that fills the bottle alone = $m_4 - m_1$
Mass of water that fills the bottle with sand = $m_3 - m_2$
:.Mass of equal volume of water to that of the sand = $(m_4 - m_1) - (m_3 - m_2)$ Hence,

$$\text{Relative density of the granular sand} = \frac{m_2 - m_1}{(m_4 - m_1) - (m_3 - m_2)}$$

$$P = \frac{m_2 - M_1}{(m_4 - m_1) - (m_3 - m_2)}$$

$$P_s = \frac{m_2 - m_1}{(m_4 - m_1) - (m_3 - M_2)} \times P_w$$

CONCLUSION

Relative density of sand = ...
Density of sand = ...kgm^{-3}

ANSWER TO ACTIVITY

SOLUTION TO ACTIVITY 1 UNIT 2

(a) 19.87 cm

(b) 3.21 s

(c) Equally precise

SOLUTION TO ACTIVITY 2 UNIT 2

(a) The relative errors are:

$$\frac{0.05}{40} = \frac{5}{4000} = \frac{1}{800}$$

$$\frac{0.05}{8} = \frac{5}{800} = \frac{1}{160}$$

Therefore the measurement 40.0 cm is more accurate. (b) 0.85 m

SOLUTION TO ACTIVITY UNIT 2

3×10^{-23} g

SOLUTION TO ACTIVITY 4 UNIT 2

S/No.	Measurement	Possible error	Relative error
1.	0.2 m	0.05 m	$\frac{0.05m}{0.2\ m} = 0.25$
2.	0.20 m	0.005 m	$\frac{0.005m}{0.20\ m} = 0.025$
3.	0.2000 m	0.00005 m	$\frac{0.00005m}{0.2\ m} = 0.00025$
4.	25 m	0.5 m	$\frac{0.5m}{25\ m} = 0.2$
5.	250 m	0.5 m	$\frac{0.5m}{250\ m} = 0.002$
6.	25000 m	0.5 m	$\frac{0.5m}{25000\ m} = 0.00002$
7.	102 m	0.5	$\frac{0.5m}{102\ m} = 0.0049$
8.	1002 m	0.5	$\frac{0.05m}{1002\ m} = 0.000499$

(a) They are significant

(b) They are also significant. The zeros are significant only if they come from a measurement. But if fifth and sixth measurements are expressed in centimeters as 2500 cm and 2500000 cm respectively, the last two zeros should not be counted as significant as these have come as a result of multiplication by the factor 100 and not from measurement.

(c) Significant

SOLUTION TO ACTIVITY 5 UNIT 2

Sometimes we take a sequence of whole number measurements such as 32, 30, 28, 26. All these measurements have two significant digits except the measurements 30. In such special cases zero can be taken as significant without any ambiguity.

SOLUTION TO ACTIVITY 6 UNIT 2

See text book

SOLUTION TO ACTIVITY 7 UNIT 2

1.4

Let us consider the multiplication of the following numbers, which have already been rounded off to significant digits.

5.2865 x 3.8 x 19.62
= 20.0887 x 19.62
= 394.14029

which must be rounded of to 3.9 x 10^2. We could have obtained the same result by rounding off these numbers first as shown below.

5.29 x 3.8 x 19.6
= 20.1 x 19.6
= 393.9

which rounds off to 3.9 x 10^2.

Here we have rounded off 20.102 (the product of 5.29 and 3.8) to 20.1 before multiplying it with 19.6. We can generalise this as a labour saving rule.

Labour Saving Rule: Before multiplying (or dividing), round off the numbers to one more significant digit than (the number of significant digits) in the least precise factor.

SOLUTION ACTIVITY 8 UNIT 2

3.0

SOLUTION TO ACTIVITY 9 UNIT 2

0.04

SOLUTION TO ACTIVITY 10 UNIT 2

HINT: In such cases we can use the following saving rule.
Labour Saving Rule: Before adding (or subtracting) round off the numbers so that they contain one more digit of precision than the number of precision digits in the least precise. Thus the addends become 2.155 m, 2.11 m and 2.125 m.

SOLUTION TO UNIT 3 ACTIVITY:

1(a)

Unlike systematic errors, random errors can be quantified by statistical analysis. Let us now learn to determine the size of such error.

2 (a) 135.0 cm

(b) 0.375 cm

3 E = AB
Taking logarithm on both sides

log E = log A + log B

Differentiating partially

$$\frac{\delta E}{E} = \frac{\delta A}{A} + \frac{\delta B}{B}$$

The statistical analysis, however, gives the following better result of the fractional error in E.

$$\frac{\delta E}{E} = \sqrt{\left(\frac{\delta A}{A}\right)^2 + \left(\frac{\delta B}{B}\right)^2}$$

4

(i) sin 90° = 1.000, sin 90.5° = cos 0.5° = 1.000, sin 89.5° = 1.000

In this case error in sin 90° is zero.