MODULE 1

UNIT 1 CONCEPT OF TEMPERATURE

CONTENTS

1.0 INTRODUCTION

You will recall that the concepts of length, mass and time are regarded as fundamental quantities during the study of elementary mechanics. You also learn about derived quantities such as force, momentum and energy. In this unit, you will be introduced to another fundamental quantity called temperature.

Temperature is a fundamental quantity in the study of heat (thermal energy) or thermodynamics. Some students usually get confused with the concept of temperature and heat. Heat is a form of energy. Temperature is a sensation of hotness and coldness. Hence in this unit we shall make an attempt to differentiate between the two.

We would then explain the concept of temperature by using the zeroth law of thermodynamic. Having done this, we would then establish how the temperature of a body is measured through the use of thermometers with emphasis on their thermometric properties. In the next unit, you will learn about the types of thermometers.

2.0 OBJECTIVES

After the end of this unit, you should be able to:

- differentiate between temperature and heat;
- define the concept of temperature;
- explain the terms used for the zeroth law of thermodynamics;
- state the law of thermodynamics;
- define temperature scales; and
- solve problems on conversion of temperatures on Celsius, Fahrenheit and Absolute scales.

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.0 MAIN CONTENT

3.1 Concept of Temperature and Heat

The concepts of temperature and heat are two kinds of concepts in physics that are so closely related that may confuse you as a learner in the study of physics. You need to be clear in your mind what exactly they mean.

You will remember that in your study of mechanics you learnt various forms of energy. Heat was among them. Heat measurement is usually referred to as Calorimetry. Also in the study of Integrated Science, you will also recall that temperature is the degree of coldness or hotness of a body. Temperature measurement in physics is referred to as thermometry. *Then the question arises: Why is Heat a form of energy?* It is a form of energy because we use it to do work for us.

In the history of Industrial Revolution, heat engines were used to perform various kinds of work as in the textiles factories and in the locomotive engines used for transport. Besides, at home we use it for cooking and ironing our clothes.

Heat is an intangible agency that causes increase of hotness of a body. A body is said to be warmer if it receives heat and colder when it losses heat. You will therefore observe that increase in heat content of a body can be caused by any of the following ways:

- direct application of mechanical work (as in friction between two bodies);
- churning of a liquid;
- contact with a hotter body.

Heat therefore flows from a body whose degree of hotness (temperature) is greater to the body of lower degree of hotness (temperature). Consequently, it is the heat energy that is transferred and not the temperature. As a rule, when two bodies of different temperatures are placed in contact with each other, by definition, the heat lost by one body is absorbed by the other body provided the two bodies neither lose nor gain heat from the surrounding. In this case we would describe such heat energy interaction as being conserved in the system.

In this unit, we would discuss in details what is meant by temperature and how it is measured using the appropriate scales. Using the zeroth law of thermodynamics will do this. The concept pf heat and its measurement will be dealt with later.

Activity 1

Distinguish between the concepts of heat and temperature.

3.2 Concept of Temperature

The concept of temperature is one of the fundamental concepts in physics. It is rather difficult to define this physical quantity as compared with, say, the length of a body. A meter is a standard length of a bar of platinum – iridium kept in Paris. We can directly copy this bar and use it to find out how many times an object is as long as this bar at any time and place. This is not the case with temperature. Temperature is measured through an indirect method, as you will observe later in our discussion. *But the question you may ask now: What is temperature? How can we measure the temperature of a body?*

Qualitatively, temperature of a body is the degree of hotness or coldness of a body. However, this answer does not lead us to the quantitative definition of temperature or its operational definition.

This is because the sensations of hotness and coldness are highly subjective. The way you feel is not the same as any other person. Other adjectives used include cold, cool, tepid, warm, etc. Using our feeling of heat to estimate degree of hotness/coldness (temperature) is very personal and very unreliable since its measurement is personal and not a standard one. Hence when we say the temperature of a body is cold, hot or lukewarm such descriptions are rather too vague to comprehend quantitatively.

Consequently, an independent scale of temperature measurement is therefore highly essential. We would now discuss how we have evolved the scale of temperature measurement.

3.2.1 Thermal Equilibrium

Two bodies may be at different temperature – one hot and the other cold. The hot one is said to possess more heat energy than the colder body. In another sense, the temperature of the hot body is higher than the colder body.

However, if the two bodies are now in contact with each other, heat energy flows from the hot body to the cold body until the temperatures of the two bodies are the same. The two bodies are then described as being in thermal equilibrium with each other.

Therefore, a thermal equilibrium exists between two bodies when they are in thermal contact with each other and there is no net flow of heat between them.

It is the temperature of a body that determines the direction of flow of heat from that body to another. It will flow until two temperatures are the same i.e. there is a thermal equilibrium. Once, there is a thermal equilibrium between two bodies, then it means that the two temperatures are the same – no net flow. *But now the basic question arising: How do we then define the temperature of a body quantitatively?* Let us discuss this concept.

3.2.2 The Zeroth Law of Temperature

The zeroth law of thermodynamics helps us to quantify the concept of temperature objectively. Quantitative definition of temperature involves terms of operations that must be independent of our sense perceptions of hotness or coldness. That is, temperature has to be measure objectively and not subjectively.

It has been observed that there are some systems in which a measurable property of the system varies with hotness or coldness of the system.

For example:

(i) the length L of a mercury column in a thin tube will change variation in temperature (fig. 1.1).

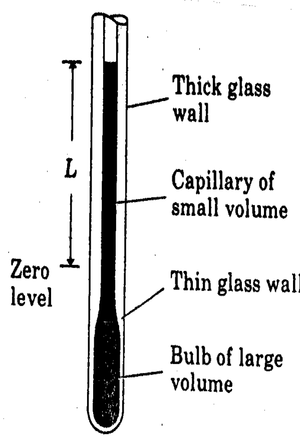


Fig. 1.1: A System Whose State is Specified by the Value L.

(ii) the pressure P of a constant volume container, measured by a pressure gauge or a manometer (fig. 1.2).

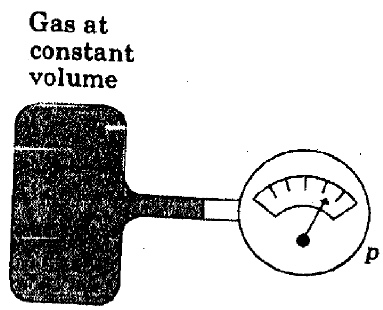


Fig. 1.2: A System Whose State is Given By the Value P (Pressure)

(iii) the electrical resistance (R) of a wire which varies with hotness or coldness as with the platinum resistance (fig. 1.3).

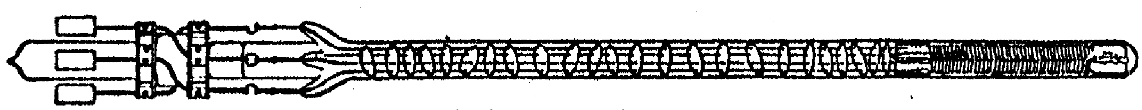


Fig 1.3: Platinum Resistance

(iv) the electromotive force (E) of a thermo-junction varies also with hotness or coldness of the system (fig. 1.4).

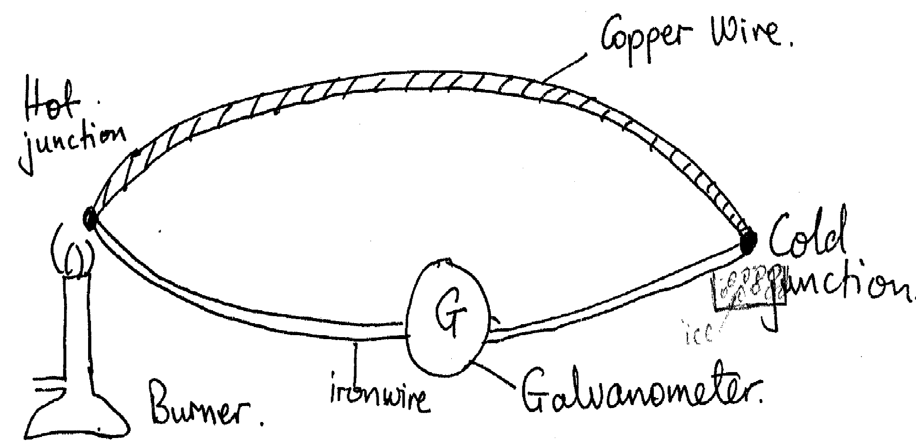


Fig. 1.4: A Thermocouple

In each case, discussed above the quantity describing the varying state of the system such as L, P, R and E are called a state of coordinate for the system.

In thermodynamics, bodies are brought into contact in order to establish the common temperature using one of these coordinates. The two bodies may be in direct contact or they may be separated by two types of wall. The types of wall are namely adiabatic and diathermic walls. Adiabatic walls are those through which no heat can be transmitted whereas the walls through which heat can be transmitted are known as diathermic walls. In thermodynamics, these two words are used to describe the process of thermal equilibrium that is, of being at thermal equilibrium. They will help you to understand the zeroth law.

Let us consider two systems A and B such that system A is a mercury- in-glass tube with a state coordinates L and system B, a constant-volume gas container with state coordinate P (fig. 1.5).

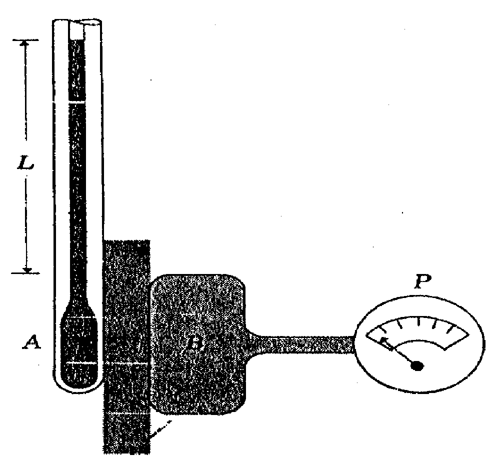


Fig. 1.5: Adiabatic Wall

If system A is at a higher temperature than system B, when brought in contact with each other, then their state coordinates change. However, if the two systems are separated by insulating such as wood, plastic or fibre glass, the change in the state coordinate will be low or none. Such an insulating material is described as an adiabatic wall. Therefore, in general, an adiabatic wall is such in which, in an ideal situation, there is no change in the state of coordinates (fig. 1.5).

The opposite of adiabatic wall will then be a wall or a partition that will allow the systems A and B to influence each other (fig. 1.6).

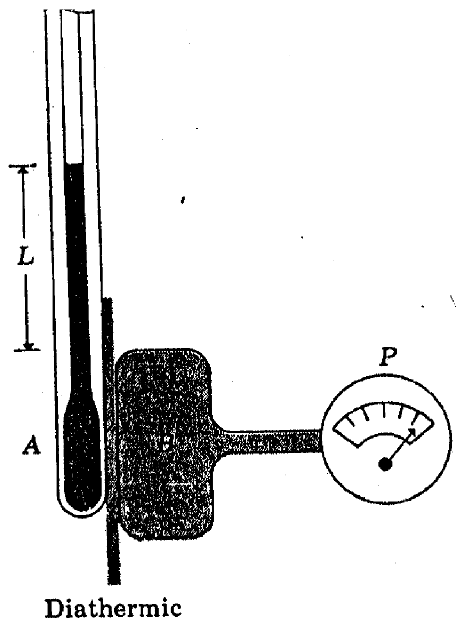


Fig. 1.3: A Diathermic Wall

In this case it will allow free exchange of heat energy. Such a wall is called a diathermic wall. Examples of such diathermic walls are copper and aluminum. Thus when the two systems A and B are separated by a diathermic wall, initially there may be no change but eventually a state is reached when no further change in the state coordinates of A and B takes place. This joint state of both systems that exist when all changes in the state coordinates have ceased is called thermal equilibrium. Note the reduction in length of L and the increase in P of the pressure at thermal equilibrium. These changes are used to measure temperature. We would use the above illustration to explain the zeroth law of thermodynamics and subsequently temperature measurement.

The Zeroth Law

Consider two systems A and B separated from each other by an adiabatic wall but each system is in contact with a third system C separated through a diathermic wall. Consider the whole systems surrounded by an adiabatic wall (fig. 1.7). This is to ensure that no heat energy is lost to or gained from the surrounding. Experiments have shown that systems A and B will attain a thermal equilibrium with C. If however the adiabatic wall is replaced by a diathermic wall as shown in (fig. 1.8).

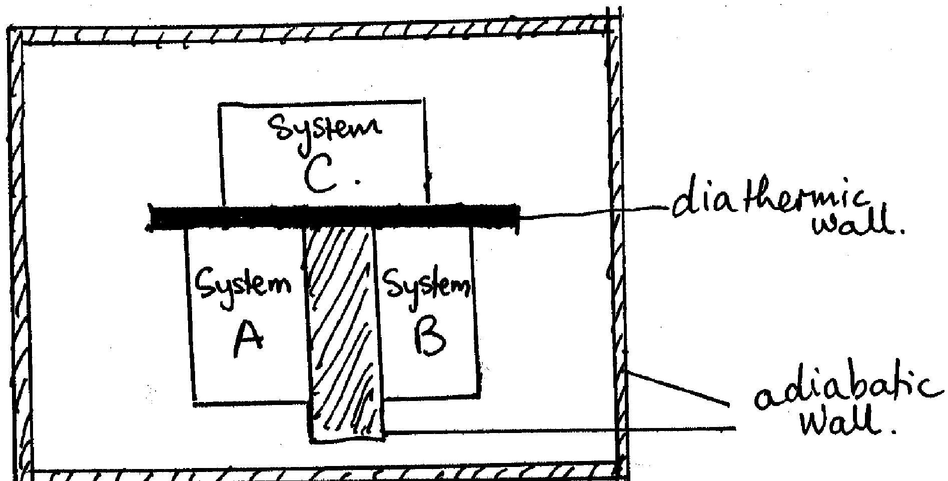


Fig. 1.7: A and B are each in Thermal Equilibrium with C

Instead of allowing both systems A and B to come to equilibrium with C at the same time, we can first have equilibrium between A and C and then equilibrium between B and C making sure that the state of C is the same in both cases, (fig. 1.7) then A and B are brought in contact through a diathermic wall, they will be found to be in thermal equilibrium (fig. 1.8). It means that no further changes occur in systems A and B

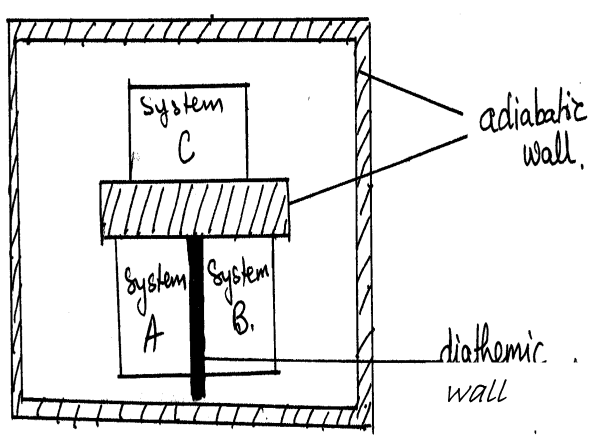


Fig. 1.8: A and B are in Thermal Equilibrium with Each Other.

That is systems A and B are already in equilibrium with each other.

The above principle is called the zeroth law of thermodynamics. The zeroth law of thermodynamics states that:

"Two thermodynamics systems A and B are separately in thermal equilibrium with a third system C, then the systems A and B are in Thermal equilibrium with each other".

It is called the zeroth law because the most important principles of thermodynamics have hitherto been identified as the first, second and third laws of thermodynamics. You will be introduced to these laws later in the course of your physics programme.

It is the property called temperature that determines whether or not given two systems are in thermal equilibrium. The temperature of a system is that property that determines whether or not it will be in thermal equilibrium with other system when two or more systems are in thermal equilibrium, they are said to have the same temperature. If the two systems are not in thermal equilibrium, their temperatures must be different.

The zeroth law is thus used in establishing the temperature of a body quantitatively and objectively. But before it is measured, a scale must be established with the aid of a physical property which varies with temperature – the thermometer.

We shall now consider the establishment of scales of temperatures in the next section.

3.3 Scale of Temperature

There are some principles underlying the establishment of temperature scales. These principles are based on the fact that when the temperature of a body changes, all the magnitudes of almost all its physical properties also change. The condition, of course, is that this variation of properties must be linear, that is, uniform. Therefore any property of any substance that varies uniformly with temperature can be used. The variation of one of these properties is chosen to represent the accompanying change of temperature.

To measure temperature, therefore, we need to select a physical property or parameter of a chosen substance which varies uniformly with temperature. A parameter or property is a variable which is assigned a constant value during a discussion or event. As we have discussed in section 3.3.2, that some of the examples of these parameters are:

(i) the volume of a liquid;
(ii) the volume of a gas at constant pressure;
(iii) the pressure of a gas at constant volume;
(iv) the electrical resistance of a conductor;
(v) the emf change of a thermocouple when there is a temperature difference between the junctions of a thermoelectric thermometer.

For the establishment of temperature scale, the following are required:

(a) specification of fixed points;
(b) specification of the method of interpolation.

Now we will discuss briefly about these concepts required for the establishment of temperature scale.

3.3.1 Specification of Fixed Points

Fixed points are temperatures chosen which are fixed and reproducible. They are useful as reference temperatures. Changes in the parameters from the fixed points are assigned numbers called degrees on a calibrated scale. Two such fixed points are:

(i) The Lower fixed point (ice point): That is the temperature of equilibrium between ice, water and air saturated at standard pressure. This temperature is) 0ºC.

(ii) The Upper fixed point (steam point): That is the temperature of steam rising from pure water boiling under standard atmospheric pressure. That is, the temperature of one standard atmosphere. This temperature is 100ºC.

(iii) The fundamental interval: This is the difference between the upper fixed point and the Lower fixed point divided into equal parts.

Other fixed points such as the sulphur point also exist for reference.

3.3.2 Factors for Changes in Fixed Points

1. Changes in the atmospheric pressure and latitude cause variation in freezing and boiling points. Changes caused by pressure in freezing point can be ignored. That due to impurities cannot be ignored.

2. Freezing point depression and Boiling point elevation are caused by impurities of slats. Hence water used in determining these points is required to be pure.

3. Daily floatation of barometric reading call for the correction of the boiling point. In the neighbourhood of standard atmospheric pressure, the boiling point rises by 0.37ºC when the height of mercury barometer increases by 1.0 cm. Therefore true boiling point is given as on the Celsius scales as:

$$6^{o}C = 100^{o}C + 0.37\ (B - 76)^{o}C \ldots\ldots\ldots\ldots\ldots (1.1)$$

Where B is any atmospheric pressure in cm of mercury.

Activity 2

Mention two factors that can change the fixed points on a temperature scale.

3.3.3 The Temperature Scales

The systems of temperature scales are:

(i) The Celsius scales whose ice point is 0ºC and the steam point is at 100ºC. Each part represents 1ºC.

(ii) The Fahrenheit scale whose ice point is 32ºF while the steam point if 212ºF. The fundamental interval is 180 divisions. Each division represents 10ºF.

(iii) The absolute scale of temperature, the thermodynamics scale. This will be discussed later.

3.4 Specification of Interpolation

The way we establish the temperature of a body on either the Celsius scale or the Fahrenheit scale is what we refer to as the specification of interpolation. This therefore establishes the scale of temperature, which decides upon the temperature below, between and above the fixed points, are to be established.

We then choose a thermometric substance and its particular property, which will serve as a temperature indicator.

3.4.1 Definition of Temperature on Celsius Scale

If X represents the property of the thermometric substance, which serves as temperature indicator, by adopting the Celsius scale. Let X_o be the values of X of the thermometric substance when surrounds by the melting ice for a long time. Let X_{100} be the value of X when the substance has reached equilibrium with steam at standard pressure (1 atmosphere).

Hence, the fundamental interval is defined as the change of X between the ice and steam points = $X_{100} – X_o$.

Consequently, the size of the Celsius degree, which results from our choice of property X, is defined at that range of temperature which causes a change in property which is Z.

$$\text{Hence } Z = \frac{X_{100} - X_0}{100} \quad \ldots\ldots (1.2)$$

If X_t is the value of X of the substance in the neighbourhood of another body whose temperature is to be determined then, the number of degrees by which the Celsius temperature t_c of the thermometric substance exceeds the temperature of melting ice 0ºC is equal to the number of items the quantity Z is contained in $(X_t – X_o)$.

$$(t_c - 0^oC) \times Z = (X_t - X_0)^oC \quad \ldots\ldots (1.3)$$

But from Eq. 1.2, you know that

$$Z = \frac{X_{100} - X_o}{100}$$

Substituting Eq. (1.2) in Eq. (1.3), we get

$$(t_c - 0) \times \frac{X_{100} - X_o}{100} = (X_t - X_o)^oC$$

$$t_c = \frac{(X_t - X_0) \times 100^oC}{(X_{100} - X_o)} \quad \ldots\ldots (1.4)$$

Activity 3
The lengths of the mercury column of a mercury thermometer are
1.06cm and 20.86cm respectively at the standard fixed points. What is the temperature of body, which produces 7.0cm of this mercury column?

3.4.2 Definition of Temperature on Fahrenheit Scale

In the case of the Fahrenheit scale, one can also state that

$$(t_F - 32)\ Z = (X_t - X_{32})\,^{o}F \quad \ldots\ldots (1.5)$$

Where, $Z = \frac{(X_{212} - X_{32})}{(212 - 32)}\,^{o}F$

$$Z = \frac{X_{212} - X_{32}}{180}\,^{o}F \quad \ldots\ldots (1.6)$$

Substituting Eq. (1.5) in Eq. (1.6), we get the expression

$$t_F - 32 = \left[\frac{X_t - X_{32}}{X_{212} - X_{32}}\right] X 180^{o} F$$

$$t_F = \left[\left[\frac{X_t - X_{32}}{X_{212} - X_{32}}\right] x\ 180 + 32\right]{}^{0}F \quad \ldots\ldots (1.7)$$

Furthermore, the value of the property of X at any definite temperature is independent of the method of numbering temperature.

Hence $X_{212} = X_{100}$ and $X_{32} = X_o$

Now inserting these parameters in Eq. (1.7), we get

$$t_F = \left[\left[\frac{X_t - X_0}{X_{100} - X_0}\right] x\ 180 + 32\right]{}^{0}F \quad \ldots\ldots (1.8)$$

Also from Eq. (1.4), we get

$$\frac{T_c}{100} = \frac{X_t - X_0}{X_{100} - X_0} \ldots\ldots (1.9)$$

Therefore, a relation between t_F and t_c can be obtained as

$$t_f = \left(\frac{t_c}{100} \times 180\right) + 32^{o}F$$

$$t_F = \frac{9}{5}t + 32^0F \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots (1.10)$$

The Eq. (1.10) enables us to convert a temperature measurement from one scale to the other.

Activity 4
Covert 50ºF to Celsius scale.

Now, let us discuss the examples of X property for different thermometers with Celsius scale.

(a) Platinum thermometer

X is in terms of resistance (R), thus

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100^0C \quad \dots\dots\dots\dots (1.11)$$

(b) Mercury thermometer

X is in terms of length of mercury L.

$$t = \frac{L_t - L_0}{L_{100} - L_0} \times 100^0C \quad \dots\dots\dots\dots (1.12)$$

(c) Constant Volume thermometer

X is in terms of the pressure P of the gas at constant volume

$$t = \frac{P_t - P_0}{P_{100} - P_0} \times 100^0C \quad \dots\dots\dots\dots (1.13)$$

(d) Constant Pressure thermometer

X is in terms of the volume of the gas at constant pressure

$$t = \frac{V_t - V_0}{V_{100} - V_0} \times 100^0C \quad \dots\dots\dots\dots (1.14)$$

Activity 5
A platinum resistance thermometer has a resistance of 10.40 ohms at
0ºC and 14.35 ohms at 100ºC. Assuming that the resistance changes uniformly with temperature, what is (a) The temperature when the resistance is 11.19 ohms? (b) The resistance of the thermometer when the temperature is 45ºC?

3.5 Thermodynamic Scale (Absolute Scale) of Temperature

The thermodynamic scale is the standard temperature scale used in scientific measurements. The symbol on this scale is T and it is measure in Kelvin after Lord Kelvin.

On the thermodynamic scale, the reference point is the triple point of water where saturated water vapour, pure water and melting ice are in equilibrium to each other. The temperature of the triple point of water has been found to be 273.16K. The ice point is 273.15K. The slight difference with the triple point is due to the pressure in the two cases.

There is variation of Pressure (P) with temperature T is shown in fig. 3.9. When the graph is extrapolated, it meets the temperature axis at -273.15ºC. This value of temperature is called absolute zero (ok) by Kelvin. It is to be noted that the value of pressure at this temperature reduce to zero.

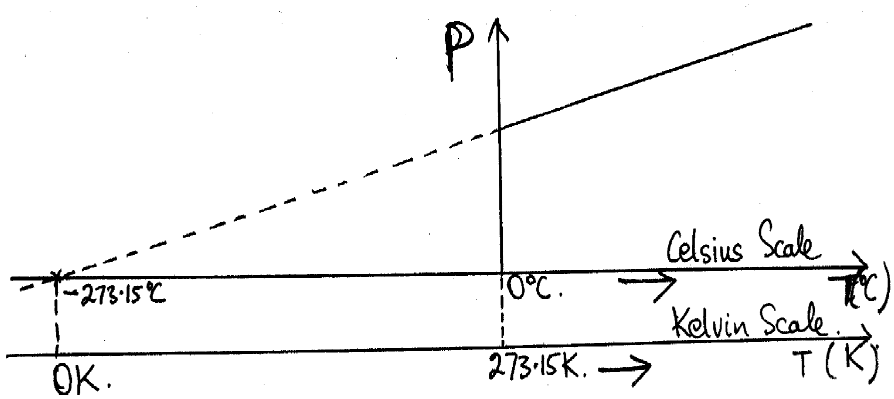


Fig. 3.9

On the Celsius temperature scale,

$$-273.15^{\circ}C = 0$$
$$0^{\circ}C = 273.15K$$

Hence the change in 1ºC on the Celsius scale is equal to the change of 1K on the Kelvin (Thermodynamics) scale.

4.0 CONCLUSION

What you have learnt in this unit concerns the concept of temperature and how it is measured. The property of a substance which varies with temperature is used to measure the temperature of a body through an appropriate scale. You have learnt about the Celsius, Fahrenheit and the Absolute scale. The property of a substance is used in the construction of a thermometer.

5.0 SUMMARY

In this unit, you have learnt:

- the difference between temperature and heat;
- the concept of temperature using the sensation of human feelings which is always sensitive;
- how the zeroth law is used to conceptualize the concept of temperature objectively by using some measurable properties such as length of a liquid, pressure on a gas at constant volume, resistance and the electromotive force;
- the scales of temperature measurements;
- that on the thermodynamics scale the triple point of water is chosen as the fixed point and is defined as 273.16K.

ANSWER TO ACTIVITY 1

- Heat is a form of energy while temperature is the degree of hotness or coldness in a body.
- Heat flows from higher temperature to a lower temperature. Thus the difference in temperature dictates the direction of transfer of the heat energy.

ANSWER TO ACTIVITY 2

See the text.

ANSWER TO ACTIVITY 3

Using the Eq. (1.4) as

$$t_c = \frac{(X_t - X_0) \times 100^0 C}{(X_{100} - X_0)}$$

Where, X_t = 7.00cm, X_{100} = 20.86cm, X_o =1.06cm

Substituting this parameter in the Eq. (1.4) we get

$$t_c = \frac{(7.00cm - 1.06cm)}{(20.86cm - 1.06cm)} \times 100^0$$

$$= \frac{5.94cm}{19.80cm} \times 100^0C$$

$$t_c = 30^0C$$

ANSWER TO ACTIVITY 4

Using equ. (1.10) as

$$t_F = \frac{9}{5}t_c + 32^0F$$

Substitute $t_f = 50^0f$

$$50^0F = \frac{9}{5}t_c + 32^0F$$

On rearranging the terms, we get

$$50^0F - 32^0F = \frac{9}{5}t_c$$

$$t_c - \frac{18^0 \times 5^0}{9}$$

$$t_c = 10^0C$$

ANSWER TO ACTIVITY 5

(a) Use Eq. (1.11) as

$$t_c = 10^0C$$

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100^0C$$

Substitute $R_t = 11.19ohms, R_{100} = 14.35 \; and \; R_0 = 100ohms.$

$$t = \frac{11.19\Omega - 10.40\Omega}{14.35\Omega - 10.40\Omega}$$

$$t = \frac{0.79ohms}{3.97ohms} \times 100^0C$$

$$t = 20^0C$$

(b) Again use Eq. (1.11)

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100^0C$$

Substituting t = 45ºC, R_o and R_{100} take their usual values

On arranging the terms, we will get,

$$45^0C = \frac{R_t - 10.40\Omega}{14.35\Omega - 10.40\Omega} \times 100^0C$$

$$45^0C = \frac{R_t - 10.40\Omega}{3.9\Omega} \times 100^0C$$

$$\frac{45}{100} \times 3.9\Omega = R_t - 10.40\Omega$$

$$1.7775 = R_t - 10.40$$

$$R_t = (1.7775 + 10.4)$$

$$R_t = 12.18$$

6.0 TUTOR-MARKED ASSIGNMENT

1. At what temperature do the Fahrenheit and the Celsius scales coincide?

2. The normal boiling point of liquid oxygen is -182.97ºC, what is this temperature on the Kelvin and Fahrenheit scales?

3. What is the body temperature of a normal human being on the Celsius scale? What will this value be on the Fahrenheit scale?

4. An ungraduated mercury thermometer attached to a millimeter scale reads 22.8mm in ice and 242mm in steam at standard pressure. What will the millimeter read when the temperature is 20ºC?

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Das Sarma, J.M. (1978). *An introduction to higher Secondary Physics*. Modern Book Agency Private Ltd.

Duncan, T. (1982). *Physics. A textbook for Advanced Level Students*. London: John Murray (Publishers) Ltd..

Fishbane, P. M, Gasiorowicz, S., & Thronton, S. T. (1996). *Physics for Scientists and Engineers. 2nd edition* Vol. 1. New Jersey: Prentice Hall Upper Saddle River.

Garg, S. C., Bansal, R. M., & Gosh, C. K. (1993). *Thermal physics.* India: Tata McGraw – Hill Publishing Company Limited.

Geddes, S. M. (1981). *Advanced physics*. London: Macmillan Education Ltd.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Indira Gandi National Open University School of Sciences. PHE 01, Elementary Mechanics

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

McKenzie, A.E.E (1973). *A second course of electricity*. Cambridge: The University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 2 HEAT MEASUREMENT

CONTENTS

1.0 INTRODUCTION

Heat and temperature are not the same but they are closely related. Heat, as you must have learnt is a form of energy while temperature is the degree of coldness or hotness of a body, may the body be solid, liquid or gas. This degree of coldness or hotness is measured objectively and quantitatively by suing the thermometer. *The question arises is: "how would you measure heat energy?"* The study of heat measurement is known as Calorimetry in physics.

The nature of substance plays an important role in the study of thermal phenomena. For example, a large iron tank and an aluminum kettle have a different heat capacity. It depends on their respective masses and on the metal need.

In this unit you will be introduced to the measurement of heat. The nature of substance plays an important role in the study of thermal phenomena. For example, a large iron tank and an aluminum kettle have a different heat capacity. It depends on their respective masses and on the metal used. Some of the major concepts crucial to the measurement of heat are: heat capacity, specific heat capacity, specific latent heat of fusion and vapourization which you will also study here. You will also learn about how temperature measurement is related to the measurement of heat energy.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the concept of heat;
- define the heat (thermal) capacity of a substance;
- define specific heat capacity of a substance;
- define (a) the specific latent heat of fusion (b) specific latent heat of vapourization; and
- solve problems involving the concepts mentioned above.

How to Study this Unit:

In this unit you are expected to:

1. Read through the course contents on your own
2. First attempt the activities, then the TMA without looking at the hints provided by the author
3. Make observations on all your difficulties to your facilitator
4. Confirm your work on the activities after you have done your best to get all correct

3.0 MAIN CONTENT

3.1 Concept of Heat

Prescott Joule first showed that heat is a form of energy in an experiment in which mechanical work (energy) was transformed into heat. Other scientists also showed that heat from fuel, such as gasoline in an engine, may be transformed to mechanical energy when the engine is used to drive carts, trains and aeroplanes. In an electric power station, heat from fuel is changed to electrical energy.

The unit of energy is Joule (J) which is also the unit of measuring mechanical energy and electrical energy. We also have larger units such as kilojoule (KJ) and megajoule (MJ).

Power, you will remember, is the rate of doing work. It is also defined as the rate at which heat energy is given out by a source. For example, the heat energy delivered per second by a gas burner is its power. This power is measured in watts (W). One watt is therefore defined as one joule per second (J/s or Js^{-1}). Other larger units are kilowatt (KW) and megawatt (MW).

Sources of heat energy are the sun, fuels such as coal, gas, oil and electricity.

Can you think of other sources of heat energy?

3.2 Heat Capacity

You would have observed that a source of heat will transfer its heat energy to another body. The source is usually at a high temperature while the other body being heated is at a lower temperature. When the source and the other body are in contact, the rise in temperature takes place in colder body.

Let $t_1{}^0C$ be the initial temperature of the body and $t_2{}^{o0}C$ be the final temperature when Q joules of heat has been supplied.

Then the change in temperature,

$$\Delta\theta = (t_2 - t_1)^oC$$

The amount of heat in joules that is capable of changing its temperature through 1^oC is $\frac{Q.J}{\Delta\theta}$

This amount of heat to change the temperature of the body is described as the heat capacity of the body. It is usually represented by the symbol H.

By definition, the heat (thermal) capacity (H) of a body, is the quantity heat (Q) in joules required to change its temperature by one degree (Celsius or one Kelvin).

$$H = \frac{QJ}{\Delta\theta^0C}$$

Where, $\Delta\theta = (t_2 - t_1)$

$$H = \frac{QJ}{(t_2 - t_1)^oC} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.1)$$

Thus the unit of heat (thermal capacity is expressed in joules per Celsius or joules per Kelvin (JK^{-1} or $J\,^oC^{-1}$).

The values of H for different bodies are not the same. They vary from one body to another.

EXAMPLE 3.1

A metal container of heat capacity $200J\,^oC^{-1}$ is heated from 15^oC to 45 ^oC. What is the total quantity of heat required to do so?

SOLUTION 3.1

Using Eq. (3.1)

$$H = \frac{Q}{\Delta\theta}$$

$Q = H(t_2 - t_1)$ Substituting the values, we get

$$= 200\ J/^{o}C \times (45 - 15)^{o}C$$
$$= 200 \times 30J$$
$$Q = 6000J$$
$$Q = 6KJ$$

3.3 Specific Heat Capacity

If there are different masses of a substance m_1, m_2 and m_3, it will be observed that to raise their temperatures through 1ºC each, they will require different quantities of heat energy Q_1, Q_2 and Q_3.

Experiments have shown that the quantities of heat (Q) required to change their temperature through 1ºC is proportional to the corresponding masses (m).

$$Q \propto m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(3.2)$$

However, if we fix the mass of the substance to 1kg an we transfer various quantities of heat Q to it, there will be various corresponding changes in temperature. Again, it will be found that the quantities of heat Q to change the temperature of 1kg mass of the body will be proportional to the corresponding changes in temperature. Hence we can write

$$Q \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.3)$$

Combining these two factors, we get

$$Q \propto m$$

Or $$Q = Cm \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.4)$$

Where C is a constant of proportionality known as the specific heat capacity of the substance

$$C = \frac{Q}{m}$$

The specific heat capacity of a substance is therefore defined as the amount of heat Q (in joules) required to raise the temperature of 1kg mass of substance through unit degree (1ºC or 1ºK).

The unit of C is J/kg ºC or $Jkg^{-10}C^{-1}$ or $Jkg^{-1}K^{-1}$. It can also be measured in cal. $g^{-1}\ k^{-1}$

The value of C differs from one substance to another.

The values of specific heat capacity for some common substances are given in Table 2.1

Table 2.1: Specific Heat Capacity for Some Substances

Substance	Specific heat capacity in $Jkg^{-1}{}^{o}C^{-1}$
Iron	460
Copper	400
Lead	120
Aluminum	800
Water	4200

EXAMPLE 3.2

How much heat is needed to bring 10g of water from 50ºC to boiling point? (Specific heat capacity of water = 4200 $Jkg^{-1}{}^{o}C^{-1}$)?

SOLUTION 3.2

Using Eq. 3.4
Q = mC
= mC (t_2 - t_1)

Substituting the values, in the Eq. 3.4 we get,

= 10g/1000kg x 4200J/kg ºC x (100 – 50) ºC

Q = 2100J

= 2.1KJ

3.3.1 Simple Method of Mixtures

In this section we shall consider exchange of heat between two bodies in such a way that one body is at a high temperature and the other is at a low temperature. In the simple method of mixtures, we are simply looking at hot and cold substances being mixed without considering the container in which they are being mixed. The principle of conservation of heat energy is being observed very closely. Here, very briefly, we will discuss the principle of conservation of heat energy.

> This principle states that *"the heat lost by a hot body is equal to the heat gained by the cold body in any system provided there is no heat exchange between the substances involved and their surrounding"*
>
> *But you may now ask: Is this always true?*

Unfortunately, this heat exchange cannot be completely true. However, the heat lost to the surrounding or gained from it can be reduced to a negligible amount by surrounding the container (calorimeter) with a bad conductor of heat (insulator). This process is called lagging.

In a laboratory, use the calorimeter as the container with which you observe exchange of heat between two substances, keeping them usually at different temperatures. The cold body is usually in form of water or any liquid and the hot body could be a solid body or liquid at a higher temperature.

Did you understand?

Let me illustrate this principle with this example,

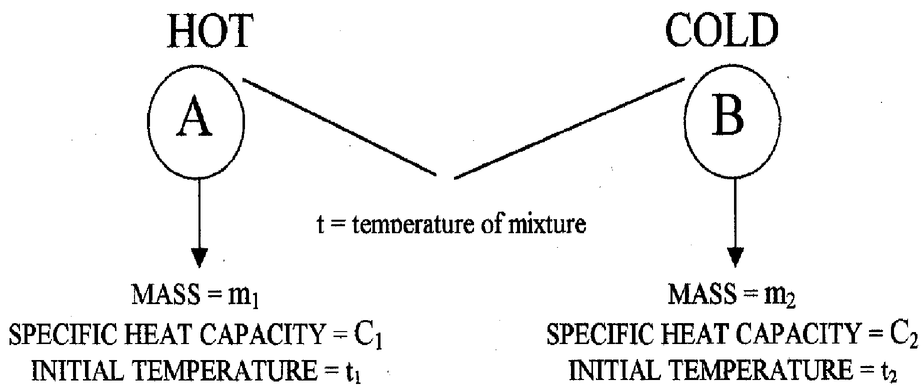


Fig. 2.1

If body A is the hot body at temperature t_1 and B is the cold body at temperature t_2, A and B are then mixed up to give a final temperature of mixture t. Let us see then what has happened?

(1) The hot body has lost some heat to the cold body B. The heat lost can be find out.

Let Q_1 be the amount of heat

$Q_1 = m_1c_1$

$Q_1 = m_1c_1 (t_1 – t)$

Or $= Q1 = -m_1c_1 (t - t_1)$ .... (3.5)

(2) The cold body B has gained some heat. The heat gained is,

Let Q_2 be the amount of heat gained.

$Q_2 = m_2c_2 \Delta\theta_2$

$Q_2 = m_2c_2 \Delta\theta (t - t_2)$ .... (3.6)

(3) How do you relate Q_1 and Q_2? We relate them together by suing the principle of conservation of heat energy, which says that

"In any heat exchange provided heat is not lost to or gained from the surrounding."

$$\text{Heat lost} = \text{Heat gained}$$
$$Q_1 = Q_2$$
$$m_1c_1\,\Delta\theta_1 = m_2c_2\,\Delta\theta_2$$
$$m_1c_1(t_1 - t) = m_2c_2\,(t - t_2) \;\ldots\ldots\ldots\ldots\ldots\; (3.7)$$

Out of all the seven quantities m_1, c_1, t_2, t, m_2, c_2 and t_1 all will be known except one. Now try to look for this only unknown quantity.

Let us solve an SAQ to understand this more clearly.

ACTIVITY 1
A piece of iron of mass 0.20kg is heated to 64°C and then dropped gently into 0.15kg of water at 16°C. If the temperature of the mixture is 22°C, what is the specific heat capacity of iron?

3.3.2 Inclusion of Calorimeter in Method of Mixtures

In section 3.3.1, we talked about simple method of mixtures in which the idea of the container taking part in the heat exchange was excluded. In this section, we are going to consider the calorimeter as one of the major players in the heat exchange (see Fig. 2.2). Once again, we shall still consider the principle of conservation of heat energy in our discussion.

Here we have the hot body A at a higher temperature than the liquid contained in the calorimeter. Thus, the liquid and the calorimeter are considered as the cold body gaining heat from the hot body.

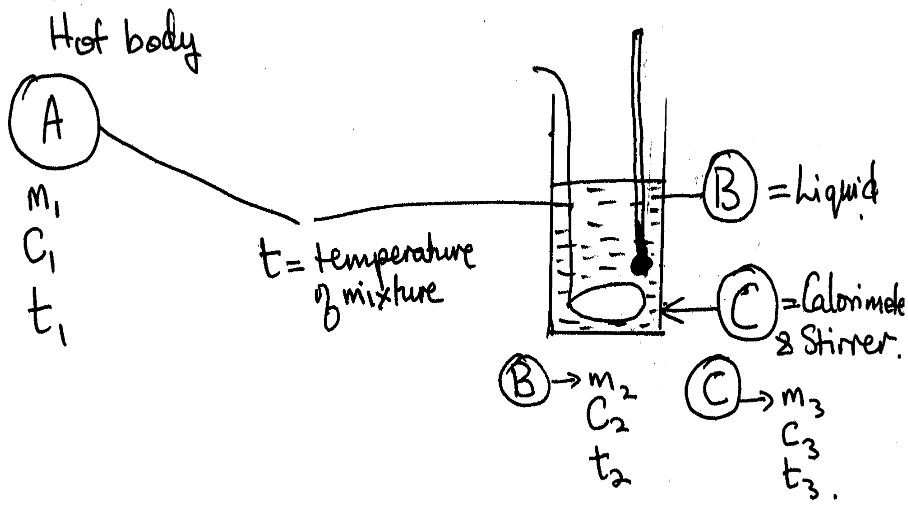


Fig. 2.2

Consider the hot body (A) with mass m_1, Specific heat capacity C_1 its initial temperature t_1 and Final temperature of mixture is = t.

Then, Heat lost by hot body:

$$Q_1 = m_1c_1 \Delta\theta_1$$
$$Q_1 = m_1c_1 (t_1 - t) \quad \text{……………………………………...…. (3.8)}$$

Again consider the cold liquid (B) with mass m_2, specific heat capacity C_2 with initial temperature t_2. The final temperature of the mixture is t.

Heat gained by cold liquid (B) = Q_2

$$Q_2 = m2C2 \Delta\theta_1$$
$$Q_2 = m_2C_2 (t - t_2) \quad \text{…………………………………...…. (3.9)}$$

Consider the cold calorimeter (C) with mass m_3, specific heat capacity C_3, Specific heat capacity = C_3 Initial temperature t_3 and Final temperature of mixture t.

Heat gained by cold calorimeter C is Q_3.

$$Q_3 = m_3C_3 \Delta\theta_3$$
$$Q_3 = m_3C_3 (t - t_3) \quad \text{…………………………………...…. (3.10)}$$

It can be noted here that the liquid (B) and calorimeter (C) are both gaining heat. Therefore their changes in temperature will be the same.

$$\Delta\theta_2 = \Delta\theta_3 \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(3.11)$$

$$(t - t_2) = (t - t_3)$$

$$Q_3 = m_3C_3\,(t - t_3) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(3.12)$$

In most cases, m_3 and C_3 may not be provided for you in the heat exchange. Rather, a property of the container, i.e. the calorimeter may be provided in the form of the thermal capacity (H) of the calorimeter. Hence, the quantity of heat gained by the calorimeter is expressed as

$$Q_3 = H\Delta\theta_3 = m_3C_3\,\Delta\theta_3 \;\ldots\ldots\ldots\ldots\ldots\ldots \;(3.13)$$

$$Q_3 = H\,(t - t_2) \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(3.14)$$

By applying the principle of conversation of heat energy, the relation between the quantities Q_1, Q_2 and Q_3 is

Heat lost = Heat gained

$$\mathbf{Q_1} = \mathbf{Q_2} + \mathbf{Q_3} \;\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \;(3.15)$$

$$m_1C_1\,(t_1 - t) = m_2C_2\,(t - t_2) + m_3C_3\,(t - t_2)$$

Or $$m_1C_1\,(t - t_1) = m_2C_2\,(t - t_2) + H\,(t - t_2) \ldots\ldots\ldots\ldots\ldots\ldots \;(3.16)$$

> **ACTIVITY 2**
> A calorimeter contains 0.30kg of water at 12^{o}C. When poured in, the temperature of the mixture is found to be 52^{o}C. What is the heat capacity of the calorimeter?

3.4 Latent Heat

When matter is heated, you will recall, there are three observable effects. Heat causes matter to:

- Expand
- Change its temperature
- Change its state

Let us look closely at the last two effects, changes in temperature and state. The first effect, expansion, will be considered later. The effect of the temperature change has been used to measure the quantity of heat absorbed or given out by a substance. The expression used in = mC (where is the change in temperature).

But what about the change in state?

We shall now look critically at the effect of heat on a solid ice-block as it is heated from say -10ºC to the boiling point 100ºC.

Fig 2.3 shows the graph of temperature versus time which allows us to understand the concept of Latent heat which is under discussion.

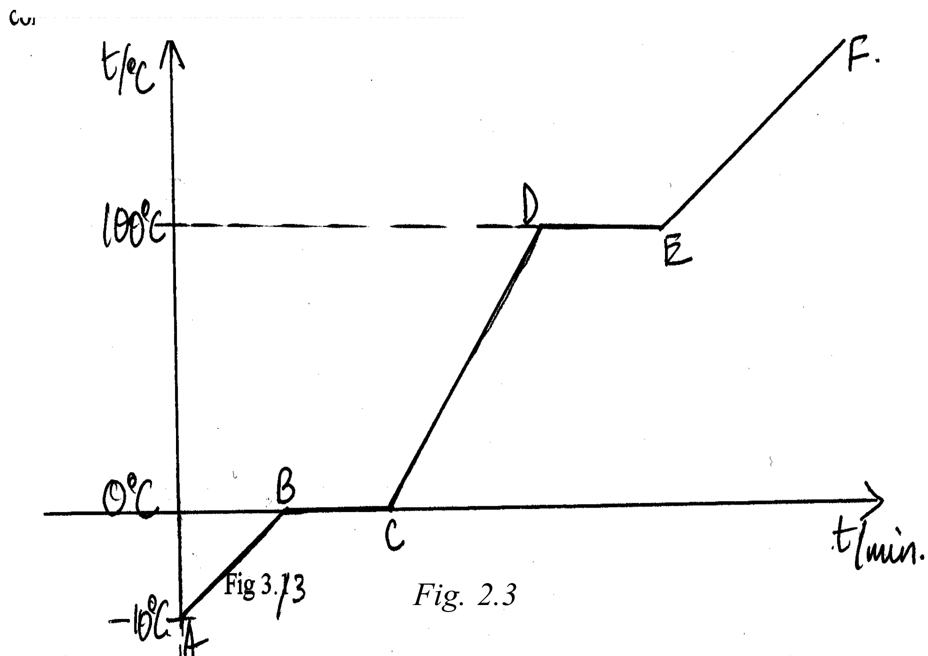


Fig. 2.3

- AB shows increase in temperature as the time increases
- BC shows no increase in temperature with respect to time
  At this point the solid ice is observed to change its state from ice (solid) to water (liquid). This takes place as 0ºC – the melting/freezing point of water. The process is called melting. The reverse process of melting is freezing.
- CD shows an increase in temperature with respect to time
- DE again shows no increase in temperature as the water is being heated. This happens at another fixed temperature of 100ºC which is the boiling point of water. The liquid water changes its state from water to steam (gas). The process is called vapourization. The reverse process of vapourization is condensation.

- EF shows that there is an increase in temperature of steam with respect to time. In this region water behaves as a gas whose behaviour will be studied later.

It is significant to note that BC and DE show regions at which water changes its state from solid to liquid and from liquid to gas respectively. The reverse could also take place by extracting heat from the system then we have condensation, the reverse of vapourization and then freezing, the reverse of melting.

But, how do we measure the heat content when water or any substance changes its state?

To answer this question will therefore be involved with energy- temperature relationship of the processes of melting and vapourization or condensation and freezing.

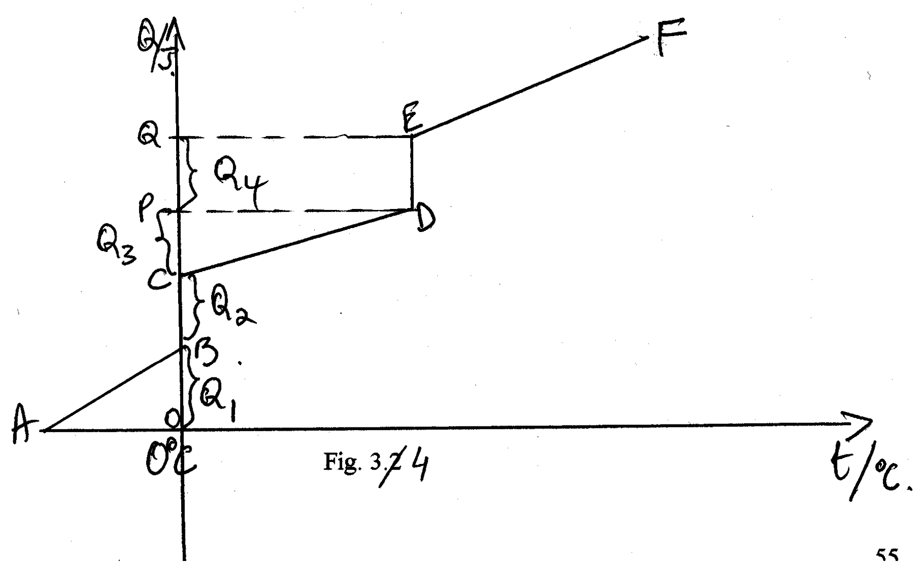


Fig. 2.4

Refer to figure 2.4

- AB indicates the absorption of heat by the solid material to change its temperature from -10ºC to 0ºC. The amount of heat supplied is OB = Q_1.

  Where $Q_1 = m_1C_1 \Delta\theta_1$ Here m_1, represents mass of ice, and m_1 = mass of ice, C_1 is the specific heat capacity of solid ice is the change in temperature from -10ºC to 0ºC

$$\Delta\theta_1 = (0 - (-10))^{o}C = 10^{o}C$$

$$\therefore \quad Q_1 = m_1C_1 \text{ x } 10J$$

BC corresponds to the absorption of heat energy without any change in temperature. The energy appears latent that is, hidden since there is no change in temperature.

The amount of heat supplied to BC is $Q_2 = mL_F$, where, m is the mass of ice and L_F is the specific heat of fusion.
Latent heat of fusion L_F for ice = 3.3 x 10^5 Jkg^{-1}

Specific latent heat of fusion is defined as the amount of heat required to change 1kg of mass of solid at its melting point to liquid at the same temperature".

- CD corresponds to the increase in temperature of substance in the liquid state. $CP = Q_3 = mC_2$ is the quantity of heat supplied to change its temperature from its melting point to 100ºC, the boiling point. Here m is the mass of the liquid converted from solid ice, C_2 is the specific heat capacity of water, and $\Delta\theta_2$ is the change in temperature from 0ºC to 100ºC.

- DE shows that there is an increase in heat energy while the temperature does not change. De therefore corresponds to the change in state from liquid to vapour. The heat energy appears latent that is, hidden, since there was no change in temperature. The quantity of heat supplied is CP.

$$CP = Q_4 = mL_V$$

Where, m is the mass of water and Lv is the specific latent heat of vapourization at its boiling point.

Therefore, we can define the specific latent heat of vapourization of a liquid.

"The specific latent heat of vaporization Lv of a liquid is the quantity of heat required to change 1kg of a liquid at its boiling point to vapour at the same temperature".

The value of Lv for water is 2.4 x $10^6 Jkg^{-1}J$ at 100ºC and external pressure of 760mmHg.

EF corresponds to the increase in temperature of the vapour. The heat absorbed by the vapour will depend on the condition under which it is treated i.e. whether it is treated at constant pressure or at constant volume. These ideas will be discussed under the concept of molar heat capacity of gases at a later stage of your physics course.

You therefore need to understand the effects of heat on matter under consideration and to know the process it is undergoing when you are solving problems involving changes in temperatures and or changes in state. For example, the total heat energy required to change the mass m of ice at -10ºC to vapour at

100ºC is Q where

$$Q = Q1 + Q2 + Q3 + Q4$$

$$Q = m_1 C_1 \Delta\theta_1 + mL_f + m_2 C_2 \Delta\theta_2 + mLv_s \text{ ………. } (3.17)$$

ACTIVITY 3

1. How much heat is needed to melt 1.5kg of ice and then to raise the temperature of the resulting water to 50ºC?

2. The water at 0ºC is changed to 50ºC

$$Q_2 = m_1C_1 \Delta\theta_1$$
$$= 1.5\text{kg} \times 4200\text{Jkg}^{-1}\,{}^{o}\text{C}^{-1} \times (50-0)^{o}\text{C}$$
$$= 1.5\text{kg} \times 4200\text{Jkg}^{-1}\,{}^{o}\text{C}^{-1} \times 50^{o}\text{C}$$
$$= 3.15 \times 10^5\text{J}$$

3. Total amount of heat required is

$$Q = Q_1 + Q_2$$
$$= 1.5\text{kg} \times 3.3 \times 10^5\text{Jkg}^{-1} + 1.5\text{kg} \times 4200\text{Jkg}^{-1}\,{}^{o}\text{C}^{-1} \times 50^{o}\text{C}$$
$$= 4.95 \times 10^5\text{J} + 3.15 \times 10^5\text{J}$$
$$= 8.1 \times 10^5\text{J}$$

It can be concluded that there are two kinds of latent heat – the one at the melting point and the other at the boiling point. Thus we define Latent heat of fusion as the heat required to melt a whole mass of solid at melting point to liquid at the same temperature.

Latent heat of vapourization is the heat required to convert a whole mass of liquid at boiling point to vapour at the same temperature. It is expressed in joules per kilogram (Jkg^{-1}).

3.5 Explanation of Latent Heat

In this section, we would like to explain the meaning of latent heat of fusion and vapourization on the basis of kinetic molecular theory of matter.

3.5.1 Specific Latent Heat of Fusion

A solid consists of atoms or molecules held in affixed structure by forces of attraction between them. These atoms or molecules vibrate about their mean position. When heat is therefore supplied to the solid, the kinetic energy of vibration increases thus increasing the temperature of the solid. The heat supplied is measure by mC . At the melting point, the heat given to the solid is used to overcome the forces of attraction between the atoms or molecules, which keep the solid in its rigid form, and then the solid melts.

At this point we define the specific latent heat of fusion (L_F) as *the quantity of heat required to change 1kg mass of a solid at its melting point to liquid at the same temperature.*

EXAMPLE 2.3

Explain the statement "the specific latent heat of fusion of ice 3.3 x 10^{-5} Jkg^{-1}

Solution 3.3
This means that 3.3 x 10^{5} joules of heat energy is required to change 1kg of solid ice at 0ºC to 1kg of water at the same 0ºC temperature.

3.5.2 Specific Latent Heat of Vapourization

Unlike solids, a liquid has no definite form; it usually takes the shape of its container. It molecules move in random manner inside although the molecules are close enough to attract each other. Some of the molecules, which have the greatest kinetic energy, are able to escape through the surface. They then exist as vapour outside the liquid. This process is called evaporation and it takes place at all temperatures.

However, boiling occurs at a definite temperature, the boiling point which depends on the external pressure. Water for example boils at 100ºC and at a pressure 760mmHg. It does so at a lower temperature when the external pressure is lower e.g. boiling point of water at the top of a mountain is less that 100ºC. Boiling occurs throughout the whole volume of the liquid whereas evaporation is a surface phenomenon.

At this point, we would define the specify latent heat of vapourization as *the quantity of heat required to change 1kg mass of liquid at boiling point to vapour at the same temperature*.

ACTIVITY 4

How much heat is given out when 50g of steam at 100ºC cool to water at 28ºC? (Specific latent heat of vapourization of water = 2.3 x 10^{6}

3.5.3 Latent Heat and Internal Energy

When a liquid reaches its boiling point, the energy needed to change it to vapour is:

(i) the energy or work needed to separate the liquid molecules from their mutual attraction until they are relatively far apart in the gaseous state.

(ii) the energy or work needed to push back the external pressure so that the molecules can escape from the liquid.

The latent heat of vapourization is used in point (i) which is needed to change the internal energy of the liquid whereas point (ii) is the external work done against the external pressure.

The work done in this case is defined as

$$W = p\Delta V \text{ ……………………………………………… (3.18)}$$

Where, p is the external pressure and ΔV is the change in volume
1g of water changes to about 1672cm^3 of steam

$$\Delta V = (1672 - 1)\text{cm}^3$$
$$= 1671 \times 10^{-6}\text{cm}^3$$

Assuming the external pressure

$$P = 1.013 \times 10^5 \text{ N m}^{-2}$$

The work done (W) = pΔV is therefore

$$w = p\Delta v\ 13\ \times 10^5 \frac{N}{m^2} \times 1671 \times 10^{-6} m^3$$

= 169.3J

The latent heat of vapourization per gram of water = 2260J
Thus the internal energy part of the latent heat of vaporization

$$= (2260 - 169.3)\text{J}$$
$$= 2090.7\text{J}$$

This is much greater than the external work done.

The latent heat of fusion is about 340J. So the energy needed to overcome the bonds between molecules in the solid state is much less than the energy to form the gaseous molecules from the liquid state.

ACTIVITY 5
1g of steam condenses to water 100 ºC. Estimate the change in potential energy per molecule, given that the latent heat of vapourization is 2240J and that Avogadro's constant is 6 x 10^{23} per mole. (Assume no heat is gained from or lost to the surrounding).

4.0 CONCLUSION

The three basic effects of heat have been identified under this unit. The effects are expansion, change in temperature and change in state. The change in temperature and change in state have been associated with the measurement of heat. With the change in temperature, the quantity of heat is expressed as $Q = mC$, whereas for the change in state the heat required is given as $Q = mL_F$ or $Q = mL_V$.

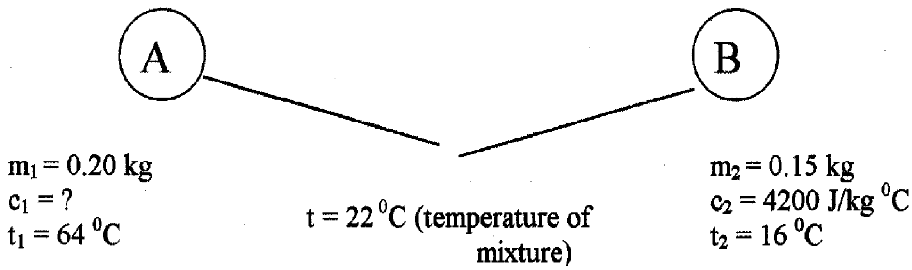
5.0 SUMMARY

In this unit you have learnt that:

- Heat is a form of energy and is measured in joules (J)
- The heat capacity of a substance is the quantity of heat required to change the temperature of the body through 1ºC.
- The specific heat capacity of a substance is the quantity of heat in joules that is required to change the temperature of 1kg mass of the substance through 1ºC.
- The specific latent heat of fusion of a solid is the quantity of heat in joules that is required to change the state of 1kg mass of the solid at the melting point to liquid at the same temperature.
- The specific latent heat of vapourization of a liquid is the quantity of heat in joules required to change 1kg mass of the liquid at its boiling point to gas at the same temperature.

ANSWER ACTIVITY 1

HOT – IRON COLD – WATER



Heat lost by iron, $Q_1 = m_1c_1\,\Delta\theta_1$

$= m_1c_1(t_1 - t)$

$= 0.20\text{kg} \times c_1 \times (64 - 22)^{\circ}\text{C}$

Heat gained by water, $Q_2 = m_2c_2\,\Delta\theta_1$

$= m_2c_2(t - t_2)$

$= 0.150\text{kg} \times 4200\,\text{Jkg}^{-1\circ}\text{C}^{-1} \times (22 - 16)^{\circ}\text{C}$

Using the principle of conversation of heat energy,

Heat lost by iron = Heat gained by water

$Q_1 = Q_2$

$0.2\text{kg} \times c_1 \times (64 - 22)^{\circ}\text{C} = 0.150\text{kg} \times 4200\,\text{Jkg}^{-1\circ}\text{C}^{-1} \times (22 - 16)^{\circ}\text{C}$

$$c_1 = \frac{0.15 \times 4200 \times 6\text{J}}{0.02\text{kg} \times 42^{\circ}\text{C}}$$

$c_1 = 450\text{Jkg}^{-10}\text{C}^{-1}$

ANSWER ACTIVITY 2

The question is centered on determining H, the heat (thermal) capacity of the container (Calorimeter).

1. Heat lost by hot water = Q_1 and

   $Q_1 = m_1C_1\,\Delta\theta_1$

   Where $m_1 = 0.4$kg,

   C_1= specific heat capacity of water = $4200\,\text{Jkg}^{-1\circ}\text{C}^{-1}$ and

   $\Delta\theta_1 = (t_1 - t) = (84 - 52)^{\circ}\text{C}$

   $Q_1 = 0.4\text{kg} \times 4200\,\text{Jkg}^{-10}\text{C}^{-1} \times (84 - 52)^{\circ}\text{C}$

   $= 0.4 \times 4200 \times 32\text{J}$

2. Heat gained by cold water in the calorimeter = Q_2 and

$Q_2 = m_2C_2$

Where m_2 = 0.3kg

C_1 = specific heat capacity of water = 4200 $Jkg^{-1}{}^oC^{-1}$ and

$_2 = (t - t_2) = (52 - 12)\,^oC = 40\,^oC$

$Q_2 = 0.3kg \times 4200\, Jkg^{-1}{}^oC^{-1} \times 40^oC$

$= 0.3 \times 4200 \times 40J$

3. Heat gained by cold water calorimeter = Q_3 and

$Q_3 = H\,\Delta\theta_3$ (where H = thermal heat capacity of the calorimeter)

$= H\,\Delta\theta_2$

$= (t - t_2)$

$= H(52 - 12)^oC$

$= H \times 40^oC$

By applying the conversation of heat energy

Heat lost = Heat gained

$Q_1 = Q_2 + Q_3$

$m_1C_1\,(t_1 - t) = m_2C_2\,(t - t_2) + m_3C_3\,(t - t_2)$

$0.4 \times 4200 \times 32J = 0.3 \times 4200 \times 40J + H40^oC$

$53760J = 50400J + H \times 40^oC$

$53760J - 50400J = H \times 40^oCH$

$H = \frac{3360J}{40^oC}$

$= 84J/\,^oC^{-1}$

The heat capacity of the calorimeter is 84J/°C

ANSWER ACTIVITY 3

SOLID $\xrightarrow{0^o\,C}$ WATER $\xrightarrow{50\,^oC}$ STEAM

1. The ice melts at

Quantity of heat used in melting = Q and

$Q_1 = mL_F$

Where, m = mass of ice and L_F = specific latent heat of fusion

$= 1.5kg \times 3.3 \times 10^5 Jkg^{-1}$

$= 4.95 \times 10^5$

ACTIVITY 4

The steam condensed at 100°C.
Therefore, heat given out is Q_1 = mLF

Where m = mass of steam = (50/1000) kg
Lv = specific latent heat of vapourization of water
= 2.3×10^6 J/kg
Q_1 = (50/1000)kg x 2.3×10^6 J/kg
= (50/1000) x 2.3×10^6 J
Q_1 = 0.155×10^6 J

To cool from 100°C to 28°C, Heat given out by steam

Q_2 = mC

Where, m = (50/1000)kg mass of water
C = specific heat capacity of water 4200 $Jkg^{-1}{}^{o}C^{-1}$
= change in temperature = (10 – 28)°C
Q_2 = (50/1000)kg x 4200 $Jkg^{-1}{}^{o}C^{-1}$ x (100 – 28)°C
= (50/1000) x 4200 x 72J
= 15120J

Total heat given out = Q = Q_1+Q_2
Q = 0.115×10^6 J + 15120J
= 1.15×10^5 J + 0.1512×10^5 J
Q = 1.30×10^5 J.

ANSWER TO ACTIVITY 5

1 mole of water (H_2O) = 18g
If 18g of H_2O contains 6×10^{23} molecules
1g of water contains $\frac{6x10^{23}}{18}$ molecules
Hence the change in Potential energy (ΔPE) per molecule

$$w = p\Delta v \; 13 \times 10^5 \frac{N}{m^2} \times 1671 \times 10^{-6} m^3$$

$$= 2240J \div \frac{6 \times 10^{23} molecules}{18}$$

$$= 2240J \; : \; \frac{18}{6 \times 10^{23} molecules}$$

$$= \frac{240 \times 18}{6} \times 10^{-3} \frac{J}{molecule}$$

$$= 6.72 \times 10^{-20} \frac{J}{molecule}$$

6.0 TUTOR-MARKED ASSIGNMENT

1. An iron casting of mass 30kg at 400°C is being cooled and it gives out heat on an average of 920J/s. Calculate its temperature after 1 hour? (Specific heat capacity of iron = 460Jkg^{-1}°C^{-1}).

2. A piece of iron of mass 0.27kg is immersed in boiling water and then dropped into 0.10kg alcohol at 27°C. If the final mixture is 50°C, what is the specific heat capacity of alcohol? (Specific heat capacity of iron = 460Jkg^{-1}°C^{-1}).

3. A copper calorimeter of mass 150g contains 100g of water at 16°C. 250g of a metal at 100°C are dropped into the water and the temperature of the mixture is 37°C. What is the specific heat capacity of the metal? (Specific heat capacity of copper = 400Jkg^{-1}°C^{-1}).

4. What mass of ice is needed to cool 60g of water from 43°C to 20°C?

5. Specific Latent Heat of Fusion of Lead is 2.1 x 10^{4}J/kg and its melting point is 328°C. How many joules will be needed to melt 7.0kg of lead at 13°C? (Specific heat capacity of lead = 120Jkg^{-1}°C^{-1})

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Das Sarma, J.M. (1978). *An introduction to higher Secondary Physics*. Modern Book Agency Private Ltd.

Duncan, T. (1982). *Physics. A textbook for Advanced Level Students* London: John Murray (Publishers) Ltd..

Fishbane, P. M, Gasiorowicz, S., & Thronton, S. T. (1996). *Physics for Scientists and Engineers. 2nd edition* Vol. 1. New Jersey: Prentice Hall Upper Saddle River.

Garg, S. C., Bansal, R. M., & Gosh, C. K. (1993). *Thermal physics.* India: Tata McGraw – Hill Publishing Company Limited.

Geddes, S. M. (1981). *Advanced physics*. London: Macmillan Education Ltd.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Indira Gandi National Open University School of Sciences. PHE 01, Elementary Mechanics

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

McKenzie, A.E.E (1973). *A second course of electricity*. Cambridge: The University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 3 THERMAL EXPANSION

CONTENTS

1.0 INTRODUCTION

In unit 3, we discussed the three effects of matter, namely, expansion, change in temperature and change in state. We were able to show how heat can be measured using the ideas of change in temperature, the specific heat capacity, the mass of the body, specific latent heat of fusion and vapourization to determine the heat absorbed or given out by the body in question.

Broadly, there are these states of matter, solids, liquids and gases. We shall consider the expansion of solid and liquids in this unit. In this unit we are going to examine the expansion/contraction of a material when it is heated or cooled. The expansion of gases will be the subject of the next unit.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain thermal expansion in solids and liquids;
- define linear, superficial and cubical expansion on a matter;
- apply the expansion of matter to day-to-day activities; and
- solve problems on the expansion of solids and liquids.

How to Study this Unit:

In this unit you are expected to :

1. Read through the course contents on your own
2. First attempt the activities, then the TMA without looking at the hints provided by the author
3. Make observations on all your difficulties to your facilitator
4. Confirm your work on the activities after you have done your best to get all correct

3.0 MAIN CONTENT

3.1 Thermal Expansion n Solids

We know, from our elementary knowledge, that matter is anything that has weight and occupies space. In that case, solids, liquids and gases are forms of matter. When they are therefore heated, experience has shown that they expand. In this unit, we shall examine the expansion of solids and liquids only. It is interesting to know that not only expansion is noticeable in matter when it is heated, but also a change in temperature is also noticeable as heat is absorbed by or removed from the body.

When solids are heated, the effect of heat on them could be found in the change of:

- the length
- the area and
- volume of the solids as temperature changes.

The changes in length, area and volume of the solids depends on:

- the material making up the solid;
- the range of the temperature change;
- the initial dimensions of the solid.

From the above three factors we could deduce that

- the expansion of solids varies from one material to the other;
- the greater the range of temperature change, the greater the expansion;
- expansion depends on the original length, area and volume of the solid.

3.1.1 Linear Expansion

Here, we shall discuss the linear i.e. straight-line expansion of the material. This means we are considering the expansion of a solid in one dimension only.

Consider a metal rod with an original length ℓ_O (fig. 3.1(i)). If such a length of material is heated from an initial temperature t_1 ℃ to t_2 ℃, the change in temperature is given as

$= \quad (t_2 - t_1)^{o}C$

It will be noticed that the length of the metal rod increased from ℓ_O to ℓ_t (fig. 3.11(ii)) ℓ_t is the new length at temperature $t\ ^{o}C_2$ and ℓ is$_0$ the original length at temperature $t\ ^{o}C._1$

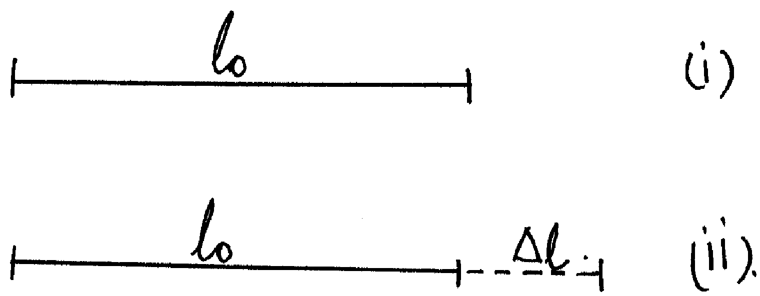


Fig. 3.1

The new length ℓ_t is therefore given as

$$\ell_t = \ell_O + \ell$$

Where ℓ is the change in length of the rod when heated from $t_1{}^{o}C$ to $t_2{}^{o}C$

$$\ell = \ell_t - \ell_O \quad .... (4.1)$$

It is found experimentally that for a given material, the increase in length, ℓ, is proportional directly to

(i) the original length ℓ_O and
(ii) the change in temperature

$$\Delta\ell = \alpha\, \ell_O \Delta\theta \quad ....$$

(4.2) Where, α is the constant of proportionality which is known as the coefficient of linear expansion of the solid or in short, linear expansivity. Consequently can be given as,

$$= \frac{\Delta \ell}{\ell_O \Delta\theta}$$

Thus by definition, α, the linear expansivity is *"The increase in length of the material ($\Delta\ell$) per the original ℓ_O length per degree Celsius change in temperature ($\Delta\theta$)."*

The unit of is $^{o}C^{-1}$ or per degree Celsius.

On comparing Eq. (4.1) and (4.2), we get

$\therefore \ell_t - \ell_O = \alpha \ell_O \Delta\theta$

$\therefore \ell_t - \ell_O = \alpha \ell_O \Delta\theta$

$\therefore \ell_t - \ell_O = \alpha \ell_O (t_2 - t_1)$………………… (4.3)

Example 1

An iron rail is 20m long. How much will it expand when heated from 10°C to 50°C (linear expansivity of iron = 1.2 x 10-5 $^{-5o}$C^{-1})?

Solution

Let the original length of iron = ℓ_O = 20m
Let the initial temperature t_1°C = 10°C
And the final temperature t_2°C = 50°C
$\therefore$ Change in temperature $\Delta\theta$ = $(t_2 - t_1)$°C
= (50 -10)°C
= 40°C
Given that α for iron = 1.2 x 10-5°C^{-1}
From $\Delta \ell$ = ℓ_O
The change in length ℓ = ℓ_O
$\therefore \Delta \ell$ = 1.2 x 20m x 40°C
= 0.0096m
$\Delta \ell$ = 0.96cm.

The iron rail would have expanded by 0.96cm.

After this example, you must have observed the values of ℓ_o and ℓ.

You would have observed that the value of is very small not only for iron but for most materials as you will further observe in Table 3.1. below.

Linear Expansivities for Some Materials

Table 3.1

Substance	Value
Pure	
Aluminum	$2.55 \times 10^{-5o}C$
Copper	$1.67 \times 10^{-5o}C$
Gold	$1.395 \times 10^{-5o}C$
Iron	$1.20 \times 10^{-5o}C$
Nickel	$1.28 \times 10^{-5o}C$
Platinum	$0.80 \times 10^{-5o}C$
Silver	$1.88 \times 10^{-5o}C$
Alloys	
Brass	$1.89 \times 10^{-5o}C$
Constantan	$1.70 \times 10^{-5o}C$
Invar	$0.10 \times 10^{-5o}C$
Phosphor – Bronze	$1.68 \times 10^{-5o}C$
Solder (2pb: 1Sn)	$2.50 \times 10^{-5o}C$
Steel	$1.10 \times 10^{-5o}C$

3.1.2 Determination of Linear Expansion (ℓ)

In the earlier section, you have learnt about linear expansion. We can measure the value of by various methods.

We have the following methods used in determining the linear expansivity (ℓ) of a metal:

- Optical lever method
- Screw gauge method
- Comparator method
- Henning's tube method
- Fizeau's method

All the above methods are different in the manner in which the increase in length is measured. The specimen to be measured is in form of a bar or tube and this involves:

- the measurement of the length of the bar,
- the rise in temperature during the experiment and
- the increase in length of the bar consequent on this rise in temperature.

The first two measurements do not present any great difficulty, but the actual measurement of the expansion that takes place. It is therefore this measurement of the increase in expansion that has called for use of elaborate vernier microscope, micrometer screw gauge and the optical lever method. We shall describe the screw gauge method here.

The Screw Gauge Method

This is one of the laboratory methods for determining the coefficient of linear expansion of a metal. The apparatus used for the determination of
is as shown in fig. 3.2.

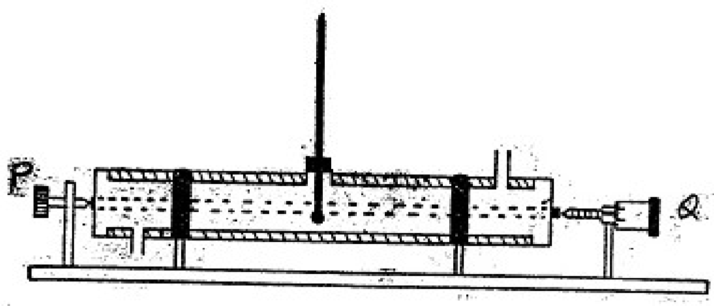


Fig. 3.2: Linear Expansivity Apparatus with Screw Gauge

The metal rod which is placed in the tune AB is about a meter in length. It is supported horizontally by pillars P and Q on affirm base.

The end A of the tube is in contact with the fixed end of the pillar of the apparatus. The screw gauge makes contact with the other end B as shown in fig. 3.2.

The reading of the screw gauge is taken when it is initially in contact with the end B at the room temperature t_1.

The screw gauge is then screwed backward to give room for the expansion of the rod inside the tube.

Steam is allowed into the tube at end A and out through end B for a considerable length of time so that the rod acquires at temperature of
100°C, the temperature of steam (t_2).

The screw is then screwed up to make contact with the rod when fully expanded. The new reading on the crew gauge is then taken. The difference of the two readings on the screw gauge gives the increase in length of the rod due to expansion.

If he original length of the rod is ℓ_O, the increase in length is ℓ and the change in temperature is = $(t_2 - t_1) = (100 - t_1)^oC$, where t_1 is the room temperature, then the coefficient of expansion is determined as

$$= \frac{\Delta\ell}{\ell_O \Delta\theta}$$

$$= \frac{\Delta\ell_{(m)}}{\ell_{O(m)}(100 - t_1)^o C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.4)}$$

3.1.3 Superficial Expansion

Under this section, we shall consider the expansion of material in two dimensions (-length and breadth) to produce an area expansion.

When a solid is heated, the area increases. In this case the expansion or the change in area is in two dimensions as shown in fig .3 3(ii) .

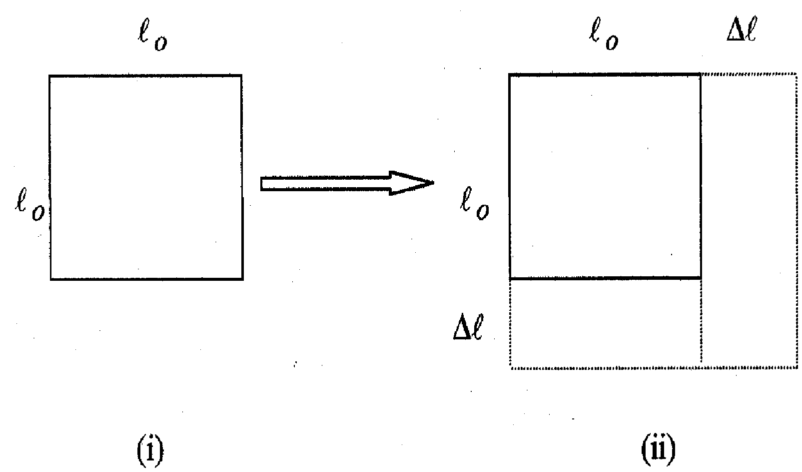


Fig. 3.3

Consider a square solid object of length ℓ_O (fig.3.3 (ii). By definition its original area at initial temperature t_1 C is A_o $(= \ell_O^2)$.

If the object is heated from temperature t_1 C to t_2 C, then each length would have increased by ℓ (fig. 3.3 (ii)). Then the

new area A_t = $(\ell_O + \ell)^2$

Thus, the change in area ℓ is given as

$$A = A_t - A_o$$

and the change in temperature is given as

$$= (t_2 - t_1)^oC$$

Again, experiments have shown that for a given material, the change in area (A) is directly proportional to the original area A_o and the change in temperature .

$$A = A_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (4.5)$$

where A_o is the constant of proportionality otherwise known as the coefficient of superficial expansion or in short area expansivity of the material.

$$A_t - A_o = \beta A_0 \Delta\theta$$
$$A_t = A_o + \beta A_o \Delta\theta$$
$$A_t = A_o \; (1 + \beta A_o \Delta\theta) \ldots\ldots\ldots\ldots\ldots\ldots.. (4.6)$$

So far we have learnt about and linear and area expansivity. Let us see, that there is any relationship between the linear expansivity (ℓ) of a material and the area expansivity (A_o) of the same material. We shall proceed to show this relationship.

Given that from Eq. (4.6), that $A_t = (\ell_O + \Delta \ell)^2$

On expanding the expansion $(\ell_{O} + \Delta \ell)^2$, we get

$$A_t = (\ell_0^{22} + 2\ell_O \Delta\ell + \Delta\ell)^2 \; ^2 \ldots\ldots\ldots\ldots\ldots.(4.7)$$

You know that $\Delta\ell = \alpha \ell_O \Delta\theta$ $\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$... (4.8)

But

Therefore, Eq. (4.7) becomes

$$A_t = \ell_O^2 + 2\ell_O \alpha \ell_O \Delta\theta + {}^2\alpha^2\ell_0^2 (\Delta\theta)^2$$
$$= \ell_O^2 + 2\alpha\ell_0^2 \Delta\theta + \alpha^2\ell_0^{2}(\Delta\theta)^2$$

But $\ell_O^2 = A_o$

On arranging the terms, we get

$$A_t = A_o + 2\alpha A_o \Delta\theta + \alpha^2 A_o(\ell o)^2 \ldots\ldots\ldots\ldots\ldots\ldots.(4.9)$$

You will recall that the value of α is very small (10^{-5}) therefore α^2 will be so small that term $\ell o^{\,2}$ the expression can be ignored.

$$A_t = A_o + 2\alpha A_o \Delta\theta \quad \text{compare with}$$

$$A_t = A_o + \beta A_o \Delta\theta$$

$$\text{Hence} \quad A_t = 2\alpha \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.\ldots (4.10)$$

That is, the coefficient of superficial expansion is twice the value of the coefficient of linear expansion.

It means that when problems are set on superficial expansion, you will not be given the value of ℓo but you will be given the value A_o of for the material. At this point, you must remember the relationship.

$$A_t = \quad 2\alpha$$

> **ACTIVITY 1**
> A square sheet of steel has a side of 15cm at 0ºC. Determine its area at 40ºC. Given: coefficient of linear expansivity of steel = 1.1 x $10^{-5}C^{-1}$)

3.1.4 Cubical Expansion

The volume of a solid increases as a result of heat. Let the original volume of the cube as shown in fig. 3.4(i) be V_o at initial temperature t_1ºC. If the cube is heated to a temperature t_2ºC, then the new volume at temperature t_2ºC is V_t (fig. 3.4(ii).

There is a change in volume given by

$$V \quad = \quad V_t \quad - \quad V_o$$

The change in volume is directly proportional to the

$$(\ell o \quad = \quad \Delta\ell)$$

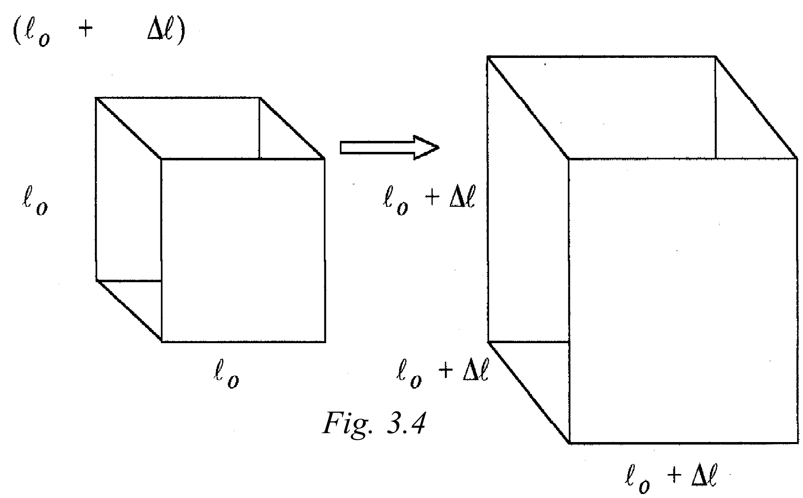


Fig. 3.4

Original volume V_o and the change in temperature

$$\Delta V = \gamma V_o \Delta\theta$$

where is a constant of proportionality the coefficient of cubical expansion, or cubical expansivity. is defined as,

$$\frac{\Delta V}{\gamma V_o \Delta\theta} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.11)}$$

The increase in volume per original volume per degree Celsius rise in temperature.

$$\text{If} \quad \Delta V = \gamma V_o \Delta\theta$$

$$\text{But} \quad V_t = V_o + \Delta V$$

$$V_t = V_o + \gamma V_o \Delta\theta$$

$$V_t = V_o \; (1 + \gamma\Delta\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.12)}$$

Again, one can establish between the cubical expansivity and the linear expansivity .

From fig. 3.4(i) we know that

$$V_o + \ell_O{}^{2\;3} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.13)}$$

and from fig. 3.4 (ii) we also note the

$$V_t = (\ell_O + \Delta\ell)^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.14)}$$

by expanding $(\ell_O + \ell)^3$ we obtain

$$V_t = \ell_0{}^3 + 3\alpha\ell_O\Delta\theta^3 + 3\ell_0{}^3\alpha^2 (\Delta\theta)^2 + \alpha^3\ell_O{}^3 \; {}^3$$

By using the previous argument that the value of is very small, therefore ^{2}, ^{3} will be so small that the expressions in terms of ^{2} and ^{3} can be ignored.

Consequently

$$V_t = \ell_O{}^3 + 3\alpha\ell_O{}^3\,\Delta\theta \quad \text{since} \quad \ell_O{}^3 = V_o$$

$$V_t = V_o + 3\alpha V_o\,\Delta\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.15)}$$

$$\text{But,} \quad V_t = V_o + \gamma V_o\,\Delta\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.16)}$$

On comparing Eq. (4.15) and Eq. (4.16), we can conclude that is three times as large as

That is the volume of expansivity is three times r =3 the linear expansivity ………………………………………………...…….. (4.17)

ACTIVITY 2
The volume of a small piece of metal is $5.000cm^3$ at 20^oC and $5.014cm^3$ at 100^oC. Determine the cubic expansivity of the metal?

3.1.5 Applications of Expansivity

The following are some practical applications of expansivity:

1. Some metals such as platinum and tungsten have their linear expanisivity very close to that of glass. Beside the fact that the linear expansivity of platinum in almost equal to that of glass, the behaviour of these solids are very much alike. This characteristics feature therefore enables us to seal electrodes through glass without the occurrence of breakage through cooling and heating processes.

2. Linear expansivity is also applied in the formation of bimetallic element which are used as:

   (i) thermostatic control switches;
   (ii) in the construction of expansion loops for use in steam lines;
   (iii) bimetallic thermometers.

3. Linear expansivity is also used in the construction of bridges where gaps are left between the girders to accommodate expansion. Such gaps are also between the iron rails in the construction of railway lines.

Example 2

Have you ever noticed that the electric wires on the public electrcity poles are always left sagging? Can you explain why this is so?

Solution

This is to allow for changes in length as the temperature changes. If they are taut further cooling in the atmosphere may make them snap.

3.2 Thermal Expansion in Liquids

As expansion takes place in solids so also in liquids. The expansion in liquids is a bulk affair hence we would talk of volume expansivity for liquids.

3.2.1 Cubical Expansion

The coefficient of volume expansion of a liquid is the fractional change in volume per the original volume fro degree change in temperature. From Eq. (4.12) given below,

$$V_t = V_o (1 + \gamma\Delta\theta)$$

It is necessary to note that for liquids is not of the same order as 3 where is the linear expansivity of solids.

In actual fact liquids 10 solid i.e. liquids is greater that solid

The observed increase in the volume f the liquid is the difference between the expansion of the liquid and that of the liquid and that of the container and it is therefore called relative or apparent change in volume.

For water, between 0ºC and 40ºC there is a decrease in volume i.e. increasing in density? But between 4ºC and 100ºC, the volume of water increases uniformly while its density decreases. This is what is being described as the anomalous behaviour of water. Such behaviour of water preserves the lives of marine creatures.

3.2.2 Real and Apparent Expansion of Liquids

Experience has shown that it is impossible to measure the real or absolute thermal expansion of a liquid by direct volume determinations. This is because liquids are contained in vessels which also expand when heated. Hence, the expansion of the content of a vessel is always relative or apparent.

Apparent expansion of the liquid is therefore less that the real expansion of the liquid. Volume dilatometers are used in the determination of thermal expansion of liquids. The mean coefficient of apparent expansion of a liquid ($_{app}$) between temperature t_1 and t_2 is given as:

$$\alpha_{app} = \frac{V_2 \quad V_1}{V_1(t_2 \quad t_1)}$$

Where V_2 is the finial volume at t_2ºC, V_1 is the initial volume and $(t_2 - t_1)$ the change in temperature.

This is a general definition of the coefficient of apparent expansion. It applies to such experiments as the volume dilatometer, weight thermometer, relative density bottle and sinker methods of determining the coefficient of apparent expansion.

In the last three examples, weights of the volumes of the liquid between t_1 and t_2 are compared which will be equal if the vessel and the sinker did not expand.

$$\alpha_{app} = \frac{\text{mass of liquid expelled}}{\text{mass remaining x temperature change}}$$

Thus $\alpha_{real} = \alpha_{app} + \gamma$

where is the coefficient of cubical expansion of the material of vessel, γ_{app} is the apparent coefficient of expansion of the liquid and γ_{real} is the real coefficient of the liquid.

4.0 CONCLUSION

When matter is heated, it expands. The ways materials expand when they are subjected to heat are described by their coefficients of expansion linear, superficial cubical expansivities.

Expansion in liquids is greater than that of solids. Besides, the study of expansion in liquids is more complicated than solids. This is because as the liquid expands, its container also expands. Thus we talk of relative expansion in liquids rather than of expansion. In the next unit we would consider the expansion in gases. This will be studied under the gas laws.

5.0 SUMMARY

You have learnt the following in this unit:

- When matter is heated, it expands;
- There are three types of coefficient of expansion – linear expansivity, superficial expansivity and cubical expansivity;
- Expansion of a material depends on its nature, temperature range and the initial dimensions of the material;
- Superficial expansivity is twice the linear expansivity of solid material;
- The cubical expansivity is thrice the linear expansivity of a solid material;
- There is no absolute expansion of a liquid because it is contained in a container, which also expands.

ACTIVITY 1

Using Eq. (4.6), we get

$A_t = A_o + \beta A_o \Delta\theta$

where, A_t = area of the plate at 40ºC = ?

A_o = original area of plate = $15cm^2$

$\beta = 2 = 2\alpha \times 1.1 \times 10^{-5}\,{}^{o}C^{-1}$

= area expansivity of steel.

$\Delta\theta$ = change in temperature = $(40 - 0)\,{}^{o}C$

= $40\,{}^{o}C$

$\therefore \quad = \quad A_o + 2\alpha A_o \Delta\theta$

$A_t = 15^2cm^{2} + 2 \times 2 \times 1.1 \times 10^{-5}\,{}^oC^{-1} \times 40^oC \times 15^2cm^2$

$= (225cm^2 + 0.198cm^2)$

$= 225.198cm^2$

ACTIVITY 2

Given:

$V_o = 5.000cm^3$

$V_t = 5.014cm^3$

and $t_1 = 20^oC$

and $t_2 = 100^oC$

$\Delta\theta = t_2 - t_1 = (100 - 20)^oC = 80\,^oC$

$\Delta V = V_t - V_o$

Using the Eq. (4.11)

$$= \frac{\Delta V}{V_o \Delta\theta}$$

$$= \frac{V_t - V_o}{V_o \Delta\theta}$$

Substituting the values, we get

$$= \frac{(5.014-5.000)cm^3}{5.000cm^3 \times 80^o C}$$

$$= \frac{0.014}{5.000 \times 80^o C}$$

$= 0.000035^oC^{-1}$

$= 3.5 \times 10^{-5\,o}C^{-1}$

Cubical expansivity of the metal is $3.5 \times 10^{-5\,o}C^{-1}$

6.0 TUTOR-MARKED ASSIGNMENT

1. A brass measuring scale is exactly two meters long at 15ºC. Determine its length at 40ºC? (for brass = 1.8 x 10^{-5}ºC^{-1}).

2. If the linear expansivity of a metal is 2.0 x 10^{-5}ºC^{-1}, calculate the approximate value of its superficial expansivity.

3. The density of aluminum at 0ºC is 2.76g (cm^{-3}). Determine its density at 200ºC (for aluminum = 2.5 x 10^{-5}ºC^{-1}).

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Das Sarma, J.M. (1978). *An introduction to higher Secondary Physics*. Modern Book Agency Private Ltd.

Duncan, T. (1982). *Physics. A textbook for Advanced Level Students* London: John Murray (Publishers) Ltd..

Fishbane, P. M, Gasiorowicz, S., & Thronton, S. T. (1996). *Physics for Scientists and Engineers. 2nd edition* Vol. 1. New Jersey: Prentice Hall Upper Saddle River.

Garg, S. C., Bansal, R. M., & Gosh, C. K. (1993). *Thermal physics.* India: Tata McGraw – Hill Publishing Company Limited.

Geddes, S. M. (1981). *Advanced physics*. London: Macmillan Education Ltd.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Indira Gandi National Open University School of Sciences. PHE 01, Elementary Mechanics

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

McKenzie, A.E.E (1973). *A second course of electricity*. Cambridge: The University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 4 GAS LAWS

CONTENTS

1.0 INTRODUCTION

When solids and liquids are heated, they are not significantly affected by changes in pressure as their temperatures change. This is why we did not consider the effect of pressure during the expansion of solids and liquids. However, for a given mass of gas, the expansion of a gas is considerably affected by pressure. It is to be noted that in describing the behaviour of gases, when subjected to heat, four variables are usually considered.

They are:

pressure (P)
volume (V)
temperature (T) and
the number of moles (n) of the gases.

These four properties or parameters are used to describe the state of a given mass of a gas. In this unit, we shall first discuss the relationship between the temperature, pressure and volume of a gas. Then we will examine the behaviour of gases using these parameters to deduce the various gas laws.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state the different gas laws;
- explain the gas laws through the use of graphs;
- distinguish between a real gas and an ideal gas;
- express the equation of state of an ideal gas;
- solve problems on these gas laws.

How to Study this Unit:

In this unit you are expected to:

1. Read through the course contents on your own
2. First attempt the activities, then the TMA without looking at the hints provided by the author
3. Make observations on all your difficulties to your facilitator
4. Confirm your work on the activities after you have done your best to get all correct

3.0 MAIN CONTENT

3.1 GAS LAWS

You will recall that four properties are used to describe the behaviour of gases; namely:

Pressure (P), Volume (V), Temperature (T), and Amount of the gas in moles (n).

When any two of these properties are kept constant, the other two are then subjected to change in order to show how the gas behaves. The first two will be considered while temperature and the number of moles of the gas is kept constant.

3.2 Boyle's Law

Boyle (1662) investigated the relationship between the pressure (P) and the volume (V) of a given mass of gas when the temperature (T) and the number of moles (n) are kept constant. Boyle's law states that:

"The pressure on a given mass of gas is inversely proportional to its volume (V) provided its temperature is kept constant".

Symbolically, this statement is written as:

$$P = \frac{1}{V} \quad \text{.. (4.1)}$$

$$P = \frac{K}{V} \text{ where, K is a constant of proportionality.}$$

$$PV = K = \text{Constant} \quad \text{.. (4.2)}$$

If you plot a graph of p against 1/v

in fig. 4.1(i) and fig. 4.1(ii). , then the graph would be as given below

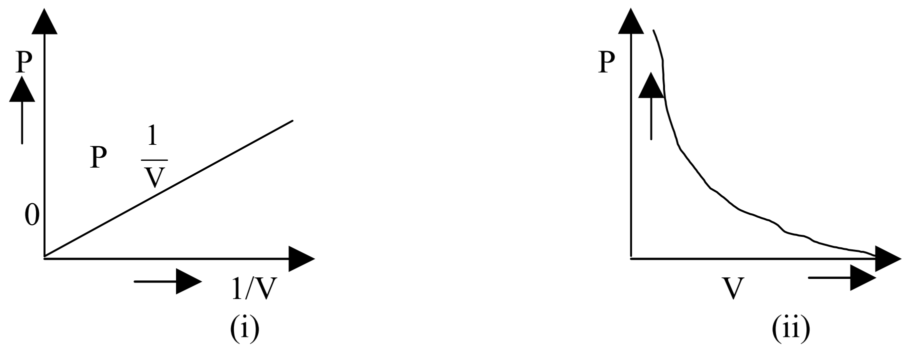


Fig. 4.1

3.2.1 Experimental Proof of Boyle's Law

Boyle's law may be demonstrated by suing a 'J' tube as shown in fig. 4.2 such that one end is opened and the other end closed. Thus AB in fig. 5.2 contains the trapped air by the column of mercury. The mercury head (h) constitutes the pressure on the trapped air in addition to the pressure due to the atmosphere (H_o). The cross sectional area of the 'J' tube is assumed to be uniform. That is, the circular area is uniform.

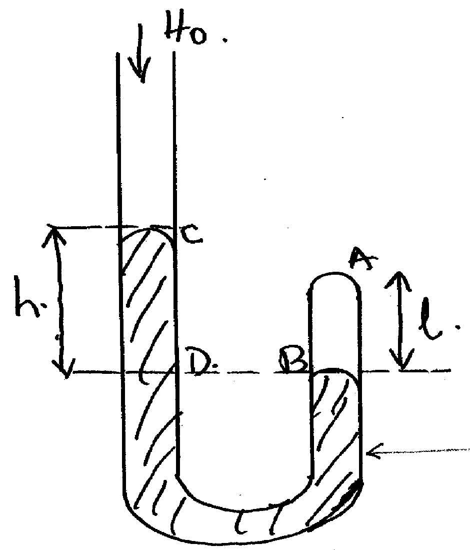


Fig. 4.2

You would have noticed that two variables can be identified in this set up.

(i) the volume of the air trapped V = A ℓ……………….(4.3) where A is the area of cross-section of the tube and ℓ is the length of the air trapped.

(ii) the pressure P on the tapped air which is made up of the atmospheric pressure H_o and the mercury column h.

From Eq. (4.3),

$V = A\ell$ we say that Volume (V) is proportional to the length of the air.

$$V \alpha \ell \text{ and } \frac{1}{v} \alpha \frac{1}{\ell} \ldots\ldots\ldots\ldots\ldots\ldots (5.4)$$

Thus the measurement of ℓ is proportional to V

As you know, the pressure P = h ρ g

Where, h is the height of the mercury column, ρ is the density of mercury, and g is the acceleration due to gravity.

ρ and g are constants.

This also means that the pressure P on the gas is proportional to the height (h) of the mercury.

P α h

That is the total pressure on the gas P (H_o + h) where H_o is the barometric height – atmospheric pressure and h is the mercury head in the 'J' tube.

Thus the measurement of H_o + h will be proportional to the pressure P. Pouring more mercury through the open end varies the length ℓ of the air column. For each measured height h of the mercury, the corresponding length ℓ of the air column is measured.

First, we then plot the graph of (H_o + h) against ℓ with the (H_o + h) on the vertical axis and ℓ on the horizontal axis. We would obtain a graph as shown in fig. 4.3(i). This shows that as (H_o + h) increases, ℓ decreases.

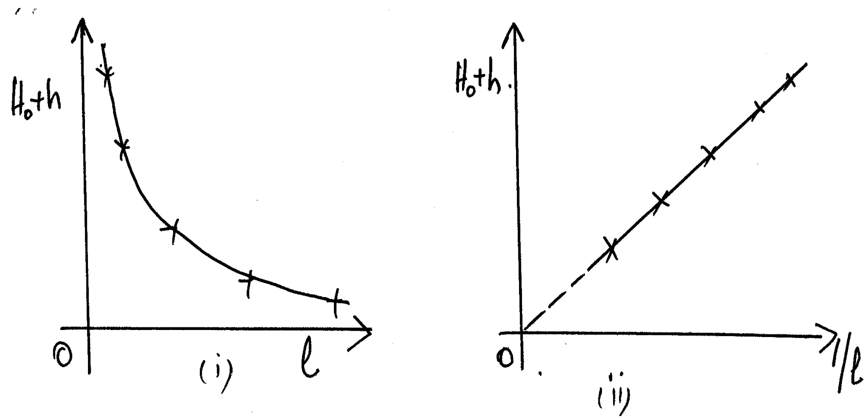


Fig. 4.3: (i) Plot of $(H_o + h)$ versus ℓ (ii) Plot of $(H_o + h)$ versus $(\frac{1}{\ell})$

Secondly, if we plot the graph of $(H_o + h)$ against $\frac{1}{\ell}$, with $(H_o + h)$ on the vertical axis and $\frac{1}{\ell}$ on the horizontal axis. A graph as shown in fig. 4.3 (ii) will be obtained. We find that the plot is a linear graph.

The two graphs are in consonance with Boyle's law. Fig. 4.3(i) says that as the pressure increases, the volume decreases while fig. 4.3(ii) is also saying that the pressure is inversely proportional to the volume of the gas.

$$(H_o + h) \quad \frac{1}{\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.5)$$

or

$$P \quad \frac{1}{V}$$

$$P = \frac{K}{V}$$

$$PV = K$$

Thus for volume V_1 and pressure P_1, $P_1V_1 = k$ and for volume V_2 and pressure P_2, $P_2V_2 = K$

The meaning of these statements is that when an amount of a gas (n) and its temperature t are kept constant we can conveniently state that

$$P_1V_1 = P_2V_2 = K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.6)$$

However, if n changes then

$P_1V_1 \quad P_2V_2$ ………….……….………………………..…… (4.7)

and the conditions of the experiment are no longer consistent with the conditions of the law even when the temperature is kept constant.

Boyle's law is applied in air compressors and exhaust (vacuum) pumps.

ACTIVITY 1
Under a Pressure of $14Nm^{-2}$, some air has a volume of $1.5m^3$. Determine its volume when its pressure is $10Nm^{-2}$. Assuming the temperature is kept constant.

3.3 Charles's Law

Charles's law deals with the behaviour of a given mass of gas at constant pressure. Under this law, we would consider the variation of volume (V) with temperature (T) when the pressure (P) and the amount of the gas (n) are kept constant.

The original Charles's law states that:

"At constant pressure, the volume of a given amount of gas increases by a constant fraction of its volume at 0^oC for each Celsius degree rise in temperature".

The Mathematical expression for this can be written as:

$V \; \alpha \; T$ (at constant n and P) ………….…………………………….. (4.8)

The above statement brings out the idea of volume coefficient, r, where r is defined as the increase in volume of a unit volume of the gas at 0^oC for each degree Celsius rise in temperature when the fixed mass of that gas is heated at constant pressure.

The volume coefficient is called volume expansivity.

If V_o is the volume of the gas at 0^oC and V_o is the volume of the gas at t^oC, then r is expressed as:

$$r = \frac{\Delta V}{V_o \Delta\theta}$$

$$= \frac{V_t - V_o}{V_o(t - 0)}$$

$$r = \frac{V_t - V_0}{V_o t}$$

$$V_t = V_o + r V_o t$$

$$V_t = V_o (1 + rt) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.9)$$

Note that V_o stands for the volume of the gas at 0ºC and not just the original volume at any selected initial temperature. And that t is the actual temperature using the Celsius scale and not for any selected temperature rise.

The valve of r for most gases is $\frac{1}{273}$. Now substituting the value of r in Eq. (4.9), we get

$$V_t = V_o \frac{273 \quad t}{273} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4.10)$$

But as you know from the absolute scale,

$$\left.\begin{array}{l} (273 + t) = T\# \text{ and that} \\ 273 = T_o \end{array}\right\} \ldots\ldots\ldots\ldots\ldots (4.11)$$

Then putting the values in Eq. (5.11) into Eq. (5.10), we get

$$V_t = \frac{V_o T}{T_o}$$

On rearranging the terms, we obtain

$$\frac{V_t}{T} = \frac{V_o}{T_o} = \text{Constant}$$

$$V_t = K\# \qquad \text{i.e. } V_t \quad T$$

Thus the volume of the gas (V) is directly proportional to its absolute temperature (T). The equation $V_t = KT$ is a deduction, or consequence of Charles's law. It is not the law.

3.3.1 Verification of Charles's Law

We shall describe here, the experimental procedure for the determination r, the coefficient if volume expansion of a gas. The experimental set up is as shown in fig. 4.4 below:

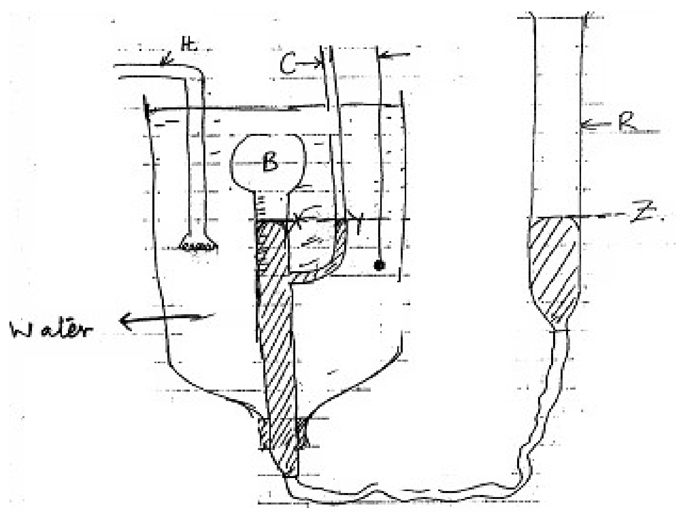


Fig. 4.4: Charles's Law Apparatus

The apparatus is made up of a glass bulb B filled with dry air. The bulb is continuous with a graduated tube. The mercury column encloses the dry air. A branch tube C is also connected to the graduated bulb B that is open to the atmosphere. The mercury levels X in bulb B and raising or lowering the reservoir R adjusts Y in tube C to the same level. In that case the pressure on the gas is the same as that of the atmosphere pressure.

The bulb B and tube C are surrounded by a water bath, which contains an electric heater H, which is also used as a stirrer. A thermometer T is inserted to measure the temperature of the bath. The initial volume V_1 and the initial temperature t_1 of the gas in the bulb are measured. The temperature of the gas is the same as that of the water in the bath. They are both recorded when the level X, Y and Z have been adjusted to be the same.

The heater is then switched on until there is difference of 20^oX rise in temperature. It is then switched off and used to stir the water thoroughly. The level X, Y and Z are then adjusted again to obtain a new volume V_1 and its new corresponding temperature t_2 C. The above procedure^{o}is repeated for another set of five or six volume measurements between the room temperature t_2^oC and 100^oC.

When then plot the graph of volumes V on the vertical axis against their corresponding temperature t on the horizontal axis. A linear graph as shown in fig. 4.5 is obtained

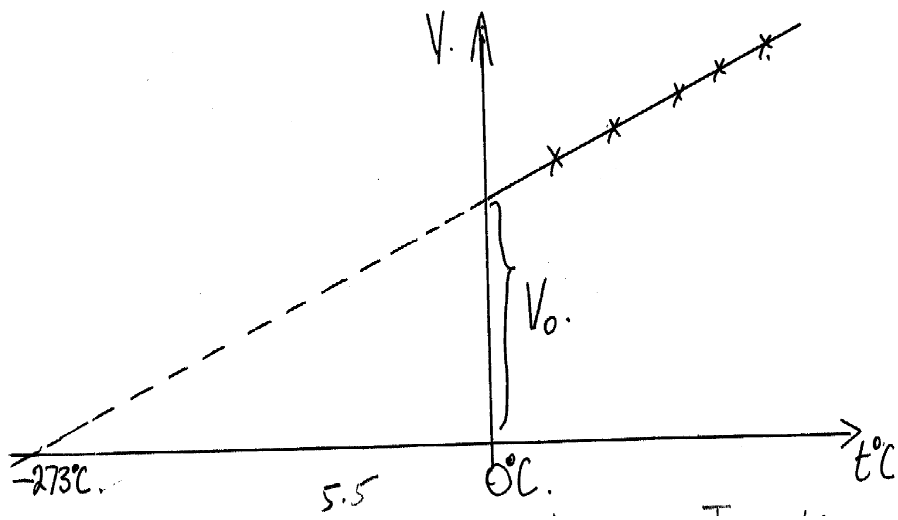


Fig. 4.5: A graph of Volume versus Temperature

The volume at 0°C, V_o may be extrapolated so that r, the volume coefficient could be determined.

$$r = \frac{V_t - V_o}{V_o \times t}$$

Further extrapolation of the graph enables us to determine the absolute zero temperature. This is found to be -273°C. In conclusion, the coefficient of the volume expansion (volume expansivity) will be found to be 0.003663, which is approximately equal to $\frac{1}{273}$.

Using the absolute scale temperature, it will be observed from the graph that $\frac{V}{T}$ Constant.

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Thus, the volume of a given mass of gas is directly proportional to its absolute temperature provided the pressure is kept constant – another form of Charles's law.

ACTIVITY 2

Some hydrogen gas a volume of 200cm^3 at 15^{o}C. If the pressure remains constant, at what temperature will its volume be 150cm^3?

3.4 Pressure Law

In this section we shall once again examine the behaviour of a gas by observing how its pressure (P) varies with temperature (t), when its volume (V) and the amount of the gas (n) are kept constant. The study under this section is described as pressure law or Gay-Lussac's law.

It will be observed that the law of increase with increase in temperature art constant volume is the same as the law of increase in volume with increase in temperature at constant pressure provided there is no change in the amount of the gas.

The pressure law states that:

"For a given mass of a gas at constant volume, its pressure increases by a constant fraction of pressure at 0^{o}C for each Celsius degree rise in temperature".

Let us consider a fixed mass of gas of volume V_1at t_1^{o}C and pressure P_1.

Suppose the gas is then heated to some temperature t_2^{o}C at which the volume is zV_1. Where z is a fraction.

We can reduce this new volume zV_1 to V_1 at higher temperature by increasing the pressure to P_2. Using Boyle's law.

$$P_2V_1 = P_1 zV_1$$
$$P_2 = zP_1$$

Thus, when the temperature is raised, the volume can be maintained at $V_1$by increasing the pressure. That is, the rise in temperature, which causes an increase in volume from V_1 to zV_1if the pressure is kept at P_1, also causes an increase in pressure from $P_1$to zP_1 if the volume is kept constant at V_1. If Boyle's law is not obeyed perfectly then the theoretical basis fails.

However, experiment have shown that when a fixed mass of gas is heated at constant volume, its pressure increase s by a constant fraction of the pressure at 0^{o}C for each degree Celsius rise in temperature.

The above statement thus defines the pressure coefficient β or pressure expansivity.

The pressure coefficient β is defined as the increase in pressure expressed as a fraction of the pressure at 0ºC for one Celsius degree rise in temperature when a fixed mass of that as is heated at constant volume.

If Po is the pressure of the gas at 0ºC and P, the pressure at tºC, then is defined as:

$$\beta = \frac{\Delta P}{P_o t}$$

$$\beta = \frac{P_t - P_o}{P_o t}$$

$$P_t - P_o = \beta P_0 t$$

$$P_t = P_o (1 + \beta t) \qquad (4.13)$$

3.4.1 Constant volume Gas Thermometer

Jolly's constant volume air thermometer s used to determine the pressure coefficient for a gas (fig. 4.6)

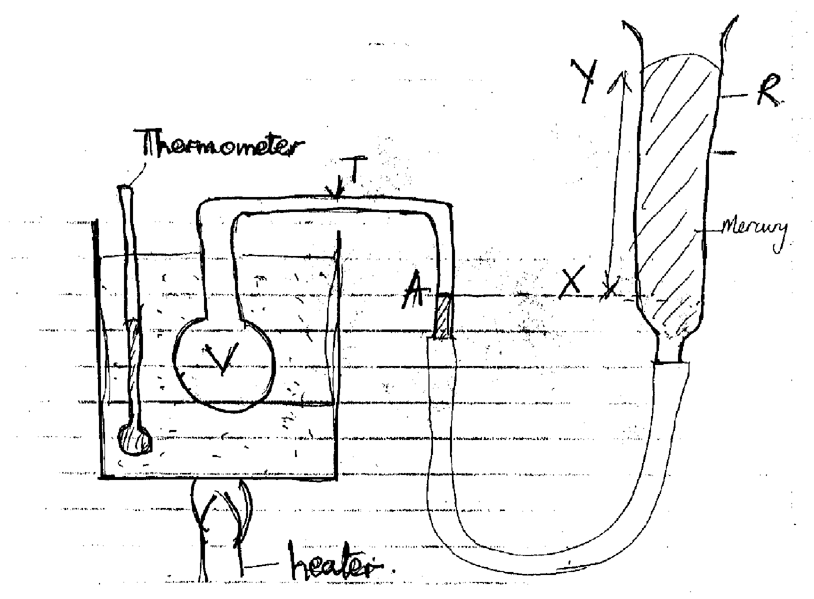


Fig. 4.6: Constant Volume Air Thermometer

It consists of a glass bulb V of volume 10cm filled with dry air. The bulb V is connected to a glass capillary tube T that in turn is connected to a mercury manometer R which measures pressure. A fixed reference mark is made on the capillary tube at A.

The moveable arm of the manometer R is usually adjusted up and down as may be necessary to ensure that the mercury level at A remains the same. This marks the constant volume before any reading is taken.

The initial pressure of the gas at the room temperature is noted from the difference in the mercury levels at X and Y. This is done by arranging the bulb in a water bath, which is well stirred, and with the mercury thermometer in the water bath. The room temperature is taken as the temperature of the water bath on which the thermometer is inserted.

The pressure of the atmosphere is read first from the barometer. Then the initial temperature of the water bath is taken. When the mercury level is first brought to level A the level Y is noted. The difference between Y and X given the mercury head, h.

Thus, the total pressure on the volume of gas is

$$P \quad \alpha \quad (H + h)\text{cm of mercury.}$$

The water bath is gently heated through, about 20°C when the heating is stopped and stirred thoroughly. The moveable arm of the manometer is adjusted, the mercury level A at a steady temperature to enable the reading of a new level of Y. The new temperature is taken and the new corresponding pressure head h is measured. The above procedure is repeated for a set of five or six readings. The barometric height is read again as a check at the end of the experiment. The graph of pressure readings is plotted against the corresponding values of the temperature readings. The graph is shown in fig. 4.7.

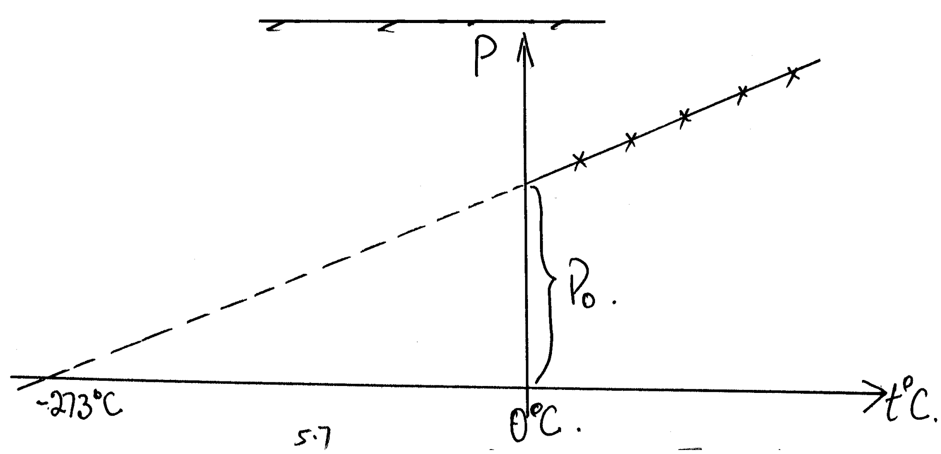


Fig. 4.7: Graph of Pressure versus Temperature

Our conclusion from this graph is that the pressure of the gas varies linearly with the temperature. When the graph is extrapolated, the pressure P_o at 0°C can be read from the graph. Further extrapolation produces the absolute temperature which will be found to be approximately equal to -273ºC.

The slope of the graph = $\beta = \frac{\Delta P}{P_o t}$

$$\beta = \frac{P_t - P_o}{P_o t} = \frac{1}{273} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.14)}$$

The following precautions are usually taken:

the mercury level is changed gradually;
the volume is maintained at constant value at A;

Sources of error could also be found in:

the dead space of the capillary tube T and
the expansion of the bulb which also introduces error.

If $= \frac{1}{273}$

From, $= \frac{\Delta P}{P_o t}$

$$= \frac{P_t - P_o}{P_o t}$$

$$P_t - P_o = P_0 t$$

$$P_t = P_o + P_0 t$$

$$P_t = P_o (1 + t) \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(4.15)}$$

Put $= \frac{1}{273}$ in Eq. (5.15), we get

$$= P_o \left[1 + \frac{1}{273} t \right]$$

$$P_t = P_o \left[\frac{273 + t}{273} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4.16)}$$

Using the absolute scale of temperature,

$$273 + t = T$$

$$P_t = \frac{P_o}{273} \text{ x } T$$

$$\frac{P_o T}{}$$

$$= \quad {}_o\,T$$

$$\frac{P}{T} = \frac{P_o}{T_o} = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.17)$$

$$P_t = KT$$

Hence, $P_t \ \alpha \ T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.18)$

This means that the pressure of the gas is directly proportional to its absolute temperature provided the volume is kept constant. This is another consequence of the pressure law. The original law states that for a fixed mass of any gas heated at constant volume, the pressure increase by $\frac{1}{273}$ of the pressure at 0ºC for each Celsius degree rise in temperature. Whereas, $\frac{P}{T}$ = constant is the deduction or the

> **ACTIVITY 3**
> The pressure in a diver's oxygen cylinder is 1.25 x $10^6 \frac{N}{m^2}$ at 20ºC. Determine the pressure in the cylinder if it is lowered into water at 10ºC

3.5 Equation of State for Ideal Gases

In physics, two kinds of gases are usually discussed. They are real gases and ideal gases. You may like to know the difference between an ideal gas and a real gas. We shall now describe the properties of real gases and ideal gases.

3.5.1 Real Gases and Ideal Gases

At extremely low pressure, all gases closely obey Boyle's law. Thus if Boyle's law is obeyed, the volume coefficient r and the pressure coefficient are the same for the same gas.

Starting with a fixed mass of a gas at 10°C at pressure P_o and volume V_o, we then heat to a temperature of t°C in two ways.

Firstly, we heat at constant pressure, then cool it down again to 0°C and then repeat the process at constant volume.

(i) at constant pressure P_o the volume changes to V_t when heated to

temperature t°C

$V_t = V_o(1 + rt)$

(1) Starting point 0°C P_o V_o 1st stage

(2) Heating the gas at Constant Pressure t°C P_o $V_t = V_o(1 + rt)$

0°C P_o V_o 2nd stage

P_o $P_t = P_o(1+ \quad t)$ V_o

(3) Heating the gas at Constant volume

At constant pressure the product $PV = P_oV_o(1 + rt)$ …..………….. (4.19)

(ii) At constant volume V_o, the pressure becomes $pt = P_o(1+ \quad t)$
The product PV then becomes $P_oV_o(1 + \quad t)$ ……………… (4.20)

If Boyle's law is obeyed, all values of the product PV at the same temperature t°C must be the same.

$P_oV_o(1 + rt) = P_oV_o(1 + \quad t)$ …………..…………… (4.21)

Hence r = 1/273

By plotting the values of r against pressure P, a graph shown in fig. 4.8 will be obtained

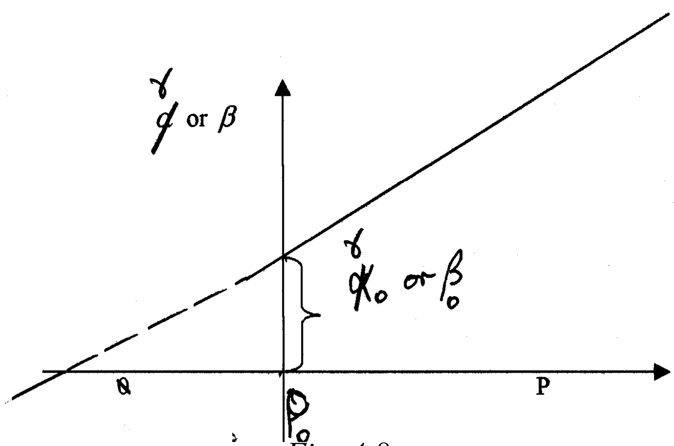


Fig. 4.8

By extrapolating to zero pressure, the pressure values of these coefficients at limiting conditions when Boyle's law is obeyed are found to be closely the same. (Table 4.1).

Table 4.1

Gas	R	
Helium	0.0036607	0.0036609
Hydrogen	0.0036611	0.0036610
Nitrogen	0.0036609	0.0036606

The Table 4.1 shows that the pressure coefficient and the volume coefficient for each individual gas are very close to one another. Also the values for different gases are all close to the mean values of 0.0036608. Hence in the limiting case at extremely low density and pressure, when Boyle's law is obeyed closely, all gases have the same volume coefficient 0.036608 and the same pressure coefficient 0.0036608.

Real gases will behave in this way only at extremely low pressures. This behaviour is what is described as being ideal.

A gas, which would behave in this way at all pressure, is called an ideal or perfect gas. In practice real gases are not ideal but we consider some approximations to ideal gas under some specified conditions.

We are now in the position to produce the equation of state for an ideal gas. At constant pressure,

$PV = P_oV_o(1 + rt)$ …………………..……………. (4.22)

At constant pressure,

$PV = P_oV_o(1 + \quad t)$ …………………..…………………. (4.23)

But $r = \quad = 0.0036608 = \frac{1}{273}$ (nearly)

Thus $PV = P_oV_o \; 1 + \frac{1}{273}t$ ………………..……….. (4.24)

$= P_oV_o \; \frac{273 + t}{273}$

where $T_o = 273$ and $273 + t = T$

$$PV = \frac{P_0V_0T}{T_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.25)$$

$$\frac{PV}{T} = \frac{P_oV_o}{T_o}$$

For a given mass of gas PV= KT

= K = Constant ……..……….. (4.26)

This is the equation of state for ideal gases.

3.5.2 Absolute Zero and Absolute Temperature

From the equation of state of an ideal gas we obtained,

$$PV = \frac{P_o V_o}{T_o}(273 + t)$$

$$PV = K(273 + t)$$

If the graph of PV is then plotted against temperature e t for a perfect, gas a graph as shown in fig. 4 59 is obtained. 4.

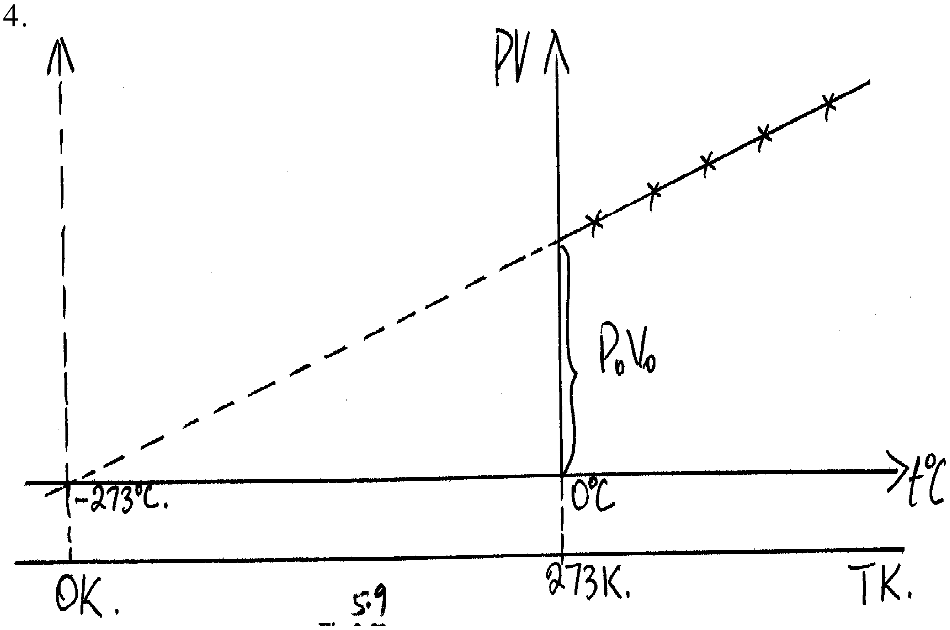


Fig. 4.9

The product PV has a value of zero when the temperature t is -273°C. At this point the product PV for a perfect gas would vanish completely. The point Z is therefore referred to as the absolute zero of temperature. If the origin of the graph 0 is now transferred to Z, then the new scale is denoted by T. This is what is referred to as the absolute scale of temperature, the SI unit of which is Kelvin. You will notice that both the Celsius scale and the Absolute scale arte related in such a way that

$$T = (273 + t)K$$

The unit of T is Kelvin while that of t is degree Celsius (°C).

3.5.3 Universal gas Constant

From the Eq. (5.27) $PV = KT$

where $K = \frac{P_oV_o}{273} = \frac{P_oV_o}{T_o}$

The pressure P_o and 273 (T_o) fix the density of the gas when the volume V_o is proportional to the mass of the gas considered. Therefore K varies directly as the mass of the gas K is constant in the sense that it has a fixed value for a given mass of an ideal gas.

There are two kinds of units of mass: the gramme and the mole (the gramme molecular weight) – gramme – mole. If you consider one mole of a gas, R replaces the constant K.

$$PV = RT$$

Hence the equation PV = RT is the ideal gas equation for one mole of the gas. Generally for n mole of a gas we would write:

$$PV = nRT \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.28)$$

Where n = molar fraction
R is the Universal gas constant for a mole of a gas.

It has been experimentally found that under standard temperature (273K) and pressure (76mm of Hg) one mole of gas occupies approximately 22.4 litres.

$$1 \text{ litre} = 1000cm^3$$
$$1\ cm^3 = 10^{-6}m^3$$
$$1000cm^3 = 1000 \times 10^{-6}m^3$$
$$22.4 \text{ litres} = 22.4 \times 10^{-3}m^3$$

The number of molecules in a mole of any gas is

6.03×10^{23} = N = Avogadro's number.

If m is the mass of gas in gramme and M is the molecular weight of the gas, the number of moles of the gas is given as:

$$n = \frac{m}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.29)$$

$$PV = \frac{mRT}{M} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4.30)$$

The value of R for 1 mole of a gas

Let P be the pressure on the gas = 76cm of Hg = standard pressure.

Put $P = h\,\rho\,g$

$$= \frac{76}{100} m \times 13600 \frac{kg}{m^3} \times 9.8 \frac{m}{s^2}$$

$= \quad 1.0129 \times 10^5 Nm^{-2}$

For a standard temperature T = 273K and standard volume V = 22.4 litres or $22.4 \times 10^3 m^3$ and for one mole of the gas, R can be obtained by suing the Eq. (4.28). The value of R

$$PV = nRT$$

$$R = \frac{PV}{nT} \quad \text{where } n = 1$$

$$R = \frac{1.0129 \times 10^5 \frac{N}{m^2} \times 22.4 \times 10^{-10} m^3}{1 \text{ mole} \times 273K}$$

$$= 8.31 \frac{J}{\text{mole K}}$$

R = 8.31 J $mole^{-1}$ K^{-1} is the molar gas constant and it is the same for all gases.

ACTIVITY 4

Some hydrogen collected at 25ºC and 740mmHg has a volume of $550cm^3$. What will be its volume at standard temperature and pressure (s.t.p.) i.e. 0ºC and 760mmHg?

3.5.5 Real Gas Equation

An ideal gas will obey Boyle's law at any temperature. However, real gases such as air, oxygen, nitrogen and other permanent gases will obey Boyle's law within less than one part in a thousand at ordinary pressures and temperatures. At higher pressures and lower temperature, the deviations are more pronounced. In other words, the relation PV is no longer valid.

Kinetic theory of gases suggest that Boyle's law should be obeyed if the molecules are themselves infinitesimally small and if they do not attract each other at all. These assumptions are not true for any real gas. Thus PV = nRT cannot be used for real gases.

In order to account for the difference between the behaviour of a gas and that of an ideal gas, we have to allow for the molecular attractions which converts the pressure P to (P + χ) and the finite volume occupies by the molecules which reduces the volume V of the gas to (V -). These corrections therefore enable us to re-express PV = nRT as:

$(P + \chi)(V - \quad) = nRT$ …..………………….………….. (4.31) We

therefore need to find suitable expressions for χ and .

It was Van der Waal (1910) a Dutch Professor of Physics who found these expression to be $\chi \quad \frac{a}{V^2}$ and $\gamma \quad b$ ………………………. (4.32)

Where, a and b are constants for a unit mass of a gas under consideration.

Consequently substituting the values of Eq. (4.32) in Eq. (4.31), we now have the equation:

$$P \quad \frac{a}{V^2} \quad (V - b) = nRT \text{ .... } (4.33)$$

This is the Van der Waal's equation of state for real gases. It is therefore known as the real equation of state.

4.0 CONCLUSION

In this unit, the three gas laws: Boyle's, Charles's and the Pressure laws have been established. We did this by observing the behaviour of the gas by using the following properties – pressure, volume, temperature and the amount of the gas in moles. Any two of these properties are held constant while we study the variation of the remaining two properties. The equation of state was also stated as PV = nRT.

Further more, we established what real and ideal gases are. We also showed how van der Waal's equation of state was used to correct for the interactive forces, which affect the pressure and the volume occupied by the molecules, which corrects for the volume of the gas. This was given as:

$$P \quad \frac{a}{V^2} \quad (V - b) = nRT$$

5.0 SUMMARY

In this unit, you have learnt about: The gas

laws such as:

(i) Boyle's law (PV = constant)

(ii) Charles's law $\frac{V}{T}$ = (constant)

(iii) Pressure law $\frac{P}{T}$ = (constant)

The three laws were combined to form the equation of state for ideal gases, which is PV = nRT.
Pressure, volume, temperature and the amount of the gas in moles have been used to describe the behaviour of gases.

The equation of state for real gases is $P \frac{a}{V^2} (V - b) = nRT$.

ANSWER TO ACTIVITY 1

First Condition

$P_1 = 14Nm^{-2}$
$V_1 = 1.5m^3$

Second Condition

$P_2 = 10Nm^{-2}$
V_2 = ? (to be found)

Using Boyle's law, Eq. (5.6)

$$V_2 = \frac{P_1V}{P_2}$$

$$V_2 = \frac{14\frac{N}{m^2} \times 1.5m^3}{10\frac{N}{m^2}}$$

We get,

$V2 = 2.1m^3$

Volume of the gas at $10Nm^{-2}$ is $2.1m^3$

ANSWER TO ACTIVITY 2

$V_1 = 200cm^3$ (given)
$T_1 = 15^oC = (273 + 15)K$ (given) $= 288K$
$V_2 = 150cm^3$
$T_2 = ?$

Using Charles's law Eq. (5.12),

$$\frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{Constant}$$

$$T_2 = \frac{V_2 T_1}{V_1}$$

$$= \frac{150cm^3 \times 288K}{200cm^3}$$

$T_2 = 216K = 273 + t$

$t = 216 – 273$
$t = -57^oC$

ANSWER TO ACTIVITY 3

$P_1 = 1.25 \times 10^6Nm^{-2}$
$T_1 = 20^oC = (273 + 20)K = 293k$
$P_2 = ?$
$T_2 = 15^oC = (273 + 10)K = 283K$

Using the pressure law:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$P_2 = \frac{P_1 T_2}{T_1}$$

$$= \frac{1.25 \times 10^6 \frac{N}{m^2} \times 283K}{293K}$$

$= 1.21 \times 10^6 Nm^{-2}$

ANSWER TO ACTIVITY 4

P_1 = 740mmHg
V_1 = $550cm^3$
T_1 = 25^oC = (273 + 25)K
= 298K

P_2 = 740mmHg
V_2 = ?
T_2 = 0^oC = 273K

Using the Eq. given below, we get

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

$$P_1 V_1 T_2 = P_2 V_2 T_1$$

$$V_2 = \frac{P_1 V_1 T_2}{P_2 T_1}$$

$$= \frac{740mmHg \times 550cm^3 \times 273K}{760mmHg \times 298K}$$

$= 490.60cm^3$

6.0 TUTOR-MARKED ASSIGNMENT

1. The density of some air at a pressure of 7720mmHg is $1.26kgm^{-3}$. Determine its density at a pressure of 600mmHg.

2. A fixed mass of gas of volume $546cm^3$ at 0^oC is heated at constant pressure. Calculate the volume of the gas at 2^oC.

3. A bottle is corked when the air inside is at 2º0C and the pressure is 1.0 x $10^5 Nm^{-2}$. If the cork blows out with a pressure of 3.0 x $10^5 Nm^{-2}$, calculate the temperature to which the bottle must be heated for this to happen. (Assume the bottle does not expand).

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Das Sarma, J.M. (1978). *An introduction to higher Secondary Physics*. Modern Book Agency Private Ltd.

Duncan, T. (1982). *Physics. A textbook for Advanced Level Students* London: John Murray (Publishers) Ltd..

Fishbane, P. M, Gasiorowicz, S., & Thronton, S. T. (1996). *Physics for Scientists and Engineers. 2nd edition* Vol. 1. New Jersey: Prentice Hall Upper Saddle River.

Garg, S. C., Bansal, R. M., & Gosh, C. K. (1993). *Thermal physics*. India: Tata McGraw – Hill Publishing Company Limited.

Geddes, S. M. (1981). *Advanced physics*. London: Macmillan Education Ltd.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Indira Gandi National Open University School of Sciences. PHE 01, Elementary Mechanics

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

McKenzie, A.E.E (1973). *A second course of electricity*. Cambridge: The University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 5 THERMODYNAMICS AND KINETIC THEORY

CONTENTS

1.0 INTRODUCTION

The behaviour of systems and gases under heat is of interest. This is extended to thermodynamics systems, first law of thermodynamics. However, for a given mass of gas, the expansion of a gas is considerably affected by pressure. It is to be noted that in describing the behaviour of gases, when subjected to heat, four variables are usually considered.

They are:

- pressure (P)
- volume (V)
- temperature (T) and
- the number of moles (n) of the gases.

These four properties or parameters are used to describe the state of a given mass of a gas. In this unit, we shall first discuss the relationship between the temperature, pressure and volume of a gas. Then we will examine the behaviour of gases when temperature changes.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state what is thermodynamics;
- explain the fundamental ideas of thermodynamics;
- Examine the changing state of system with heat and work;
- Explain first law of thermodynamics;
- Discuss kinetic theory of gas.

How to Study this Unit:

In this unit you are expected to :

5. Read through the course contents on your own
6. First attempt the activities, then the TMA without looking at the hints provided by the author
7. Make observations on all your difficulties to your facilitator
8. Confirm your work on the activities after you have done your best to get all correct

3.0 MAIN CONTENT

3.1 What is thermodynamics?

Thermodynamics is a branch of physics concerned with heat and temperature and their relation to energy and work. It defines macroscopic variables, such as internal energy, entropy, and pressure, that partly describe a body of matter or radiation (Wikipedia, 2014). In classical thermodynamics, the **entropy** of a system is defined only if it is in **thermodynamic equilibrium**. Thermodynamics is the systematic study of transformations of matter and energy in systems as they approach **equilibrium**.

Thermodynamics is a science and, more importantly, an engineering tool used to describe processes that involve changes in temperature, transformation of energy, and the relationships between heat and work. It can be regarded as a generalization of an enormous body of empirical evidence. It is extremely general: there are no hypotheses made concerning the structure and type of matter that we deal with. It is used to describe the performance of propulsion systems, power generation systems, and refrigerators, and to describe fluid flow, combustion, and many other phenomena.

The focus of thermodynamics in aerospace engineering is on the production of work, often in the form of kinetic energy (for example in the exhaust of a jet engine) or shaft power, from different sources of heat. For the most part the heat will be the result of combustion processes, but this is not always the case. The course content can be viewed in terms of a ``propulsion chain" as shown in Figure 5.1, where we see a progression from an energy source to useful propulsive work (thrust power of a jet engine). In terms of the different blocks, Parts I and II are mainly about how to progress from the second block to the third, Part III takes us from the third to the fourth, and a chapter in Part IV takes us from the first to the second. We will start with the progression from heat to work, examples of which are given in Figure 5.1.

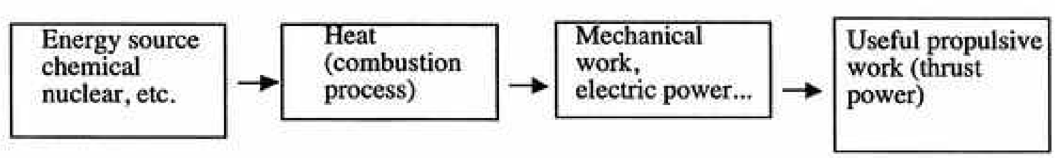


Figure 5.1: The propulsion chain

3.2 Definitions and Fundamental Ideas of Thermodynamics

As with all sciences, thermodynamics is concerned with the mathematical modeling of the real world. In order that the mathematical deductions are consistent, we need some precise definitions of the basic concepts. The following is a discussion of some of the concepts we will need. Several of these will be further amplified in the lectures and in other handouts. If you need additional information or examples concerning these topics, they are described clearly and in-depth.

3.2.1 The Continuum Model

Matter may be described at a molecular (or microscopic) level using the techniques of statistical mechanics and kinetic theory. For engineering purposes, however, we want ``averaged" information, i.e., a macroscopic, not a microscopic, description. There are two reasons for this. First, a microscopic description of an engineering device may produce too much information to manage. For example, 1 mm^3 of air at standard temperature and pressure contains 10^{16} molecules, each of which has a position and a velocity. Typical engineering applications involve more than 10^{20} molecules. Second, and more importantly, microscopic positions and velocities are generally not useful for determining how macroscopic systems will act or react unless, for instance, their total effect is integrated. We therefore neglect the fact that real substances are composed of discrete molecules and model matter from the start as a smoothed-out **continuum**. The information we have about a continuum represents the microscopic information averaged over a volume. **Classical thermodynamics** is concerned only with continua.

3.2.2 The Concept of a ``System"

A thermodynamic **system** is a quantity of matter of fixed identity, around which we can draw a boundary (see Figure 5.2 for an example). The boundaries may be fixed or moveable. Work or heat can be transferred across the system boundary. Everything outside the boundary is the **surroundings**.

When working with devices such as engines it is often useful to define the system to be an identifiable volume with *flow* in and out. This is termed a **control volume**. An example is shown in Figure 5.4.

A **closed system** is a special class of system with boundaries that matter cannot cross. Hence the principle of the conservation of mass is automatically satisfied whenever we employ a closed system analysis. This type of system is sometimes termed a **control mass**.

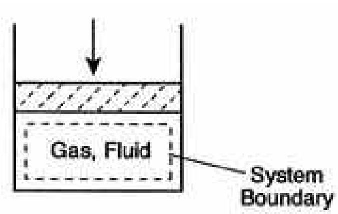


Figure 5.2: Piston (boundary) and gas (system)

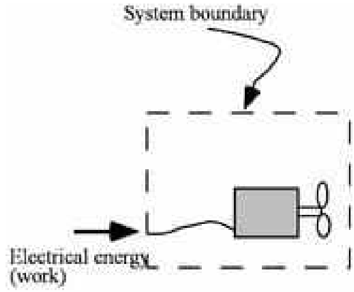


Figure 5.3: Boundary around electric motor (system)

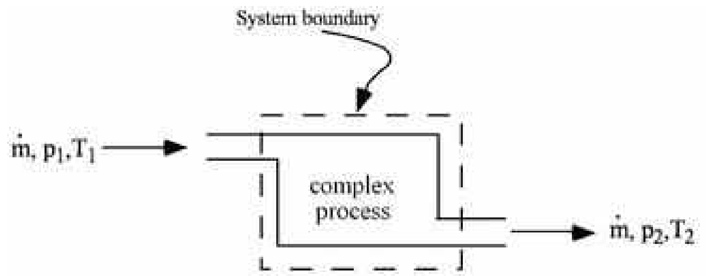


Figure 5.4: Sample control volume

3.2.3 The Concept of a ``State''

The **thermodynamic state** of a system is defined by specifying values of a set of measurable **properties** sufficient to determine all other properties. For fluid systems, typical properties are pressure, volume and temperature. More complex systems may require the specification of more unusual properties. As an example, the state of an electric battery requires the specification of the amount of electric charge it contains.

Properties may be **extensive** or **intensive**. Extensive properties are additive. Thus, if the system is divided into a number of sub-systems, the value of the property for the whole system is equal to the sum of the values for the parts. *Volume* is an extensive property. Intensive properties do not depend on the quantity of matter present. *Temperature* and *pressure* are intensive properties.

Specific properties are extensive properties per unit mass and are denoted by lower case letters. For example:

$$\text{specific volume} = V/m = v.$$

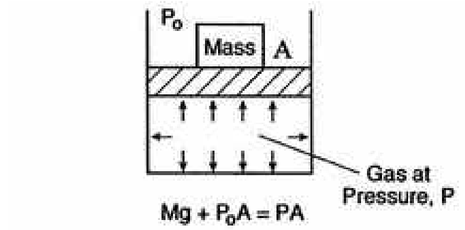
Specific properties are intensive because they do not depend on the mass of the system.

The properties of a **simple system** are uniform throughout. In general, however, the properties of a system can vary from point to point. We can usually analyze a general system by sub-dividing it (either conceptually or in practice) into a number of simple systems in each of which the properties are assumed to be uniform.

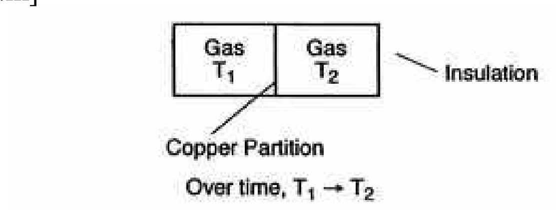
It is important to note that properties describe states *only* when the system is in equilibrium.

3.2.4 The Concept of ``Equilibrium''

The state of a system in which properties have definite, unchanged values as long as external conditions are unchanged is called an equilibrium state.



[Mechanical Equilibrium]



[Thermal Equilibrium]

Figure 5.5: Equilibrium

A system in thermodynamic equilibrium satisfies:

1. mechanical equilibrium (no unbalanced forces)
2. thermal equilibrium (no temperature differences)
3. chemical equilibrium.

3.2.5 The Concept of a ``Process''

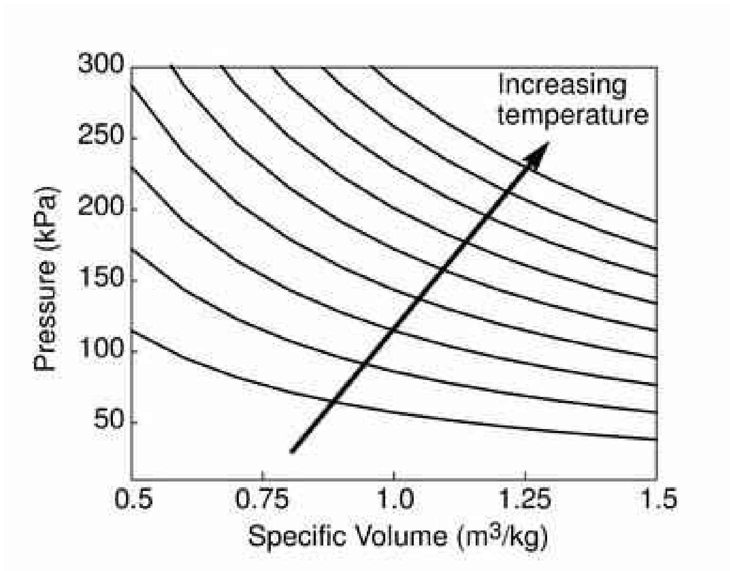
If the state of a system changes, then it is undergoing a **process**. The succession of states through which the system passes defines the **path** of the process. If, at the end of the process, the properties have returned to their original values, the system has undergone a **cyclic process** or a **cycle**. Note that even if a system has returned to its original state and completed a cycle, the state of the surroundings may have changed.

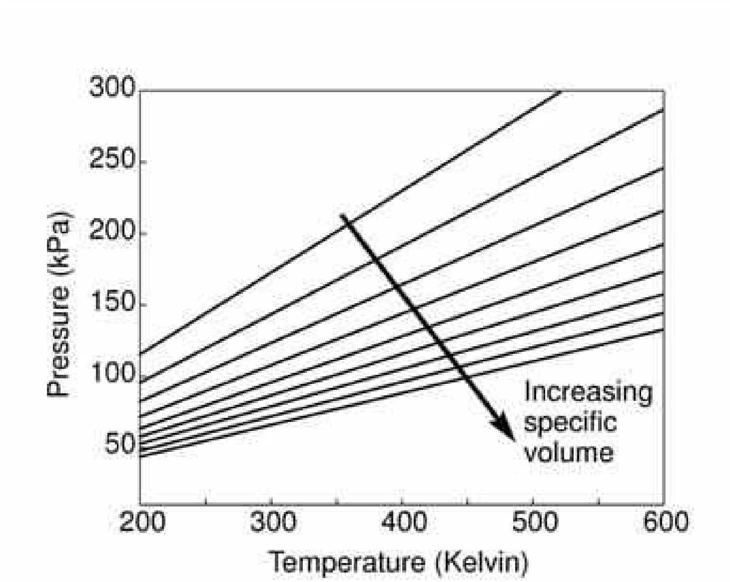
3.2.6 Quasi-Equilibrium Processes

We are often interested in charting thermodynamic processes between states on thermodynamic coordinates. Recall that properties define a state only when a system is in equilibrium. If a process involves finite, unbalanced forces, the system can pass through non-equilibrium states, which we cannot treat. An extremely useful idealization, however, is that only ``infinitesimal'' unbalanced forces exist, so that the process can be viewed as taking place in a series of ``quasi-equilibrium'' states. (The term *quasi* can be taken to mean ``as if;'' you will see it used in a number of contexts such as quasi-one-dimensional, quasi-steady, etc.) For this to be true the process must be slow in relation to the time needed for the system to come to equilibrium internally. For a gas at conditions of interest to us, a given molecule can undergo roughly 10^{10} molecular collisions per second, so that, if ten collisions are needed to come to equilibrium, the equilibration time is on the order of 10^{-9} seconds. This is generally much shorter than the time scales associated with the bulk properties of the flow (say the time needed for a fluid particle to move some significant fraction of the length of the device of interest). Over a large range of parameters, therefore, it is a very good approximation to view the thermodynamic processes as consisting of such a succession of equilibrium states, which we can chart.

The figures below demonstrate the use of thermodynamics coordinates to plot isolines, lines along which a property is constant. They include constant temperature lines, or **isotherms**, on a p - v diagram, constant volume lines, or **isochors** on a T - p diagram, and constant pressure lines, or **isobars**, on a T - v diagram for an ideal gas.

Real substances may have phase changes (water to water vapor, or water to ice, for example), which we can also plot on thermodynamic coordinates. We will see such phase changes plotted and used for liquid-vapour power generation cycles.





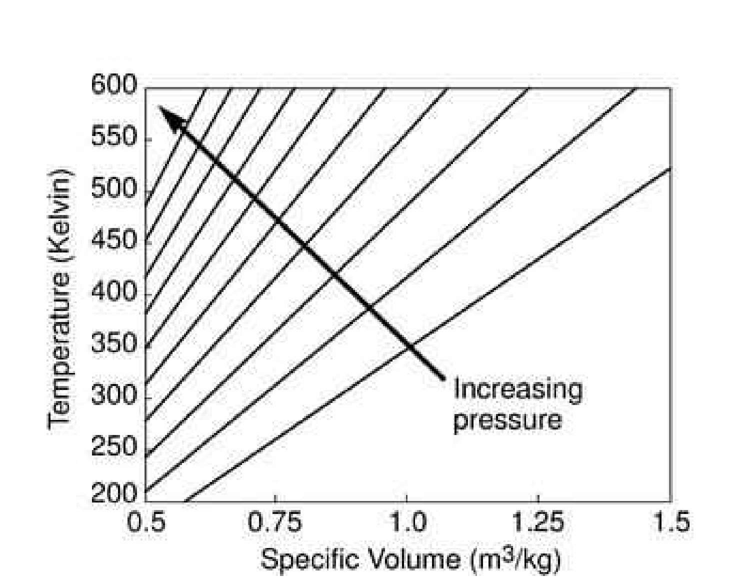


Figure: p - v diagram

Figure: p - T diagram

Figure: T - v diagram

Figure 5.6: Thermodynamics coordinates and isolines for an ideal gas

5.2.7 Equations of state

It is an experimental fact that two properties are needed to define the state of any pure substance in equilibrium or undergoing a steady or quasi-steady process. Thus for a simple compressible gas like air,

$$P = P(v,T), \quad \text{or} \quad v = v(P,T), \quad \text{or} \quad T = T(P,v),$$

where v is the volume per unit mass, $1/\rho$. In words, if we know v and T we know P, etc.

Any of these is equivalent to an equation $f(P,v,T) = 0$, which is known as an equation of state. The equation of state for an ideal gas, which is a very good approximation to real gases at conditions that are typically of interest for aerospace applications, is

$$P\bar{v} = \mathcal{R}T,$$

where $\bar{v}$ is the volume per mol of gas and $\mathcal{R}$ is the ``Universal Gas Constant," $8.31\ \text{kJ/kmol-K}$.

A form of this equation which is more useful in fluid flow problems is obtained if we divide by the molecular weight, $\mathcal{M}$:

$$Pv = RT, \quad \text{or} \quad P = \rho RT$$

where R is $\mathcal{R}/\mathcal{M}$, which has a different value for different gases due to the different molecular weights. For air at room conditions, $R = 0.287 \text{ kJ/kg-K}$.

ACTIVITY 1

Show mathematically that Pv = RT

3.3 Changing the State of a System with Heat and Work

Changes in the state of a system are produced by interactions with the environment through *heat* and *work*, which are two different modes of energy transfer. During these interactions, equilibrium (a static or quasi-static process) is necessary for the equations that relate system properties to one-another to be valid.

3.3.1 Heat

Heat is energy transferred due to temperature differences only.

1. Heat transfer can alter system states;
2. Bodies don't ``contain'' heat; heat is identified as it comes across system boundaries;
3. The amount of heat needed to go from one state to another is path dependent;
4. *Adiabatic processes* are ones in which no heat is transferred.

3.3.2 Zeroth Law of Thermodynamics

With the material we have discussed so far, we are now in a position to describe the Zeroth Law. Like the other laws of thermodynamics we will see, the Zeroth Law is based on observation. We start with two such observations:

1. If two bodies are in contact through a thermally-conducting boundary for a sufficiently long time, no further observable changes take place; *thermal equilibrium* is said to prevail.
2. Two systems which are individually in thermal equilibrium with a third are in thermal equilibrium with each other; all three systems have the same value of the property called *temperature*.

These closely connected ideas of temperature and thermal equilibrium are expressed formally in the ``Zeroth Law of Thermodynamics:''

Zeroth Law: There exists for every thermodynamic system in equilibrium a property called temperature. Equality of temperature is a necessary and sufficient condition for thermal equilibrium.

The Zeroth Law thus *defines a property* (temperature) and *describes its behaviour*.

Note that this law is true regardless of how we measure the property temperature. (Other relationships we work with will typically require an absolute scale, so in these notes we use either the Kelvin $K = 273.15 +^{\circ} C$ or Rankine $R = 459.9 +^{\circ} F$ scales. Temperature scales will be discussed further in Section 6.2.) The zeroth law is depicted schematically in Figure 1.8.

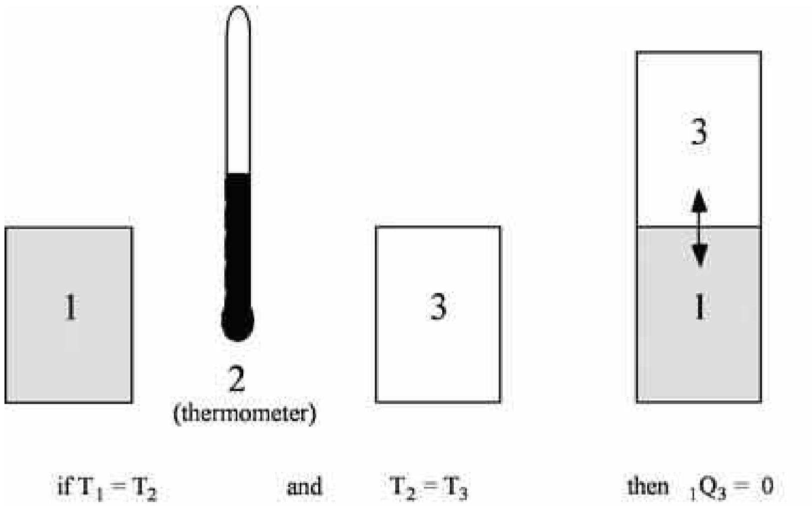


Figure 5.7: The zeroth law schematically

3.3.3 Work

We have stated that *heat* is a way of changing the energy of a system by virtue of a temperature difference only. Any other means for changing the energy of a system is called *work*. We can have push-pull work (e.g. in a piston-cylinder, lifting a weight), electric and magnetic work (e.g. an electric motor), chemical work, surface tension work, elastic work, etc. In defining work, we focus on the effects that the system (e.g. an engine) has on its surroundings. Thus we define work as being positive when the system does work on the surroundings (energy leaves the system). If work is done on the system (energy added to the system), the work is negative.

Consider a simple compressible substance, for example, a gas (the system), exerting a force on the surroundings via a piston, which moves through some distance, l (Figure 1.9). The work done *on* the surroundings, $W_{\text{on surr.}}$, is

$$dW_{\text{on surr.}} = \frac{\text{Force on surr.}}{\text{Area}} \times (\text{Area} \times dl)$$

$$dW_{\text{on surr.}} = \text{pressure of surr.} \times d\text{Volume}$$

$$dW_{\text{on surr.}} = p_x \times dV$$

therefore

$$= \int_{V_1}^{V_2} p_x dV.$$

Why is the pressure p_x instead of p_s ? Consider $p_x = 0$ (vacuum). *No work* is done *on the surroundings* even though p_s changes and the system volume changes.

Use of p_x instead of p_s is often inconvenient because it is usually the state of the system that we are interested in. The external pressure can only be related to the system pressure if $p_x \approx p_s$. For this to occur, there cannot be any friction, and the process must also be slow enough so that pressure differences due to accelerations are not significant. In other words, we require a ``quasi-static" process, $p_s \approx p_x$. Consider $p_x = p_s \pm dp$.

$$W = \int_{V_1}^{V_2} p_x dV \qquad = \int_{V_1}^{V_2} (p_s \pm dp) dV \qquad = \int_{V_1}^{V_2} p_s dV \pm dpdV.$$

Therefore, when dp is small (the process is quasi-static),

$$W = \int_{V_1}^{V_2} p_s dV,$$

and the work done *by* the system is the same as the work done *on* the surroundings.

Under these conditions, we say that the process is ``reversible." The conditions for reversibility are that:

1. If the process is reversed, the system *and the surroundings* will be returned to the original states.
2. To reverse the process we need to apply only an infinitesimal dp. A reversible process can be altered in direction by infinitesimal changes in the external conditions (see Van Ness, Chapter 2).

Remember this result, that we can only relate work done on surroundings to system pressure for quasi-static (or reversible) processes. In the case of a ``free expansion," where $p_x = 0$ (vacuum), p_s is not related to p_x (and thus, not related to the work) because the system is not in equilibrium.

We can write the above expression for work done by the system in terms of the specific volume, v ,

$$W = m \int_{v_1}^{v_2} p_s dv.$$

where m is the mass of the system. Note that if the system volume *expands* against a force, work is done *by the system*. If the system volume *contracts* under a force, work is done *on the system*.

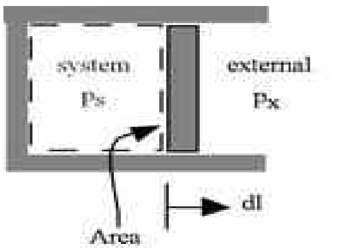
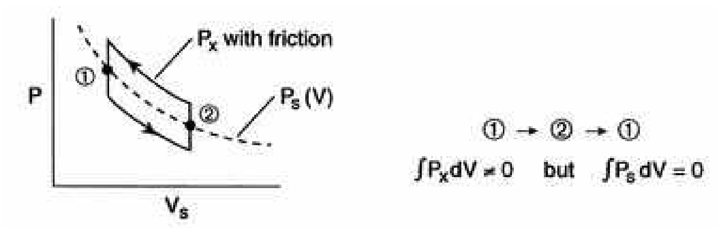


Figure 5.8: A closed system (dashed box) against a piston, which moves into the surroundings

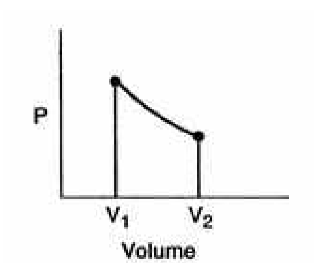


$\neq \int P_s dV.$

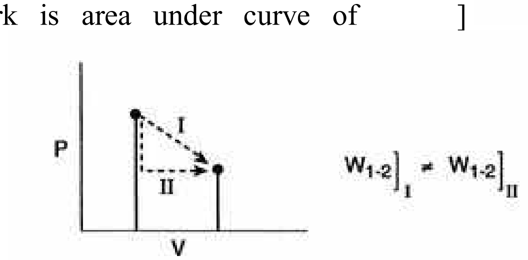
Figure 5.9: Work during an irreversible process

For simple compressible substances *in reversible processes*, the work done can be represented as the area under a curve in a pressure-volume diagram, as in Figure 5.10(a).

[Work is area under curve of $P(V)$]



[Work depends on



path]

Figure 5.10: Work in P -V coordinates

Key points to note are the following:

1. Properties only depend on states, but work is path dependent (depends on the path taken between states); therefore work is not a property, and not a state variable.
2. When we say $W_{1\text{-}2}$, the work between states 1 and 2, we need to specify the path.
3. For *irreversible* (non-reversible) processes, we cannot use $\int pdV$; either the work must be given or it must be found by another method.

3.3.3.1 Example: Work on Two Simple Paths

Consider Figure 5.11, which shows a system undergoing quasi-static processes for which we can calculate work interactions as $\int pdV$.

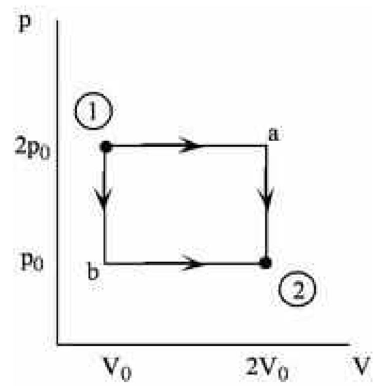


Figure 5.11: Simple processes

Along Path a:
$$W = 2p_0(2V_0 - V_0) = 2p_0V_0$$

Along Path b:
$$W = p_0(2V_0 - V_0) = p_0V_0$$

ACTIVITY 2

Given a piston filled with air, ice, a bunsen burner, and a stack of small weights, describe

1. how you would use these to move along either path a or path b above, and
2. how you would physically know the work is different along each path.

3.3.3.2 Example: Work Done During Expansion of a Gas

Consider the quasi-static, isothermal expansion of a thermally ideal gas from p_1, V_1 to p_2, V_2, as shown in Figure 5.12. To find the work we must know the path. Is it specified? Yes, the path is specified as isothermal.

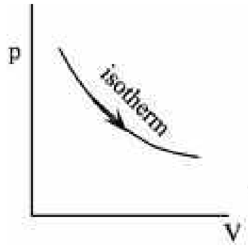


Figure 5.12: Quasi-static, isothermal expansion of an ideal gas

The equation of state for a thermally ideal gas is

$$pV = n\mathcal{R}T,$$

where n is the number of moles, $\mathcal{R}$ is the Universal gas constant, and V is the total system volume. We write the work as above, substituting the ideal gas equation of state,

$$W = \int_{V_1}^{V_2} \frac{n\mathcal{R}T}{V} dV \qquad = n\mathcal{R}T \int_{V_1}^{V_2} \frac{dV}{V} \qquad = n\mathcal{R}T \ln\left(\frac{V_2}{V_1}\right)$$

also for $T = \text{constant}$, $p_1V_1 = p_2V_2$, so the work done by the system is

$$W = n\mathcal{R}T \ln\left(\frac{V_2}{V_1}\right) \qquad = n\mathcal{R}T \ln\left(\frac{p_1}{p_2}\right)$$

or in terms of the specific volume and the system mass,

$$W = mRT \ln\left(\frac{v_2}{v_1}\right) \qquad = mRT \ln\left(\frac{p_1}{p_2}\right).$$

3.3.4 Work vs. Heat

We can have one, the other, or both: it depends on what crosses the system boundary (and thus, on how we define our system). For example consider a resistor that is heating a volume of water (Figure 5.13):

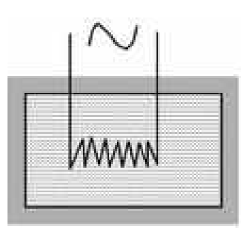
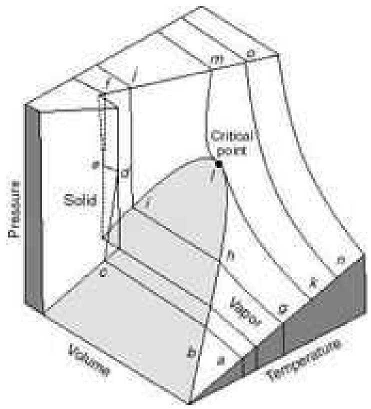
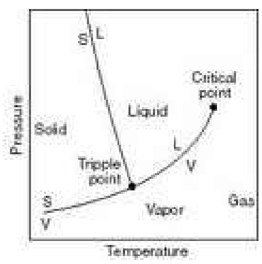


Figure 5.13: A resistor heating water

1. If the water is the system, then the state of the system will be changed by heat transferred from the resistor.
2. If the system is the water *and* the resistor combined, then the state of the system will be changed by electrical work.



[] []


[]

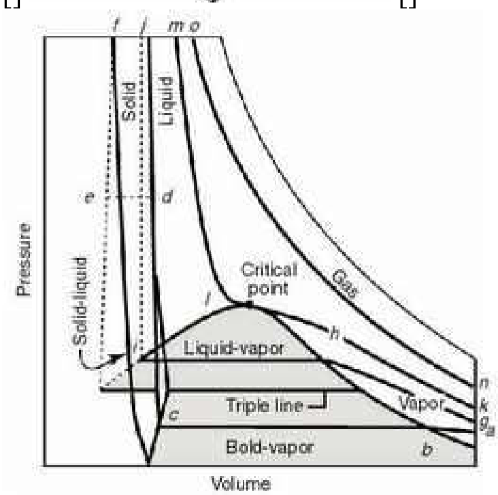


Figure 5.14: Pressure-temperature-volume surface for a substance that expands on freezing

3.4 First Law of Thermodynamics

Observation leads to the following two assertions:

1. *There exists for every system a property called energy,* E . The system energy can be considered as a sum of internal energy, kinetic energy, potential energy, and chemical energy.
    1. Like the Zeroth Law, which defined a useful property, ``temperature,'' the First Law defines a useful property called ``energy.''
    2. The two new terms (compared to what you have seen in physics and dynamics, for example) are the internal energy and the chemical energy. For most situations in this class, we will neglect the chemical energy. We will generally not, however, neglect the internal energy, u . It arises from the random or disorganized motion of molecules in the system, as shown in Figure 5.15. Since this molecular motion is primarily a function of temperature, the internal energy is sometimes called ``thermal energy.''

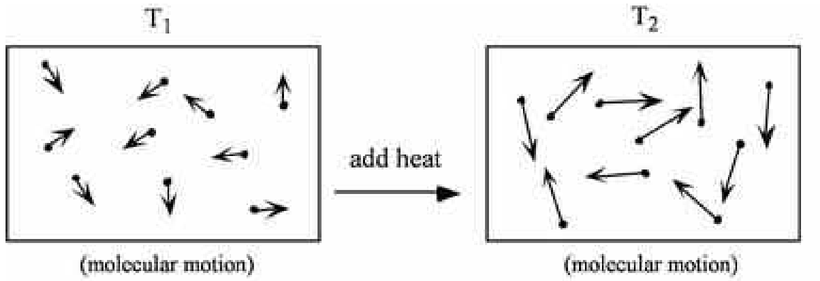


Figure 5.15: Random motion is the physical basis for internal energy

3. The internal energy, u, is a function of the state of the system. Thus $u = u(p,T)$, or $u = u(p,v)$, or $u = u(v,T)$. Recall that for pure substances the entire state of the system is specified if any *two* properties are specified. (We will discuss the equations that relate the internal energy to these other variables as the course progresses.)

2. *The change in energy of a system is equal to the difference between the heat added to the system and the work done by the system*,

$$\Delta E = Q - W \qquad \text{(units are Joules, J)}, \tag{2..1}$$

To give an example of where the first law is applied, consider the device shown in Figure 5.16. We heat a gas, it expands against a weight, some force (pressure times area) is applied over a distance, and work is done. The change in energy of the system supplies the connection between the heat added and work done. We will spend most of the course dealing with various applications of the first law -- in one form or another.

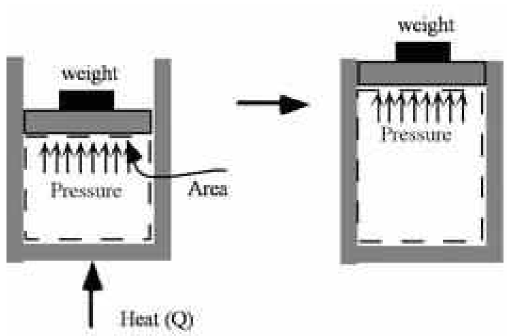


Figure 5.16: The change in energy of a system relates the heat added to the work done

The form of the first law we have given here is sometimes called the ``control mass'' form, because it is well suited to dealing with systems of a fixed mass.

3.4.1 Adiabatic, steady, throttling of a gas (flow through a valve or other restriction)

Figure 2.5 shows the configuration of interest. We wish to know the relation between properties upstream of the valve, denoted by ``1" and those downstream, denoted by ``2".

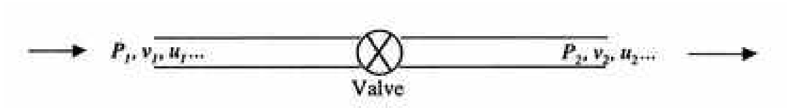


Figure 5.17: Adiabatic flow through a valve, a generic throttling process

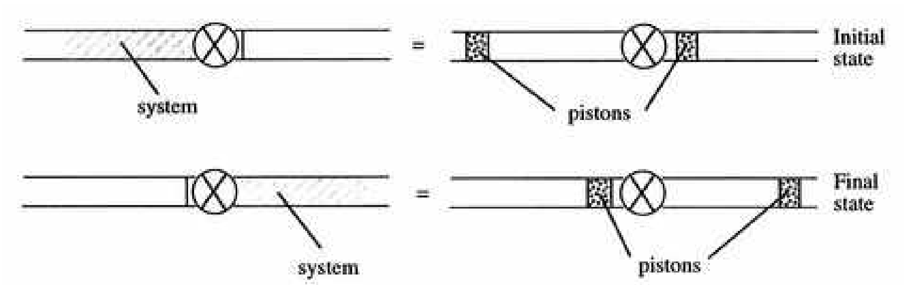


Figure 5.18: Equivalence of actual system and piston model

To analyze this situation, we can define the system (choosing the appropriate system is often a critical element in effective problem solving) as a unit mass of gas in the following two states. Initially the gas is upstream of the valve and just through the valve. In the final state the gas is downstream of the valve plus just before the valve. The figures on the left of Figure 5.18 show the actual configuration just described. In terms of the system behavior, however, we could replace the fluid external to the system by pistons which exert the same pressure that the external fluid exerts, as indicated schematically on the right side of Figure 5.18.

The process is adiabatic, with changes in potential energy and kinetic energy assumed to be negligible. The first law for the system is therefore

$$\Delta U = -W.$$

The work done by the system is

$$W = P_2V_2 - P_1V_1.$$

Use of the first law leads to

$$U_2 + P_2V_2 = U_1 + P_1V_1.$$

In words, the initial and final states of the system have the same value of the quantity $U + PV$. For the case examined, since we are dealing with a unit mass, the initial and final states of the system have the same value of $u + Pv$.

We define this quantity as the ``enthalpy," usually denoted by H,

$H = U + PV.$

In terms of the specific quantities, the enthalpy per unit mass is

$h = u + Pv = u + P/\rho.$

It is a function of the state of the system. H has units of Joules, and h has units of Joules per kilogram.

The utility and physical significance of enthalpy will become clearer as we work with more flow problems. For now, you may wish to think of it as follows (Levenspiel, 1996). When you evaluate the energy of an object of volume V, you have to remember that the object had to push the surroundings out of the way to make room for itself. With pressure p on the object, the work required to make a place for itself is pV. This is so with any object or system, and this work may not be negligible. (The force of one atmosphere pressure on one square meter is equivalent to the force of a mass of about 10 tons.) Thus the total energy of a body is its internal energy plus the extra energy it is credited with by having a volume V at pressure p. We call this total energy the enthalpy, H.

3.4.3. The First Law in Terms of Enthalpy

We start with the first law in differential form and substitute pdV for dW by assuming a quasi-static or reversible process:

$$dU = \delta Q - \delta W \qquad \text{(true for any process, neglecting } \Delta KE \text{ and } \Delta PE\text{)}$$

$$dU = \delta Q - pdV \qquad \text{(true for any quasi-static process, no } \Delta KE \text{ or } \Delta\text{PE)}$$

The definition of enthalpy,

$$H = U + pV,$$

can be differentiated (applying the chain rule to the pV term) to produce

$$dH = dU + pdV + Vdp.$$

Substituting the dU above for the dU in the First Law, we obtain

$$dH = \delta Q - \delta W + pdV + Vdp \qquad \text{(valid for any process)}$$

or

$$dH = \delta Q + V dp \qquad \text{(valid for any quasi-static process).}$$

3.4.4 Specific Heats: the relation between temperature change and heat

How much does a given amount of heat transfer change the temperature of a substance? It depends on the substance. In general

$$Q = C\Delta T, \tag{2..4}$$

where C is a constant that depends on the substance. We can determine the constant for any substance if we know how much heat is transferred. Since heat is path dependent, however, we must specify the process, i.e., the path, to find C .

Two useful processes are constant pressure and constant volume, so we will consider these each in turn. We will call the specific heat at constant pressure C_p , and that at constant volume C_v , or c_p and c_v per unit mass.

1. The Specific Heat at Constant Volume

   Remember that if we specify any two properties of the system, then the state of the system is fully specified. In other words we can write $u = u(T, v)$, $u = u(p, v)$ or $u = u(p, T)$. Consider the form $u = u(T, v)$, and use the chain rule to write how u changes with respect to T and v :

$$du = \left(\frac{\partial u}{\partial T}\right)_v dT + \left(\frac{\partial u}{\partial v}\right)_T dv. \tag{2..5}$$

   For a constant volume process, the second term is zero since there is no change in volume, $dv = 0$. Now if we write the First Law for a quasi-static process, with $dW = pdv$,

$$du = \delta q - pdv, \tag{2..6}$$

   we see that again the second term is zero if the process is also constant volume. Equting (2.5) and (2.6) with dv canceled in each,

$$\delta q = \left(\frac{\partial u}{\partial T}\right)_v dT,$$

and rearranging

$$\left(\frac{\partial u}{\partial T}\right)_v = \left(\frac{\partial q}{\partial T}\right)_v .$$

In this case, any energy increase is due only to energy transfer as heat. We can therefore use our definition of specific heat from Equation (2.4) to define the specific heat for a constant volume process,

$$c_v \equiv \left(\frac{\partial u}{\partial T}\right)_v .$$

2. The Specific Heat at Constant Pressure

If we write $h = h(T, p)$, and consider a constant pressure process, we can perform a similar derivation to the one above and show that

$$c_p \equiv \left(\frac{\partial h}{\partial T}\right)_p .$$

In the derivation of c_v, we considered only a constant volume process, hence the name, ``specific heat at constant volume.'' It is more useful, however, to think of c_v in terms of its definition as a certain partial derivative, which is a thermodynamic property, rather than as a quantity related to heat transfer in a special process. In fact, the derivatives above are defined *at any point in any quasi-static process* whether that process is constant volume, constant pressure, or neither. The names ``specific heat at constant volume'' and ``specific heat at constant pressure'' are therefore unfortunate misnomers; c_v and c_p are thermodynamic properties of a substance, and by definition depend only the state. They are extremely important values, and have been experimentally determined as a function of the thermodynamic state for an enormous number of simple compressible substances[2.1].

To recap:

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p \quad \text{and} \quad c_v = \left(\frac{\partial u}{\partial T}\right)_v .$$

or

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p \quad \text{and} \quad C_v = \left(\frac{\partial U}{\partial T}\right)_v .$$

ACTIVITY 3

Throw an object from the top tier of the lecture hall to the front of the room. Estimate how much the temperature of the room has changed as a result. Start by listing what information you need to solve this problem.

.3.5 The Kinetic Theory of Gases

3.5.1 Avogadro constant

The laws of classical thermodynamics do not show the direct dependence of the observed macroscopic variables on microscopic aspects of the motion of atoms and molecules. It is however clear that the pressure exerted by a gas is related to the linear momentum of the atoms and molecules, and that the temperature of the gas is related to the kinetic energy of the atoms and molecules. In relating the effects of the motion of atoms and molecules to macroscopic observables like pressure and temperature, we have to determine the number of molecules in the gas. The **mole** is a measure of the number of molecules in a sample, and it is defined as " the amount of any substance that contains as many atoms/molecules as there are atoms in

a 12-g sample of ^{12}C "

Laboratory experiments show that the number of atoms in a 12-g sample of ^{12}C is equal to 6.02×10^{23} mol^{-1}. This number is called the **Avogadro constant**, N_A. The number of moles in a sample, n, can be determined easily:

$$n = \frac{\text{Number of molecules in sample}}{N_A} = \frac{\text{Mass of sample}}{\text{Molecular mass}}$$

3.5.2 The Ideal Gas

Avogadro made the suggestion that all gases - under the same conditions of temperature and pressure - contain the same number of molecules. Reversely, if we take 1 mole samples of various gases, confine them in boxes of identical volume and hold them at the same temperature, we find that their measured pressures are nearly identical. Experiments showed that the gases obey the following relation (the **ideal gas law**):

$$p V = n R T$$

where n is the number of moles of gas, and R is the **gas constant**. R has the same value for all gases:

$$R = 8.31 \text{ J/mol K}$$

The temperature of the gas must always be expressed in absolute units (Kelvin).

Using the ideal gas law we can calculate the work done by an ideal gas. Suppose a sample of n moles of an ideal gas is confined in an initial volume V_i. The gas expands by moving a piston. Its final volume is V_f. During the expansion the temperature T of the gas is kept constant (this process is called **isothermal expansion**). The work done by the expanding gas is given by

$$W = \int_{V_i}^{V_f} p \, dV$$

The ideal gas law provides us with a relation between the pressure and the volume

$$p = \frac{n R T}{V}$$

Since T is kept constant, the work done can be calculated easily

$$W = \int_{V_i}^{V_f} \frac{n R T}{V} dV = n R T \int_{V_i}^{V_f} \frac{1}{V} dV = n R T \ln\left(\frac{V_f}{V_i}\right)$$

Note:

$$\ln\left(\frac{V_f}{V_i}\right) > 0 \quad \text{if} \quad V_f > V_i$$

$$\ln\left(\frac{V_f}{V_i}\right) < 0 \quad \text{if} \quad V_f < V_i$$

Sample Problem

A cylinder contains oxygen at 20°C and a pressure of 15 atm. at a volume of 12 l. The temperature is raised to 35°C , and the volume is reduced to 8.5 l. What is the final pressure of the gas ?

The ideal gas law tells us that

$$\frac{p\,V}{T} = \text{constant}$$

The initial state of the gas is specified by V_i, p_i and T_i; the final state of the gas is specified by V_f, p_f and T_f. We conclude that

$$\frac{p_i V_i}{T_i} = \frac{p_f V_f}{T_f}$$

Thus

$$p_f = p_i \frac{V_i}{V_f} \frac{T_f}{T_i}$$

The temperature T in this formula must be expressed in Kelvin:

$$T_i = 293 \text{ K}$$

$$T_f = 308 \text{ K}$$

The units for the volume and pressure can be left in l and atm. since only their ratio enter the equation. We conclude that $p_f = 22$ atm.

3.5.3. Pressure and Temperature

Let n moles of an ideal gas be confined to a cubical box of volume V. The molecules in the box move in all directions with varying speeds, colliding with each other and with the walls of the box. Figure 18.1 shows a molecule moving in the box. The molecule will collide with the right wall. The result of the collision is a reversal of the direction of the x-component of the momentum of the molecule:

$$p_{ix} = +m v_x$$

$$p_{fx} = -m v_x$$

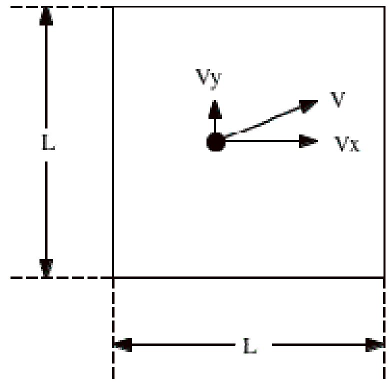


Figure 18.1. Molecule moving in box.

The y and z components of the momentum of the molecule are left unchanged. The change in the momentum of the particle is therefore

$$\Delta p = p_{fx} - p_{ix} = -2 m v_x$$

After the molecule is scattered of the right wall, it will collide with the left wall, and finally return to the right wall. The time required to complete this path is given by

$$\Delta t = \frac{2L}{v_x}$$

Each time the molecule collides with the right wall, it will change the momentum of the wall by Δp. The force exerted on the wall by this molecule can be calculated easily

$$F = \frac{\Delta p}{\Delta t} = \frac{2 m v_x}{\left(\frac{2L}{v_x}\right)} = \frac{m v_x^2}{L}$$

For n moles of gas, the corresponding force is equal to

$$F = \frac{m}{L}\left(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\right)$$

The pressure exerted by the gas is equal to the force per unit area, and therefore

$$p = \frac{F}{L^2} = \frac{m}{L^3}\left(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\right)$$

The term in parenthesis can be rewritten in terms of the average square velocity:

$$\overline{v_x^2} = \frac{\left(v_{1x}^2 + v_{2x}^2 + v_{3x}^2 + \ldots\ldots\right)}{n N_A}$$

Thus, we conclude that

$$p = \frac{n\,m\,N_A}{L^3}\overline{v_x^{\,2}} = \frac{n\,M}{L^3}\overline{v_x^{\,2}}$$

where M is the molecular weight of the gas. For every molecule the total velocity can be calculated easily

$$v^2 = v_x^{\,2} + v_y^{\,2} + v_z^{\,2}$$

Since there are many molecules and since there is no preferred direction, the average square of the velocities in the x, y and z-direction are equal

$$\overline{v_x^{\,2}} = \overline{v_y^{\,2}} = \overline{v_z^{\,2}}$$

and thus

$$\overline{v^2} = \overline{v_x^{\,2}} + \overline{v_y^{\,2}} + \overline{v_z^{\,2}} = 3\,\overline{v_x^{\,2}}$$

Using this relation, the expression for the pressure p can be rewritten as

$$p = \frac{n\,M}{3\,V}\overline{v^2} = \frac{n\,M}{3\,V}v_{rms}^{\,2}$$

where v_{rms} is called the **root-mean-square speed** of the molecule. The ideal gas law tells us that

$$p = \frac{n\,R\,T}{V}$$

Combining the last two equations we conclude that

$$\frac{n\,R\,T}{V} = \frac{n\,M}{3\,V}v_{rms}^{\,2}$$

and

$$v_{rms} = \sqrt{\frac{3\,R\,T}{M}}$$

For H at 300 K v_{rms} = 1920 m/s; for $_{14}$N v_{rms} = 517 m/s. The speed of sound in these two gases is 1350 m/s and 350 m/s, respectively. **The speed of sound in a gas will always be less than v_{rms} since the sound propagates through the gas by disturbing the motion of the molecules**. The disturbance is passed on from molecule to molecule by means of collisions; **a sound wave can therefore never travel faster than the average speed of the molecules**.

3.5.4 Translational Kinetic Energy

The average translational kinetic energy of the molecule discussed in the previous section is given by

$$\bar{K} = \overline{\frac{1}{2} m v^2} = \frac{1}{2} m \overline{v^2} = \frac{1}{2} m v_{rms}^{2}$$

Using the previously derived expression for v_{rms}, we obtain

$$\bar{K} = \frac{1}{2} m \frac{3RT}{M} = \frac{3}{2} \frac{R}{N_A} T = \frac{3}{2} k T$$

The constant k is called the **Boltzmann constant** and is equal to the ratio of the gas constant R and the Avogadro constant N_A

$$k = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K}$$

The calculation shows that **for a given temperature, all gas molecules - no matter what their mass - have the same average translational kinetic energy, namely (3/2)kT**. When we measure the temperature of a gas, we are measuring the average translational kinetic energy of its molecules.

3.5.5 Mean Free Path

The motion of a molecule in a gas is complicated. Besides colliding with the walls of the confinement vessel, the molecules collide with each other. A useful parameter to describe this motion is the **mean free path** λ. The mean free path λ is the average distance traversed by a molecule between collisions. The mean free path of a molecule is related to its size; the larger its size the shorter its mean free path.

Suppose the gas molecules are spherical and have a diameter d. Two gas molecules will collide if their centers are separated by less than 2d. Suppose the average time between collisions is Δt. During this time, the molecule travels a distance v . Δt, and sweeps a volume equal to

$$V = \left(\pi d^2\right)\left(v \Delta t\right)$$

If on average it experiences one collision, the number of molecules in the volume V must be 1. If N is the number of molecules per unit volume, this means that

$$1 = NV = N\left(\pi d^2\right)\left(v\Delta t\right)$$

or

$$v\,\Delta t = \frac{1}{N\left(\pi d^2\right)}$$

The time interval Δt defined in this manner is the mean time between collisions, and the mean free path λ is given by

$$\lambda = v\,\Delta t = \frac{1}{N\left(\pi d^2\right)}$$

Here we have assumed that only one molecule is moving while all others are stationary. If we carry out the calculation correctly (all molecules moving), the following relation is obtained for the mean free path:

$$\lambda = \frac{1}{\sqrt{2}\,\pi N d^2}$$

The relation derived between the macroscopic pressure and the microscopic aspects of molecular motion only depend on the average root-mean-square velocity of the molecules in the gas. Quit often we want more information than just the average root-mean-square velocity. For example, questions like what fraction of the molecules have a velocity larger than v_0 can be important (nuclear reaction cross sections increase dramatically with increasing velocity). It can be shown that the distribution of velocities of molecules in as gas is described by the so-called **Maxwell velocity distribution**

$$P(v) = 4\pi\left(\frac{M}{2\pi R T}\right)^{3/2} v^2 \exp\left(-\frac{M v^2}{2 R T}\right)$$

The product P(v)dv is the fraction of molecules whose speed lies in the range v to v + dv. The distribution is normalized, which means that

$$\int_{v=0}^{v=\infty} P(v)\,dv = 1$$

The **most probable speed**, v_p, is that velocity at which the speed distribution peaks. The most probable speed is obtained by requiring that dP/dv = 0

$$\frac{dP(v)}{dv} = 4\pi\left(\frac{M}{2\pi R T}\right)^{3/2}\left(2v - \frac{M}{RT}v^3\right)\exp\left(-\frac{M v^2}{2 R T}\right)$$

We conclude that dP/dv = 0 when

$$2 - \frac{M}{RT} v^2 = 0$$

and thus

$$v_p = \sqrt{\frac{2RT}{M}}$$

The **average speed** of the gas molecules can be calculated as follows

$$\bar{v} = \int_{v=0}^{v=\infty} v\, P(v) dv = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \int_{v=0}^{v=\infty} v^3 \exp\left(-\frac{Mv^2}{2RT}\right) P(v)\, dv =$$

$$= 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \frac{1}{2} \left(\frac{M}{2RT}\right)^{-2} = \sqrt{\frac{8RT}{\pi M}}$$

The **mean square speed** of the molecules can be obtained in a similar manner.

$$\overline{v^2} = \int_{v=0}^{v=\infty} v^2 P(v) dv = 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \int_{v=0}^{v=\infty} v^4 \exp\left(-\frac{Mv^2}{2RT}\right) P(v) dv =$$

$$= 4\pi \left(\frac{M}{2\pi RT}\right)^{3/2} \frac{3}{8} \left(\frac{M}{2RT}\right)^{-5/2} \sqrt{\pi} = \frac{3RT}{M}$$

The **root-mean-square speed**, v_{rms}, can now be obtained

$$v_{rms} = \sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{M}}$$

We observe that $v_p < v_{av} < v_{rms}$.

3.5.6 Heat Capacity of Ideal Gas

The internal energy of a gas is related to the kinetic energy of its molecules. Assume for the moment that we are dealing with a monatomic gas. In this case, the average translational kinetic energy of each gas molecule is simply equal to 3kT/2. If the sample contains n moles of such a gas, it contains nN_A molecules. The total internal energy of the gas is equal to

$$U = (n N_A)\overline{K} = (n N_A)(\frac{3}{2} k T) = \frac{3}{2} n R T$$

We observe that the total internal energy of a gas is a function of only the gas temperature, and is independent of other variables such as the pressure and the density. **For more complex molecules (diatomic N_2 etc.) the situation is complicated by the fact that the kinetic energy of the molecules will consist not only out of translational motion, but also out of rotational motion**.

3.5.6.1. Molar Heat Capacity at Constant Volume

Suppose we heat up n moles of gas while keeping its volume constant. The result of adding heat to the system is an increase of its temperature

$$\Delta Q = n\, C_V\, \Delta T$$

Here, Cv is the **molar heat capacity at constant volume**, ΔQ is the heat added, and ΔT is the resulting increase in the temperature of the system. The first law of thermodynamics shows that

$$\Delta U = \Delta Q - \Delta W = n\, C_V\, \Delta T - p\, \Delta V$$

Since the volume is kept constant (ΔV = 0) we conclude that

$$\Delta U = n\, C_V\, \Delta T$$

and

$$C_V = \frac{1}{n}\frac{\Delta U}{\Delta T}$$

Using the previously derived equation for U in terms of T we can show that

$$\frac{\Delta U}{\Delta T} = \frac{3}{2} n\, R$$

and thus

$$C_V = \frac{3}{2} R$$

3.5.6.2. Molar heat Capacity at Constant Pressure

Suppose that, while heat is added to the system, the volume is changed such that the gas pressure does not change. Again, the change in the internal energy of the system is given by

$$\Delta U = \Delta Q - \Delta W = n\,C_p\,\Delta T - p\,\Delta V$$

where C_p is the **molar heat capacity at constant pressure**. This expression can be rewritten as

$$\Delta U + p\,\Delta V = n\,C_p\,\Delta T$$

For an ideal gas ($pV = nRT$) we can relate ΔV to ΔT, if we assume a constant pressure

$$\Delta V = \frac{nR}{p}\Delta T$$

Using this relation, the first law of thermodynamics can be rewritten as

$$\Delta U + nR\,\Delta T = n\,C_p\,\Delta T$$

or

$$\frac{\Delta U}{\Delta T} + nR = n\,C_p$$

However, the internal energy U depends only on the temperature and not on how the volume and/or pressure is changing. Thus, $\Delta U/\Delta T = 3/2\ n\ R = n\ C_v$. The previous equation can therefore be rewritten as

$$n\,C_V + nR = n\,C_p$$

or

$$C_p - C_V = R$$

We see that $C_p \neq C_v$.

6.0 TUTOR-MARKED ASSIGNMENTS

1a. What is the difference between extensive and intensive properties?

1b. How do we know when work is done?

2.Explain the concept of entropy, thermodynamic process and closed system.

ANSWER TO ACTIVITIES 1, 2 &3

Refer to the text to answer activities 1, 2 and 3.

REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Das Sarma, J.M. (1978). *An introduction to higher Secondary Physics*. Modern Book Agency Private

Duncan, T. (1982). *Physics. A textbook for Advanced Level Students* London: John Murray (Publishers) Ltd..

Fishbane, P. M, Gasiorowicz, S., & Thronton, S. T. (1996). *Physics for Scientists and Engineers. 2nd edition* Vol. 1. New Jersey: Prentice Hall Upper Saddle River.

Garg, S. C., Bansal, R. M., & Gosh, C. K. (1993). *Thermal physics.* India: Tata McGraw – Hill Publishing Company Limited.

Geddes, S. M. (1981). *Advanced physics*. London: Macmillan Education Ltd.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Indira Gandi National Open University School of Sciences. PHE 01, Elementary Mechanics

McKenzie, A.E.E (1973). *A second course of electricity*. Cambridge: The University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://web.mit.edu/16.unified/www/FALL/thermodynamics/notes/node10.html
eb.mit.edu/16.unified/www/FALL/thermodynamics/notes/node15.html

http://physics.bu.edu/~duffy/py105/

UNIT 1 REFLECTION AT PLANE SURFACES

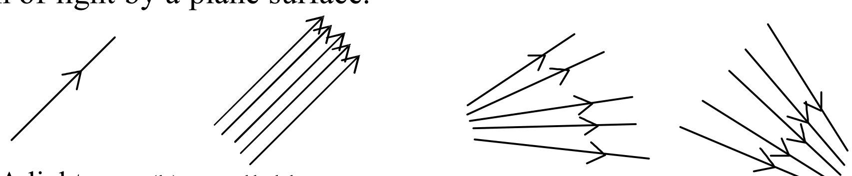
CONTENTS

1.0 INTRODUCTION

We see objects either by the light they produce or by the light they reflect from other objects. Objects that produce their own light are said to be luminous. Examples are the sun, candle light, electric light bulbs etc. Whereas, non-luminous objects do not produce their own light. They are seen only when light from other sources fall on them and is thrown back or "reflected" into our eyes. For example the moon shines in the night because it reflects light coming from the sun and not because it is luminous.

i) The narrowest of light is a ray which is usually diagrammatically represented by a thin line (as shown in Fig. 1.1a) with an arrow head on it. The arrow head represents the direction of propagation of the light.

ii) A group of rays gives rise to a beam of which can be parallel or convergent or divergent as shown in Fig. 1.1. Light rays can be reflected or refracted on plane or curved surfaces depending on the nature of the surfaces, including their material make up. In this unit we shall only look at reflection of light by a plane surface.



(a) A light ray (b) Parallel beams (c) Divergent beams (d) Convergent beams

Fig. 1.1: A ray and type of beams of light.

2.0 OBJECTIVES

After studying this unit, you will be able to:

1. recognize incident and reflected rays
2. recognize angle of incident and angle of reflection
3. state the laws of reflection
4. experimentally verify the laws of reflection.

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.1 The Law of Reflection

When a ray of light strikes a plane mirror, the light ray reflects off the mirror. Reflection involves a change in direction of the light ray. The convention used to express the direction of a light ray is to indicate the angle which the light ray makes with a normal line drawn to the surface of the mirror. The angle of incidence is the angle between this normal line and the incident ray; the angle of reflection is the angle between this normal line and the reflected ray. According to the law of reflection, the angle of incidence equals the angle of reflection.

3.1.1 The Role of Light to Sight

The bottom line is: without light, there would be no sight. The visual ability of humans and other animals is the result of the complex interaction of light, eyes and brain. We are able to see because light from an object can move through space and reach our eyes. Once light reaches our eyes, signals are sent to our brain, and our brain deciphers the information in order to detect the appearance, location and movement of the objects we are sighting at. The whole process, as complex as it is, would not be possible if it were not for the presence of light. Without light, there would be no sight.

If you were to turn off the room lights for a moment and then cover all the windows with black construction paper to prevent any entry of light into the room, then you would notice that nothing in the room would be visible. There would be objects present that were capable of being seen. There would be eyes present that would be capable of detecting light from those objects. There would be a brain present that would be capable of deciphering the information sent to it. But there would be no light! The room and everything in it would look black. The appearance of black is merely a sign of the absence of light. When a room full of objects (or a table, a shirt or a sky) looks black, then the objects are neither generating nor reflecting light to your eyes. And without light, there would be no sight.

3.1.2 Luminous versus Illuminated Objects

The objects that we see can be placed into one of two categories: luminous objects and illuminated objects. **Luminous objects** are objects that generate their own light. **Illuminated objects** are objects that are capable of reflecting light to our eyes. The sun is an example of a luminous object, while the moon is an illuminated object. During the day, the sun generates sufficient light to illuminate objects on Earth. The blue skies, the white clouds, the green grass, the colored leaves of fall, the neighbor's house, and the car approaching the intersection are all seen as a result of light from the sun (the luminous object) reflecting off the illuminated objects and traveling to our eyes. Without the light from the luminous objects, these illuminated objects would not be seen. During the evening when the Earth has rotated to a position where the light from the sun can no longer reach our part of the Earth (due to its inability to bend around the spherical shape of the Earth), objects on Earth appear black (or at least so dark that we could say they are nearly black). In the absence of a porch light or a street light, the neighbor's house can no longer be seen; the grass is no longer green, but rather black; the leaves on the trees are dark; and were it not for the headlights of the car, it would not be seen approaching the intersection. Without luminous objects generating light that propagates through space to illuminate non-luminous objects, those non-luminous objects cannot be seen. Without light, there would be no sight.

A common Physics demonstration involves the directing of a laser beam across the room. With the room lights off, the laser is turned on and its beam is directed towards a plane mirror. The presence of the light beam cannot be detected as it travels towards the mirror. Furthermore, the light beam cannot be detected after reflecting off the mirror and traveling through the air towards a wall in the room. The only locations where the presence of the light beam can be detected are at the location where the light beam strikes the mirror and at the location where the light beam strikes a wall. At these two locations, a portion of the light in the beam is reflecting off the objects (the mirror and the wall) and traveling towards the students' eyes. And since the detection of objects is dependent upon light traveling from that object to the eye, these are the only two locations where one can detect the light beam. But in between the laser and the mirror, the light beam cannot be detected. There is nothing present in the region between the laser and the mirror that is capable of reflecting the light of the beam to students' eyes.

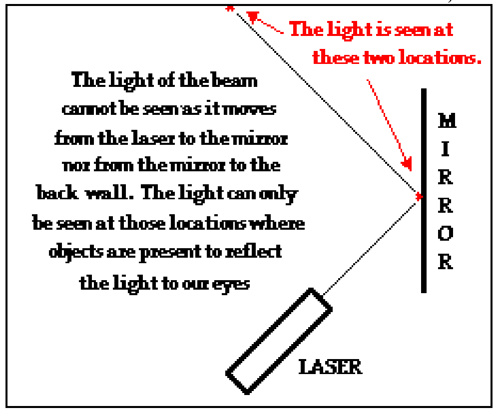


Figure 1.1 Light reflection

A mister is used to spray water into the air in the region where the light beam is moving. Small suspended droplets of water are capable of reflecting light from the beam to your eye. It is only due to the presence of the suspended water droplets that the light path from the laser to the mirror could be detected. When light from the laser (a luminous object) strikes the suspended water droplets (the illuminated object), the light is reflected to students' eyes. The path of the light beam can now be seen. With light, there can be sight. But without light, there would be no sight.

None of us generate light in the visible region of the electromagnetic spectrum. We are not brilliant objects (please take no offense) like the sun; rather, we are illuminated objects like the moon. We make our presence visibly known by reflecting light to the eyes of those who look our way. It is only by reflection that we, as well as most of the other objects in our physical world, can be seen. And if reflected light is so essential to sight, then the very nature of light reflection is a worthy topic of study among students of physics. And in this lesson and the several that follow, we will undertake a study of the way light reflects off objects and travels to our eyes in order to allow us to view them.

3.1.3 The Line of Sight

In the first section it was stated, "without light, there would be no sight." Everything that can be seen is seen only when light from that object travels to our eyes. Whether it be a luminous object (that generates light of its own) or an illuminated object (that reflects the light that is incident upon it), you can only view the object when light from that object travels to your eye. As you look at Mary in class, you are able to see Mary because she is illuminated with light and that light reflects off of her and travels to your eye. In the process of viewing Mary, you are directing your sight along a line in the direction of Mary. If you wish to view the top of Mary's head, then you direct your sight along a line towards the top of her head. If you wish to view Mary's feet, then you direct your sight along a line towards Mary's feet. And if you wish to view the image of Mary in a mirror, then you must direct your sight along a line towards the location of Mary's image. This directing of our sight in a specific direction is sometimes referred to as the **line of sight**.

In order to view an object, you must sight along a line at that object; and when you do light will come from that object to your eye along the line of sight.

A luminous object emits light in a variety of directions; and an illuminated object reflects light in a variety of directions. Although this light diverges from the object in a variety of directions, your eye only sees the very small diverging cone of rays that is coming towards it. If your eye were located at a different location, then you would see a different cone of rays. Regardless of the eye location, you will still need to sight along a line in a specific direction in order to view the object.

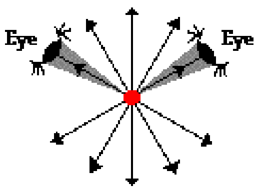


Figure 1.2 Line of sight

While simple, this concept of the line of sight is also profound. This very principle of the line of sight will assist us in understanding the formation of images in both this unit (reflection) and the next unit (refraction).

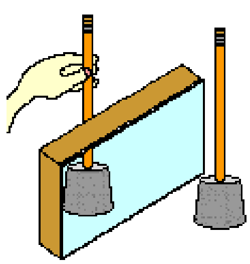


Figure 1.3 Viewing Image

A common Physics lab involves the determination of the image location of a pencil (or some object) as formed by a plane mirror. In the process of determining the image location, the manner in which light from the object travels to your eye is investigated. First, the method of parallax is used to locate the image of the object. Two pencils are inserted into rubber stoppers; one stoppered pencil serves as the object and the other serves to assist the student in locating the image. The *object pencil* is placed in front of a plane mirror. Then the student sights at the image of the *object pencil* in the mirror. As a student sights along a line (the line of sight) at the image of the pencil, the second pencil is placed behind the mirror along the same line of sight; this is called the *image pencil*. When placed along the line of sight, the portion of the image pencil that extends above the mirror will be aligned with the image that is seen in the mirror. Then the eye location is repositioned to the other side of the object pencil and the process is repeated. The precise image location of the object is the location where all lines of sight intersect regardless of where the eye is located. Two important ideas are gleaned from such a lab: one pertains to how light travels from the object to the eye and one pertain to the location of the image of an object.

As you sight at the image of an object in the mirror (whether it be a stoppered pencil or any object), light travels along your line of sight towards your eye. The object is being illuminated by light in the room; a countless number of rays of light are reflecting off the object in a variety of directions. When viewing the image of the object in a plane mirror, one of these rays of light originates at the object location and first travels along a line towards the mirror (as represented by the blue ray in the diagram below). This ray of light is known as the **incident ray -** the light ray approaching the mirror. The incident ray intersects the mirror at the same location where your line of sight intersects the mirror. The light ray then reflects off the mirror and travels to your eye (as represented by the red ray in the diagram below); this ray of light is known as the **reflected ray**.

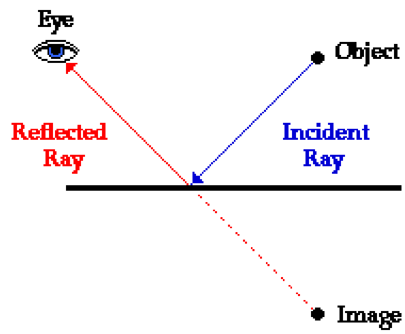


Figure 1.4 Looking into a mirror

So the manner in which light travels to your eye as you view the image of an object in a mirror can be summarized as follows.

To view the image of an object in a mirror, you must sight along a line at the image. One of the many rays of light from the object will approach the mirror and reflect along your line of sight to your eye.

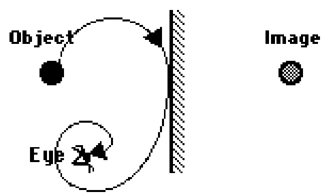
The second important idea that can be gleaned from this stoppered pencil lab pertains to the location of the image. Observe in the diagram above that the image is positioned directly across the mirror along a line that runs perpendicular to the mirror. The distance from the mirror to the object (known as the **object distance**) is equal to the distance from the mirror to the image (known as the **image distance**). For all plane mirrors, this equality holds true:

Object distance = Image distance

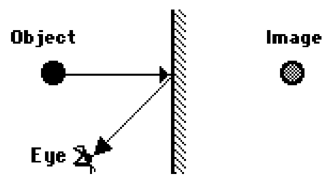
ACTIVITY 1

The following diagrams depict some ideas about how light might travel from an object location to an eye location when the image of the object is viewed in a mirror. Comment on the incorrectness of the following diagrams. Discuss what makes them incorrect.

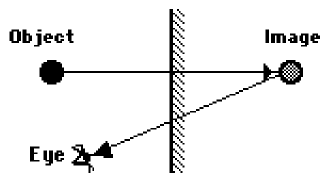
1.



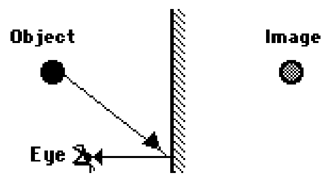
2.



1.



4.



Light is known to behave in a very predictable manner. If a ray of light could be observed approaching and reflecting off of a flat mirror, then the behavior of the light as it reflects would follow a predictable *law* known as the **law of reflection**. The diagram below illustrates the law of reflection.

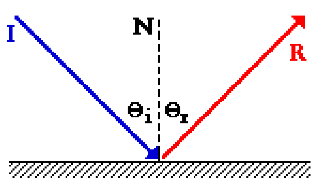


Figure 1.5 Laws of reflection

In Figure 1.5, the ray of light approaching the mirror is known as the **incident ray** (labeled **I** in the diagram). The ray of light that leaves the mirror is known as the **reflected ray** (labeled **R** in the diagram). At the point of incidence where the ray strikes the mirror, a line can be drawn perpendicular to the surface of the mirror. This line is known as a **normal line** (labeled **N** in the diagram). The normal line divides the angle between the incident ray and the reflected ray into two equal angles. The angle between the incident ray and the normal is known as the **angle of incidence**. The angle between the reflected ray and the normal is known as the **angle of reflection**. (These two angles are labeled with the Greek letter "theta" accompanied by a subscript; read as "theta-i" for angle of incidence and "theta-r" for angle of reflection.) The law of reflection states that when a ray of light reflects off a surface, the angle of incidence is equal to the angle of reflection.

1.1.5 Reflection and the Locating of Images

It is common to observe this law at work in a Physics lab. To view an image of a pencil in a mirror, you must sight along a line at the image location. As you sight at the image, light travels to your eye along the path shown in Figure 1.5. The diagram shows that the light reflects off the mirror in such a manner that the angle of incidence is equal to the angle of reflection.

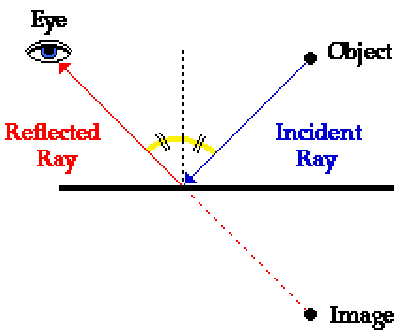


Figure 1.6 Reflection and image location

It just so happens that the light that travels along the line of sight to your eye follows the law of reflection. (The reason for this will be discussed later). If you were to sight along a line at a different location than the image location, it would be impossible for a ray of light to come from the object, reflect off the mirror according to the law of reflection, and subsequently travel to your eye. Only when you sight at the image, does light from the object reflect off the mirror in accordance with the law of reflection and travel to your eye. This truth is depicted in Figure 1.6.

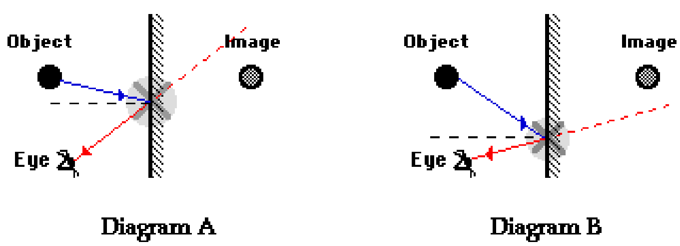
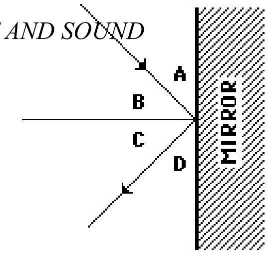


Figure 1.7 Reflection and images

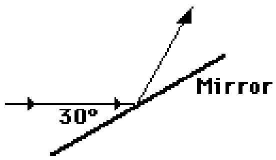
For example, in Diagram A above, the eye is sighting along a line at a position *above* the actual image location. For light from the object to reflect off the mirror and travel to the eye, the light would have to reflect in such a way that the angle of incidence is less than the angle of reflection. In Diagram B above, the eye is sighting along a line at a position *below* the actual image location. In this case, for light from the object to reflect off the mirror and travel to the eye, the light would have to reflect in such a way that the angle of incidence is more than the angle of reflection. Neither of these cases would follow the law of reflection. In fact, in each case, the image is not seen when sighting along the indicated line of sight. It is because of the law of reflection that an eye must sight at the image location in order to see the image of an object in a mirror.

ACTIVITY 2

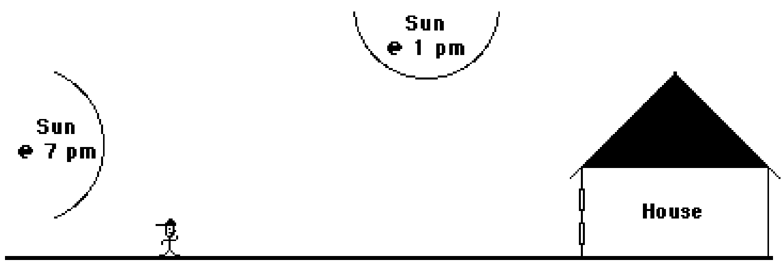
1. Consider the diagram at the right. Which one of the angles (A, B, C, or D) is the angle of incidence? ______ Which one of the angles is the angle of reflection? ______



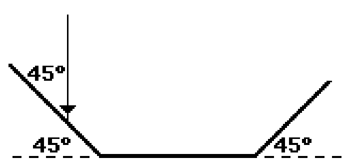
2. A ray of light is incident towards a plane mirror at an angle of 30-degrees with the mirror surface. What will be the angle of reflection?



3. Perhaps you have observed the image of the sun in the windows of distant buildings near the time that the sun is rising or setting. However, the image of the sun is not seen in the windows of distant building during midday. Use the diagram below to explain, drawing appropriate light rays on the diagram.



4. A ray of light is approaching a set of three mirrors as shown in the diagram. The light ray is approaching the first mirror at an angle of 45-degrees with the mirror surface. Trace the path of the light ray as it bounces off the mirror. Continue tracing the ray until it finally exits from the mirror system. How many times will the ray reflect before it finally exits?



1.1.6 Why is an Image Formed?

:In order to view an object, you must sight along a line at that object; and when you do light will come from that object to your eye along the line of sight.

This very principle can be extended to the task of viewing the image of an object in a plane (i.e., flat) mirror:

In order to see the image of an object in a mirror, you must sight at the image; when you sight at the image, light will come to your eye along that line of sight.

The image location is thus located at that position where observers are sighting when viewing the image of an object. It is the location behind the mirror where all the light appears to diverge from. In the diagram below, three individuals are sighting at the image of an object along three different lines of sight. Each person sees the image due to the reflection of light off the mirror in accordance with the law of reflection. When each line of sight is extended backwards, each line will intersect at the same point. This point is the image point of the object.

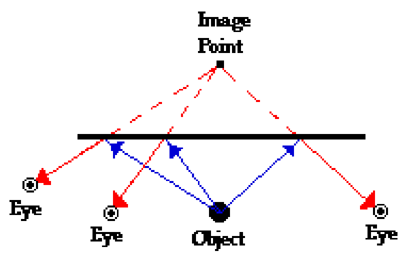


Figure 1.8 Image location

1.1.7 Locating an Image via Parallax

This principle can be illustrated in a Physics class using a 5-foot plane mirror and a pair of large cylinders. One cylinder is placed in front of the mirror and students from different locations in the room are asked to sight at its image. The second cylinder is then aligned along the line of sight and readjusted until it is in line with each person's line of sight. Regardless of who is viewing the image and from where they are viewing the image, each sight line must intersect in the same location. It is possible that the second cylinder is aligned with one student's line of sight but not with another student's. If this is so, then the cylinder is not placed at the exact location of the image. This is depicted in Figure 1.8

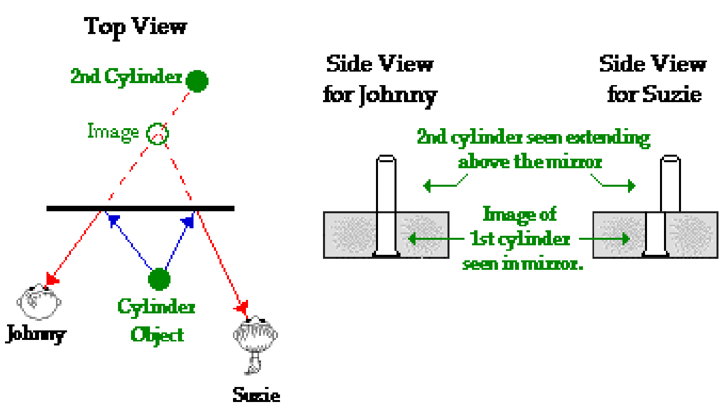


Figure 1.9 Image location

In a case such as this, the cylinder position is adjusted until it is located at the position where all students in the classroom can see it extending above the mirror and in line with the image that each student sees when looking in the mirror. Only, then can we conclude the cylinder is located at the image position.

Since there is only one image for an object placed in front of a plane mirror, it is reasonable that every sight line would intersect in a single location. This location of intersection is known as the image location. The **image location** is simply the one location in space where it seems to every observer that the light is diverging from. Regardless of where the observer is located, when the observer sights at the image location, the observer is sighting along a line towards the same location that all other observers are sighting. And as mentioned earlier, the perpendicular distance from this image location to the mirror is equal to the perpendicular distance from the object location to the mirror. In fact, the image location is directly across the mirror from the object location and an equal distance from the mirror.

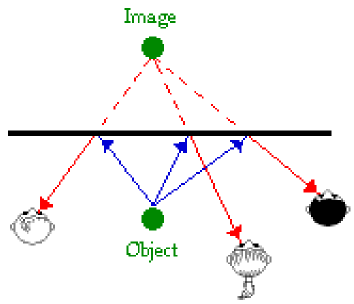


Figure 1. 10 Image location

Of course, it is possible that certain individuals in the room will be unable to view the image of an object in a plane mirror. Because of the person's position relative to the image position and to the extremities of the mirror, the person is unable to detect a ray of light reflecting to their eye as they sight at the image location. This does not mean that there is no image. Indeed, any object positioned in front of a plane mirror (or even to the side of the plane mirror) has an image regardless of whether there are people positioned in an appropriate location to view it.

4.0 CONCLUSION

Any polished surface is capable of becoming a reflector of light. Where a reflection occurs, the incident ray, the reflected ray and the normal all lie in the same plane. Also the angle of incident, i, is equal the angle of reflection r. These two laws constitute the laws of reflection.

5.0 SUMMARY

A ray is a fundamental component of light in a given direction and is represented by a thin line with an arrow, while a beam of light consists of several rays.

There are two laws of reflection:

1st Law: The incident ray, the reflected ray and the normal at the point of incidence all lie in the same plane.

2nd Law: The angle of incident is equal to the angle of reflection.

6.0 TUTOR-MARKED ASSIGNMENT

Activity

Place or fix a sheet of white A4 size paper with a thumbtack at each edge of the paper. With the help of two empty match boxes with vertical slot (holes) cut into them, support the mirror in a vertical position on the paper as shown in Fig. 1.5 below.

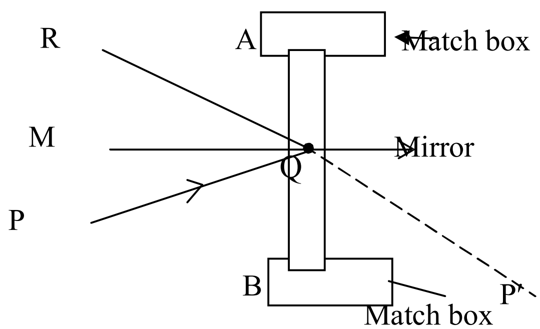


Fig. 1.5: A mirror in a vertical position on a paper

Then, trace with your pencil the surface of the mirror, line AB on the paper. Place a point P and place another one as point Q as shown in fig. 1.5. Move your head to the left of P, looking into the mirror as you move your head, until you see the image of P in line with P appearing to be along line P′Q. Use a third pin R to a line with P′ and Q.

That is, until when pins P′, Q and R appear to be on the same straight line. When this occurs, fix pin R on the paper and remove the mirror. Then, draw line M Q such that line M Q is 90^o to line AB. Measure the angle between MQ and PQ and then, the angle between MQ and QR.

Questions

1. What is line MQ called?
2. What is line PQ called?
3. What is line QR called?
4. What is angle PQM called and what is angle MQR called?
5. What can you deduce about the magnitude of angle PQR and MQR?
6. From 1-5, deduce the laws of reflection.

ANSWER TO ACTIVITY 1

1. This diagram depicts light moving in curved lines. The light by which we view objects on Earth moves in straight lines.

2. This diagram depicts the eye looking at a location that does not correspond to the image location. The eye must sight along a line at the image location.
3. This diagram depicts light passing through the mirror on the way to the mirror and on the way to the eye. Light always reflects off the mirror; and never passes through it.
4. This diagram depicts the eye looking at a location that does not correspond to the image location. The eye must sight along a line at the image location.

ANSWER TO ACTIVITY 2

1. Angle B is the angle of incidence (angle between the incident ray and the normal). Angle C is the angle of reflection (angle between the reflected ray and the normal).
2. The angle of reflection is 60 degrees. (Note that the angle of incidence is not 30 degrees; it is 60 degrees since the angle of incidence is measured between the incident ray and the normal.)
3. A ray of light drawn from the sun's position at 7 pm to the distant window reflects off the window and travel to the observer's eye. On the other hand, a ray of light drawn from the 1 pm sun position to the window will reflect and travel to the ground, never making it to the distant observer's eye.
4. The light reflects twice before it finally exits the system. Draw a normal at the point of incidence to the first mirror; measure the angle of incidence (45 degrees); then draw a reflected ray at 45 degrees from the normal. Repeat the process for the second mirror.

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 2 IMAGES FORMED FROM REFLECTION AT PLANE SURFACES

CONTENTS

1.0 INTRODUCTION

In the last Unit, you studied reflections at plane (flat) surfaces. In this unit you will study images formed due to reflections at plane surfaces.

When light is incident on a plane surface of mirror, depending on the type of object along its path, images are formed.

2.0 OBJECTIVES

After studying this unit, you will be able to:

1. Draw ray diagram of images from plane surfaces
2. Identify and explain with ray diagrams characteristics of images from reflections at plane surfaces

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.1 Reflection and Image Formation

So why is an image formed by a plane mirror? An image is formed because light emanates from an object in a variety of directions. Some of this light (which we represent by rays) reaches the mirror and reflects off the mirror according to the law of reflection. Each one of these rays of light can be extended backwards behind the mirror where they will all intersect at a point (the image point). Any person who is positioned along the line of one of these reflected rays can sight along the line and view the image - a representation of the object.

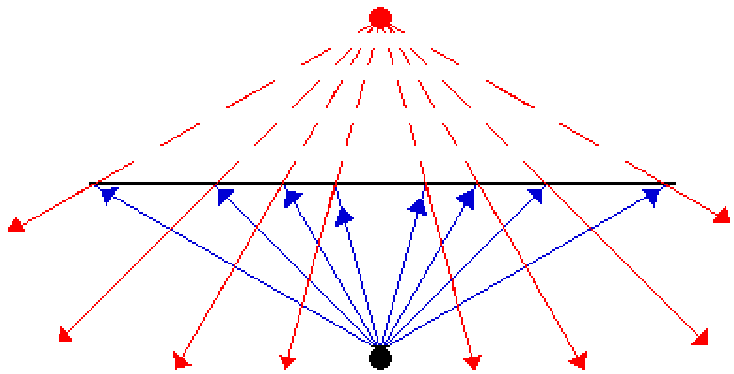


Figure 2.1 Light ray from a variety of directions

This principle of image formation is often applied in a Physics lab. Suppose that a mirror is placed on a sheet of paper that is placed on top of a piece of cardboard. A pin is positioned in an upright position (and held in place by the cardboard) at a location in front of the mirror. A student can sight along a line at the image of the pin from a variety of locations. With one eye closed, a straightedge is used to assist in drawing the lines of sight. These lines of sight are drawn from a variety of sighting locations. Each line of sight can be extended backwards beyond the mirror. If the sight lines are drawn correctly, then each line will intersect at the same location. The location of intersection of all sight lines is the image location. Validation of the accuracy of your sighting and ray tracing can be accomplished by measuring angles of incidence and angles of reflection on the diagram. These should be equal for each individual sight line. That is, angle A should equal angle B; angle C should equal angle D; and angle E should equal angle F. Finally, the object distance can be compared to the image distance; these should also be equal.

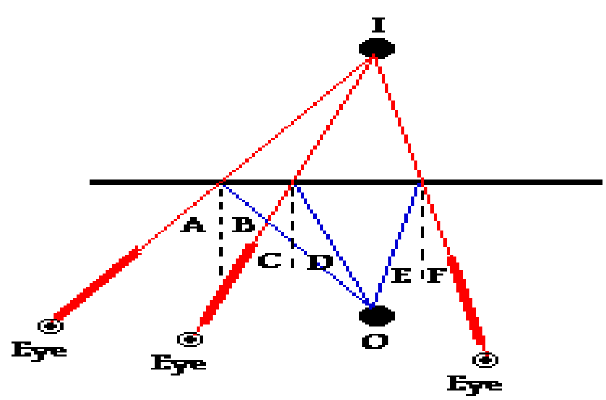


Figure 2.2 How the eye sees an image

3.2 Image Characteristics

3.2.1 Virtual vs. Real Images

In the case of plane mirrors, the image is said to be a **virtual image**. Virtual images are images that are formed in locations where light does not actually reach. Light does not actually pass through the location on the other side of the mirror; it only appears to an observer as though the light is coming from this location. Whenever a mirror (whether a plane mirror or otherwise) creates an image that is virtual, it will be located behind the mirror where light does not really come from. Later in this unit, we will study instances in which **real images** are formed by curved mirrors. Such images are formed on the same side of the mirror as the object and light passes through the actual image location.

3.2.2 Apparent Left-Right Image Reversal

Besides the fact that plane mirror images are virtual, there are several other characteristics that are worth noting. The second characteristic has to do with the orientation of the image. If you view an image of yourself in a plane mirror (perhaps a bathroom mirror), you will quickly notice that there is an apparent left-right reversal of the image. That is, if you raise your left hand, you will notice that the image raises what would seem to be it's right hand. If you raise your right hand, the image raises what would seem to be its left hand. This is often termed **left-right reversal**. This characteristic becomes even more obvious if you wear a shirt with lettering. For example, a shirt displaying the word "NIKE" will read "EKIN" when viewed in the mirror; a shirt displaying the word "ILLINOIS" will read "SIONILLI;" and a shirt displaying the word "BOB" will read "BOB." (NOTE: Not only will the order of letters appear reversed, the actual orientation of the letters themselves will appear reversed as well. Of course, this is a little difficult to do when typing from a keyboard.) While there is an apparent left-right reversal of the orientation of the image, there is no top-bottom vertical reversal. If you *stand on your feet* in front of a plane mirror, the image does not *stand on its head*. Similarly, the *ceiling* does not become the *floor*. The image is said to be **upright**, as opposed to **inverted**.

Students of Physics are usually quite intrigued by this apparent left-right reversal. Exactly what is happening to cause ILLINOIS to read as SIONILLI? And why is the reversal observed in the left to right direction and not in the head to toe direction? These questions urge us to ponder the situation more deeply. Let's suppose for a moment that we could print the name of your favorite school subject on your shirt and have you look in the mirror. We all know that when you look in the mirror, you will see the letters SCISYHP written on the shirt of your image - the reversed form of PHYSICS. But can we really say that the word appearing on your shirt is the word PHYSICS (with the letters un-reversed)? The answer is no! (But you don't have to believe it yet. Keep reading ... and pondering.)

To further explore the reason for this appearance of left-right reversal, let's suppose we write the word PHYSICS on a transparency and hold it in front of us in front of a plane mirror. If we look at the image of the transparency in the mirror, we would observe the expected - SCISYHP. The letters are written *reversed* when viewed in the mirror. But what if we look at the letters on the transparency? How are those letters oriented? When we face the mirror and look at the letters on the transparency, we observe the unexpected - SCISYHP. When viewed from the perspective of the person holding the transparency (and facing the mirror, the letters exhibit the same left-right reversal as the mirror image. The letters appear reversed on the image because they are actually reversed on the shirt. At least they are reversed when viewed from the perspective of a person who is facing the mirror. Imagine that! All this time you thought the mirror was reversing the letters on your shirt. But the fact is that the letters were already reversed on your shirt; at least they were reversed from the person who *stands behind the T-shirt*. The people who view your shirt from the front have a different reference frame and thus do not see the letters as being reversed. The apparent left-right reversal of an image is simply a frame of reference phenomenon. When viewing the image of your shirt in a plane mirror (or any part of the world), you are viewing your shirt from the front. This is a switch of reference frames.

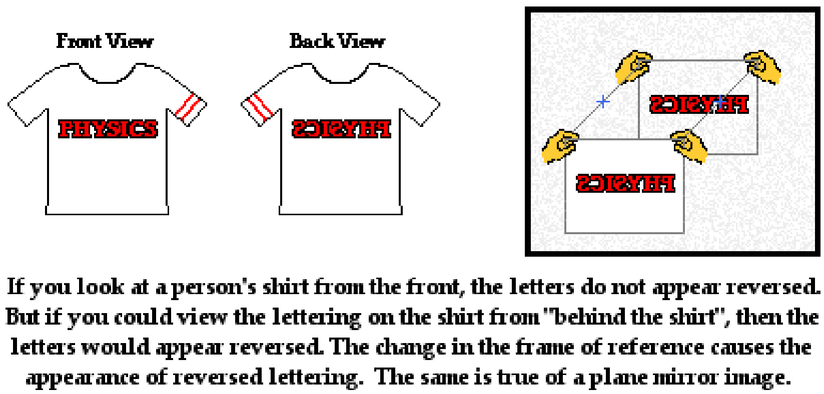


Figure 2. 3 Write up seen from plane mirror

3.2.3 Object Distance and Image Distance

A third characteristic of plane mirror images pertains to the relationship between the object's distance to the mirror and the image's distance to the mirror. For plane mirrors, the object distance (often represented by the symbol d_o) is equal to the image distance (often represented by the symbol d_i). That is the image is the same distance behind the mirror as the object is in front of the mirror. If you stand a distance of 2 meters from a plane mirror, you must focus at a location 2 meters behind the mirror in order to view your image.

3.2.4 Relative Size of Image and Object

A fourth and final characteristic of plane mirror images is that the dimensions of the image are the same as the dimensions of the object. If a 1.6-meter tall person stands in front of a mirror, he/she will see an image that is 1.6-meters tall. If a penny with a diameter of 18-mm is placed in front of a plane mirror, the image of the penny has a diameter of 18 mm. The ratio of the image dimensions to the object dimensions is termed the **magnification**. Plane mirrors produce images that have a magnification of 1.

ACTIVITY 1

1. You might have noticed that emergency vehicles such as ambulances are often labeled on the front hood with reversed lettering (e.g., ECNALUBMA). Explain why this is so.

2. If Suzie stands 3 feet in front of a plane mirror, how far from the person will her image be located?

3. If a toddler crawls towards a mirror at a rate of 0.25 m/s, then at what speed will the toddler and the toddler's image approach each other?

Ray Diagrams

The line of sight principle suggests that in order to view an image of an object in a mirror, a person must sight along a line at the image of the object. When sighting along such a line, light from the object reflects off the mirror according to the law of reflection and travels to the person's eye. This process was discussed and explained earlier in this lesson. One useful tool that is frequently used to depict this idea is known as a ray diagram. A **ray diagram** is a diagram that traces the path that light takes in order for a person to view a point on the image of an object. On the diagram, rays (lines with arrows) are drawn for the incident ray and the reflected ray. Complex objects such as people are often represented by stick figures or arrows. In such cases it is customary to draw rays for the extreme positions of such objects.

Drawing Ray Diagrams - a Step-by-Step Approach

This section of Lesson 2 details and illustrates the procedure for drawing ray diagrams. Let's begin with the task of drawing a ray diagram to show how Suzie will be able to see the image of the green *object arrow* in the diagram below. For simplicity sake, we will suppose that Suzie is viewing the image with her left eye closed. Thus, we will focus on how light travels from the two extremities of the object arrow (the left and right side) to the mirror and finally to Suzie's right eye as she sights at the image. The four steps of the process for drawing a ray diagram are listed, described and illustrated below.

1. Draw the image of the object.

Use the principle that the object distance is equal to the image distance to determine the exact location of the object. Pick one *extreme* on the object and carefully measure the distance from this *extreme point* to the mirror. Mark off the same distance on the opposite side of the mirror and mark the image of this *extreme point*. Repeat this process for all extremes on the object until you have determined the complete location and shape of the image. Note that all distance measurements should be made by measuring along a segment that is perpendicular to the mirror.

2. Pick one extreme on the image of the object and draw the reflected ray that will travel to the eye as it sights at this point.

Use the line of sight principle: the eye must sight along a line at the image of the object in order to see the image of the object. It is customary to draw a bold line for the reflected ray (from the mirror to the eye) and a dashed line as an extension of this reflected ray; the dashed line extends behind the mirror to the location of the image point. The reflected ray should have an arrowhead upon it to indicate the direction that the light is traveling. The arrowhead should be pointing towards the eye since the light is traveling from the mirror to the eye, thus enabling the eye to see the image.

3. Draw the incident ray for light traveling from the corresponding extreme on the object to the mirror.

The incident ray reflects at the mirror's surface according to the law of reflection. But rather than measuring angles, you can merely draw the incident ray from the *extreme* of the object to the point of incidence on the mirror's surface. Since you drew the reflected ray in step 2, the point of incidence has already been determined; the point of incidence is merely the point where the line of sight intersects the mirror's surface. Thus draw the incident ray from the *extreme point* to the point of incidence. Once more, be sure to draw an arrowhead upon the ray to indicate its direction of travel. The arrowhead should be pointing towards the mirror since light travels from the object to the mirror.

4. Repeat steps 2 and 3 for all other extremities on the object.

After completing steps 2 and 3, you have only shown how light travels from a single *extreme* on the object to the mirror and finally to the eye. You will also have to show how light travels from the other *extremes* on the object to the eye. This is merely a matter of repeating steps 2 and 3 for each individual extreme. Once repeated for each extreme, your ray diagram is complete.

Uses of Ray Diagrams

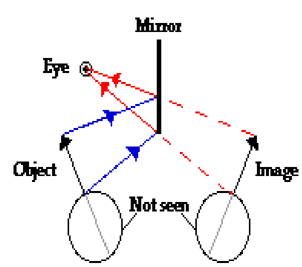
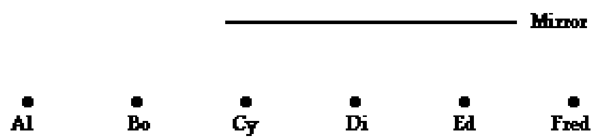


Figure 2.4 Non visible images

Ray diagrams can be particularly useful for determining and explaining why only a portion of the image of an object can be seen from a given location. The ray diagram at the right shows the lines of sight used by the eye in order to see a portion of the image in the mirror. Since the mirror is not long enough, the eye can only view the topmost portion of the image. The lowest point on the image that the eye can see is that point in line with the line of sight that intersects the very bottom of the mirror. As the eye tries to view even lower points on the image, there is not sufficient mirror present to reflect light from the lower points on the object to the eye. The portion of the object that cannot be seen in the mirror is shaded green in the diagram below.

Similarly, ray diagrams are useful tools for determining and explaining what objects might be viewed when sighting into a mirror from a given location. *For example, suppose that six students - Al, Bo, Cy, Di, Ed, and Fred sit* in front of *a plane mirror and attempt to see each other in the mirror. And suppose the exercise involves answering the following questions: Whom can Al see? Whom can Bo see? Whom can Cy see? Whom can Di see? Whom can Ed see? And whom can Fred see?*



The task begins by locating the images of the given students. Then, Al is isolated from the rest of the students and lines of sight are drawn to see who Al can see. The leftward-most student whom Al can see is the student whose image is to the right of the line of sight that intersects the left edge of the mirror. This would be Ed. The rightward-most student who Al can see is the student whose image is to the left of the line of sight that intersects the right edge of the mirror. This would be Fred. Al could see any student positioned between Ed and Fred by looking at any other positions along the mirror. However in this case, there are no other students between Ed and Fred; thus, Ed and Fred are the only students whom Al can see? The diagram below illustrates this using lines of sight for Al.

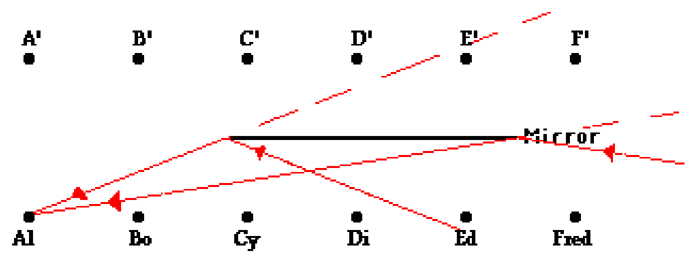


Figure 2.4 Puzzle to be sorted out

Of course the same process can be repeated for the other students by observing their lines of sight. Perhaps you will want to try to determine whom Bo, Cy, Di, Ed, and Fred can see?

Answer:

Bo can see Di, Ed, and Fred

Cy can see Cy, Di, Ed, and Fred

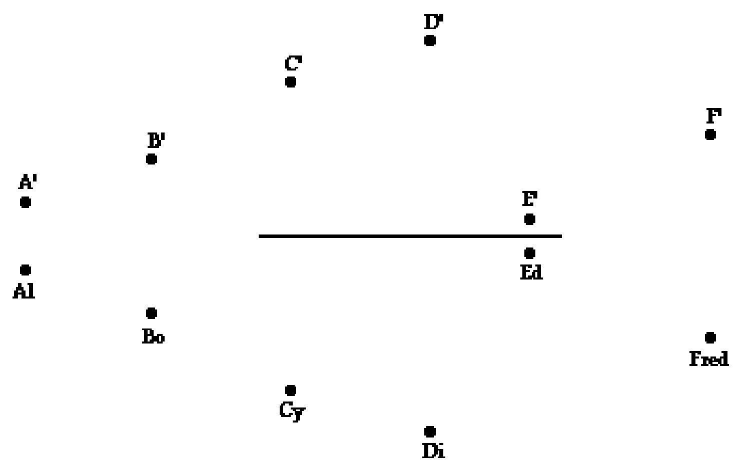
Di can see Cy, Di, Ed, and Fred

Ed can see Bo, Cy, Di, and Eve

Fred can see Al, Bo, Cy, and Di

ACTIVITY 2

1. Six students are arranged in front of a mirror. Their positions are shown below. The image of each student is also drawn on the diagram. Make the appropriate line of sight constructions to determine that students each individual student can see.



4.0 CONCLUSION

Plane mirrors produce images with a number of distinguishable characteristics. Images formed by plane mirrors are virtual, upright, left-right reversed, the same distance from the mirror as the object's distance, and the same size as the object.

.

5.0 SUMMARY

The image formed by a plane mirror due to reflection of light by the plane mirror is such that the distance of the mirror object from the surface of the mirror and the distance of the image from the surface of the mirror are equal.

Major Characteristics of images formed by plane mirror are as follows:

i) It is upright, that is, the image is oriented in the same direction as the object.
ii) It is virtual, that is, it cannot be received on the screen.
iii) It is of the same size as the object.
iv) It is laterally inverted

6.0 TUTOR-MARKED ASSIGNMENT

1. List and explain with aid of diagram any four characteristics of images formed by a plane mirror.
2. How is a virtual image different from a real image?

ANSWER TO ACTIVITY 1

1. Most drivers will view the ambulance in their rear-view mirrors. As such, they will be viewing an image of the lettering. Such images appear with left-right reversal and so will be viewed with the proper orientation - AMBULANCE.
2. Answer = **6 feet**

   Suzie (the object) is located 3 feet from the mirror. Suzie's image will be located 3 feet behind the mirror. Thus, the distance between Suzie and the image will be 6 feet.
3. Answer = **0.50 m/s**

   In one second, the toddler has moved towards the mirror by a distance of 0.25 meters. In the same second, the image will be 0.25 meters closer as well. Thus, the toddler and its image have become 0.50 meters closer in 1 second.

ANSWER TO ACTIVITY 2

Al can see Ed and Fred

Bo can see Ed, and Fred

Cy can see Cy, Di, Ed, and Fred

Di can see Cy, Di, and Ed,

Ed can see Al, Bo, Cy, Di, and Ed

Fred can see Bo, Cy, Di, and Ed

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 3 REFLECTION AT CURVED SURFACES

CONTENTS

1.0 INTRODUCTION

In the last Unit, you studied reflections at plane (flat) surfaces. In this unit you will study reflection at curved surfaces. Such surfaces include concave and convex mirrors.

When light is incident on a curved surface of mirror, the reflected rays either diverge or converge depending on the direction of curvature of the surface. We could produce a curved surface by cutting out a part of a hollow spherical shell. A concave mirror is a curved surface which is silvered inside while a convex mirror is a curved surface that is silvered side is outside, as shown in Fig. 3.1 (a) and 3.1 (b) respectively. Therefore, a concave or converging mirror reflect light from its inside while a convex or diverging mirror reflect light from its outside as shown in Fig. 3.2 (a) and Fig. 3.2 (b).

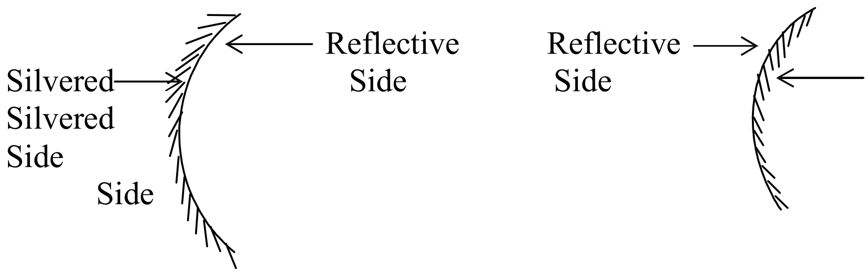


Fig. 3.1 (a) concave mirror (b) convex mirror

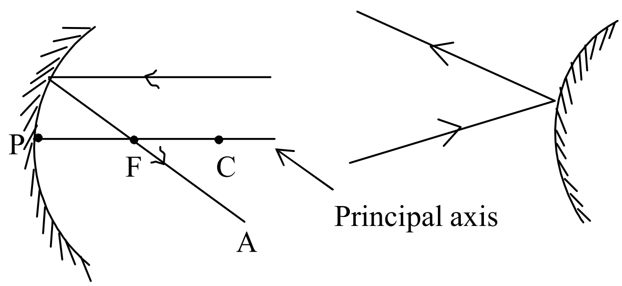


Fig. 3.2 (a) **Fig. 3.2 (b)**

Because the convex mirror or the concave mirror is part of a sphere, it has a center C called the center of curvature, and a radius (r) called radius of curvature. And it also has a Principal Focus F_1, whose distance from the pole P to the mirror is half the radius of curvature. These parameters are shown in Fig. 3.2 for the convex mirror respectively.

2.0 OBJECTIVES

After studying this unit, you will be able to:

- distinguish between reflection at curved surface and that at a plane surface
- identify the principal focus of a curved mirror
- obtain images formed by a curved mirror using ray diagrams state the mirror formula
- apply the mirror formula to obtain either image distance or object distance or the focal length and solve problems involving a curved mirror
- define magnification.

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

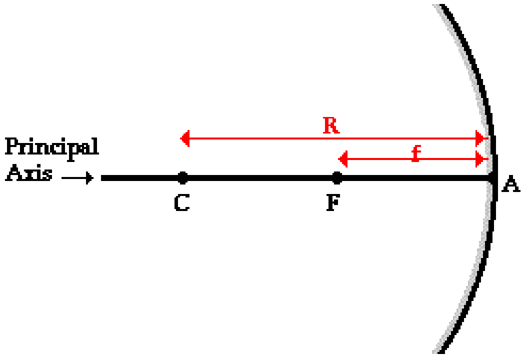
3.1 The Curved Mirror

Thus far in this unit, our focus has been the reflection of light off flat surfaces and the formation of images by plane mirrors. In Unit 3 we will turn our attention to the topic of curved mirrors, and specifically curved mirrors that have a *spherical* shape. Such mirrors are called **spherical mirrors**. The two types of spherical mirrors are shown in the diagram on the right. Spherical mirrors can be thought of as a portion of a sphere that was sliced away and then silvered on one of the sides to form a reflecting surface. **Concave mirrors** were silvered on the inside of the sphere and **convex mirrors** were silvered on the outside of the sphere.

To start a study of spherical mirrors demands that you first become acquainted with some terminology that will be periodically used. The internalized understanding of the following terms will be essential in this unit.

Principal axis	Center of Curvature	Vertex
Focal Point	Radius of Curvature	Focal Length

If a concave mirror were thought of as being a slice of a sphere, then there would be a line passing through the center of the sphere and attaching to the mirror in the exact center of the mirror. This line is known as the **principal axis**. The point in the center of the sphere from which the mirror was sliced is known as the **center of curvature** and is denoted by the letter **C** in the diagram below. The point on the mirror's surface where the principal axis meets the mirror is known as the **vertex** and is denoted by the letter **A** in the diagram below. The vertex is the geometric center of the mirror. Midway between the vertex and the center of curvature is a point known as the **focal point**; the focal point is denoted by the letter **F** in the diagram below. The distance from the vertex to the center of curvature is known as the **radius of curvature** (represented by **R**). The radius of curvature is the radius of the sphere from which the mirror was cut. Finally, the distance from the mirror to the focal point is known as the **focal length** (represented by **f**). Since the focal point is the midpoint of the line segment adjoining the vertex and the center of curvature, the focal length would be one-half the radius of curvature.



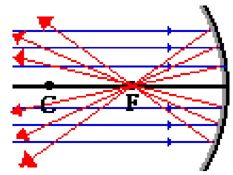


Figure 3.3 Concave mirror

Figure 3.4 The principal focus

The focal point is the point in space at which light incident towards the mirror and traveling parallel to the principal axis will meet after reflection. The diagram at the right depicts this principle. In fact, if some light from the sun were collected by a concave mirror, then it would converge at the focal point. Because the sun is such a large distance from the Earth, any light rays from the sun that strike the mirror will essentially be traveling parallel to the principal axis. As such, this light should reflect and pass through the focal point. A common Physics demonstration involves using a large demonstration mirror to set a pencil aflame in a matter of seconds. In the demonstration, the pencil is placed at the focal point and the concave mirror is *pointed* upwards towards the sun. Whatever rays of light from the sun that hit the mirror are focused at the point where the pencil is located. To the surprise of many, the heat is sufficient to ignite the pencil.

A large concave mirror is used to focus light from the sun upon a pencil. Depending on the object location, the image could be enlarged or reduced in size or even the same size as the object; the image could be inverted or upright; and the image will be located in a specific region along the principal axis. To understand these relationships between object and image, you may need to review the vocabulary terms described on this page.

ACTIVITY 1

1. The surface of a concave mirror is *pointed* towards the sun. Light from the sun hits the mirror and converges to a point. How far is this *converging point* from the mirror's surface if the radius of curvature (R) of the mirror is 150 cm?

2. It's the early stages of the Concave Mirror Lab. Your teacher hands your lab group a concave mirror and asks you to find the focal point. What procedure would you use to do this?

3.2 Reflection of Light and Image Formation

Light always follows the law of reflection, whether the reflection occurs off a curved surface or off a flat surface. The task of determining the direction in which an incident light ray would reflect involves determining the normal to the surface at the point of incidence. For a concave mirror, the normal at the point of incidence on the mirror surface is a line that extends through the center of curvature. Once the normal is drawn the angle of incidence can be measured and the reflected ray can be drawn with the same angle. This process is illustrated with two separate incident rays in the diagram at the right.

Lesson 2 discussed the formation of images by plane mirrors. In Lesson 2, it was emphasized the image location is the location where reflected light appears to diverge from. For plane mirrors, virtual images are formed. Light does not actually pass through the virtual image location; it only appears to an observer as though the light is emanating from the virtual image location. In this lesson we will begin to see that concave mirrors are capable of producing **real images** (as well as virtual images). When a real image is formed, it still appears to an observer as though light is diverging from the real image location. Only in the case of a real image, light is actually passing through the image location.

3.2.1 How is image Formed?

Suppose that a light bulb is placed in front of a concave mirror at a location somewhere *behind* the center of curvature (C). The light bulb will emit light in a variety of directions, some of which will strike the mirror. Each individual ray of light that strikes the mirror will reflect according to the law of reflection. Upon reflecting, the light will converge at a point. At the point where the light from the object converges, a replica, likeness or reproduction of the actual object is created. This replica is known as the **image**. Once the reflected light rays reach the image location, they begin to diverge. The point where all the reflected light rays converge is known as the image point. Not only is it the point where light rays converge, it is also the point where reflected light rays appear to an observer to be diverging from. Regardless of the observer's location, the observer will see a ray of light passing through the real image location. To view the image, the observer must line her sight up with the image location in order to see the image via the reflected light ray. The diagram below depicts several rays from the object reflecting from the mirror and converging at the image location. The reflected light rays then begin to diverge, with each one being capable of assisting an individual in viewing the image of the object.

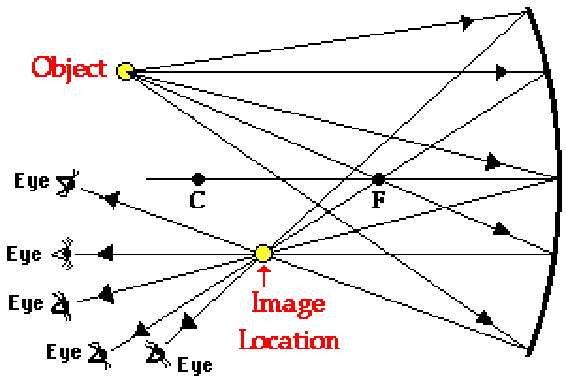


Figure 3.5 Image location in concave mirror

If the light bulb is located at a different location, the same principles apply. The image location is the location where reflected light appears to diverge from. By determining the path that light from the bulb takes after reflecting from the mirror, the image location can be identified. The diagram below depicts this concept.

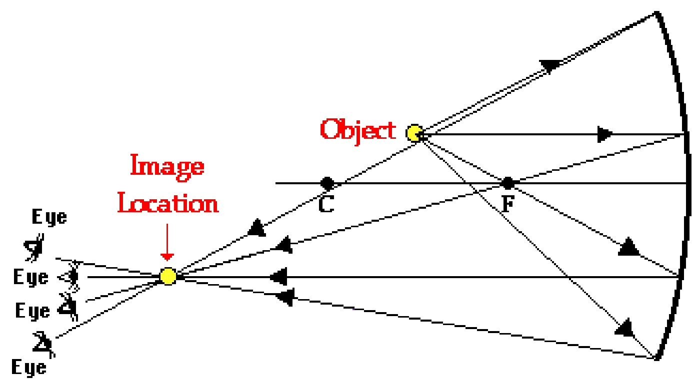


Figure 3.6 Diagram for object between f & C

You might notice that while the same principle applies for determining the image location, a different result is obtained. When the object is located *beyond* the center of curvature (C), the image is located between the center of curvature (C) and the focal point (F). On the other hand, when the object is located between the center of curvature (C) and the focal point (F), the image is located *beyond* the center of curvature (C). Unlike plane mirrors, the object distance is not necessarily equal to the image distance. The actual relationship between object distance and image distance is dependent upon the location of the object. These ideas will be discussed in more detail later in this lesson.

Rules of Reflection for Concave Mirrors

Light always reflects according to the law of reflection, regardless of whether the reflection occurs off a flat surface or a curved surface. Using reflection laws allows one to determine the image location for an object. The image location is the location where all reflected light appears to diverge from. Thus to determine this location demands that one merely needs to know how light reflects off a mirror. In the previous section, the image of an object for a concave mirror was determined by tracing the path of light as it emanated from an object and reflected off a concave mirror. The image was merely that location where all reflected rays intersected. The use of the law of reflection to determine a reflected ray is not an easy task. For each incident ray, a normal line at the point of incidence on a curved surface must be drawn and then the law of reflection must be applied. A simpler method of determining a reflected ray is needed.

The simpler method relies on two rules of reflection for concave mirrors. They are:

- Any incident ray traveling parallel to the principal axis on the way to the mirror will pass through the focal point upon reflection.
- Any incident ray passing through the focal point on the way to the mirror will travel parallel to the principal axis upon reflection.

These two rules of reflection are illustrated in Figure 3.7a&b.

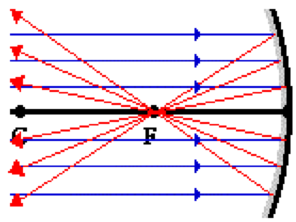


Figure 3.7a

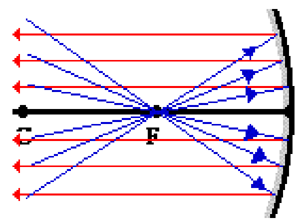


Figure 3.7b

These two rules will greatly simplify the task of determining the image locations for objects placed in front of concave mirrors. In the next section of Lesson 3, these two rules will be applied to determine the location, orientation, size and type of image produced by a concave mirror. As the rules are applied in the construction of ray diagrams, do not forget the fact that the law of reflection holds for each of these rays. It just so happens that when the law of reflection is applied for a ray (either traveling parallel to the principal axis or passing through F) that strikes the mirror at a location near the principal axis, the ray will reflect in close approximation with the above two rules.

3.2.2 Ray Diagrams - Concave Mirrors

The theme of this unit has been that we see an object because light from the object travels to our eyes as we sight along a line at the object. Similarly, we see an image of an object because light from the object reflects off a mirror and travel to our eyes as we sight at the image location of the object. From these two basic premises, we have defined the image location as the location in space where light appears to diverge from. Ray diagrams have been a valuable tool for determining the path taken by light from the object to the mirror to our eyes. In this section of Lesson 3, we will investigate the method for drawing ray diagrams for objects placed at various locations in front of a concave mirror.

To draw these diagrams, we will have to recall the two rules of reflection for concave mirrors:

- Any incident ray traveling parallel to the principal axis on the way to the mirror will pass through the focal point upon reflection.

- Any incident ray passing through the focal point on the way to the mirror will travel parallel to the principal axis upon reflection.

Earlier in this lesson, the following diagram was shown to illustrate the path of light from an object to mirror to an eye.

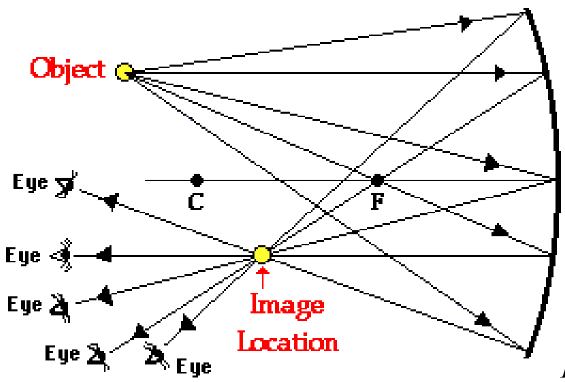


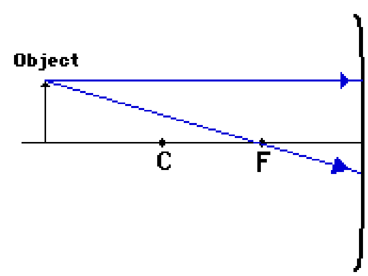
Figure 3.8 Object beyond C

In this diagram five incident rays are drawn along with their corresponding reflected rays. Each ray intersects at the image location and then diverges to the eye of an observer. Every observer would observe the same image location and every light ray would follow the law of reflection. Yet only two of these rays would be needed to determine the image location since it only requires two rays to find the intersection point. Of the five incident rays drawn, two of them correspond to the incident rays described by our two rules of reflection for concave mirrors. Because they are the easiest and most predictable pair of rays to draw, these will be the two rays used through the remainder of this lesson

3.2.3 Step-by-Step Method for Drawing Ray Diagrams

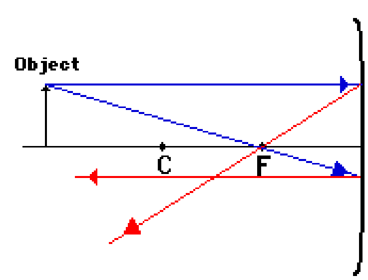
The method for drawing ray diagrams for concave mirror is described below. The method is applied to the task of drawing a ray diagram for an object located *beyond* the center of curvature (C) of a concave mirror. Yet the same method works for drawing a ray diagram for any object location.

1. Pick a point on the top of the object and draw two incident rays traveling towards the mirror.



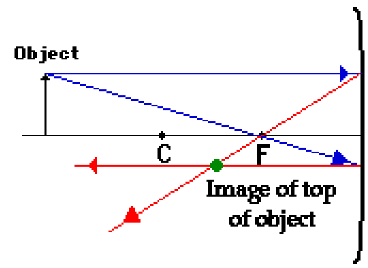
Using a straight edge, accurately draw one ray so that it passes exactly through the focal point on the way to the mirror. Draw the second ray such that it travels exactly parallel to the principal axis. Place arrowheads upon the rays to indicate their direction of travel.

2. Once these incident rays strike the mirror, reflect them according to the two rules of reflection for concave mirrors.



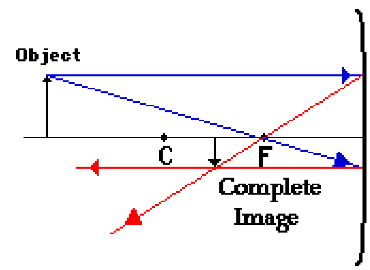
The ray that passes through the focal point on the way to the mirror will reflect and travel parallel to the principal axis. Use a straight edge to accurately draw its path. The ray that traveled parallel to the principal axis on the way to the mirror will reflect and travel through the focal point. Place arrowheads upon the rays to indicate their direction of travel. Extend the rays past their point of intersection.

3. Mark the image of the top of the object.



The image point of the top of the object is the point where the two reflected rays intersect. If your were to draw a third pair of incident and reflected rays, then the third reflected ray would also pass through this point. This is merely the point where all light from the top of the object would intersect upon reflecting off the mirror. Of course, the rest of the object has an image as well and it can be found by applying the same three steps to another chosen point.

4. Repeat the process for the bottom of the object.



The goal of a ray diagram is to determine the location, size, orientation, and type of image that is formed by the concave mirror. Typically, this requires determining where the image of the upper and lower extreme of the object is located and then tracing the entire image. After completing the first three steps, only the image location of the top extreme of the object has been found. Thus, the process must be repeated for the point on the bottom of the object. If the bottom of the object lies upon the principal axis (as it does in this example), then the image of this point will also lie upon the principal axis and be the same distance from the mirror as the image of the top of the object. At this point the entire image can be filled in.

Some students have difficulty understanding how the entire image of an object can be deduced once a single point on the image has been determined. If the object is a vertically aligned object (such as the arrow object used in the example below), then the process is easy. The image is merely a vertical line. In theory, it would be necessary to pick each point on the object and draw a separate ray diagram to determine the location of the image of that point. That would require a lot of ray diagrams as illustrated below.

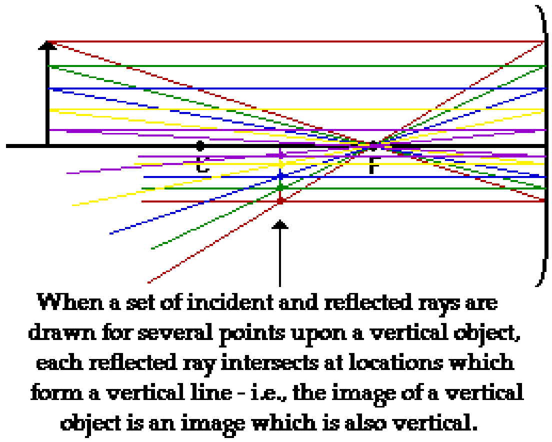


Figure 3.9 Object beyond C

Fortunately, a shortcut exists. If the object is a vertical line, then the image is also a vertical line. For our purposes, we will only deal with the simpler situations in which the object is a vertical line that has its bottom located upon the principal axis. For such simplified situations, the image is a vertical line with the lower extremity located upon the principal axis.

The ray diagram above illustrates that when the object is located at a position *beyond* the center of curvature, the image is located at a position between the center of curvature and the focal point. Furthermore, the image is inverted, reduced in size (smaller than the object), and real. This is the type of information that we wish to obtain from a ray diagram. These characteristics of the image will be discussed further.

Once the method of drawing ray diagrams is practiced a couple of times, it becomes as natural as breathing. Each diagram yields specific information about the image. The two diagrams Figure 3.10a & b show how to determine image location, size, orientation and type for situations in which the object is located at the center of curvature and when the object is located between the center of curvature and the focal point.

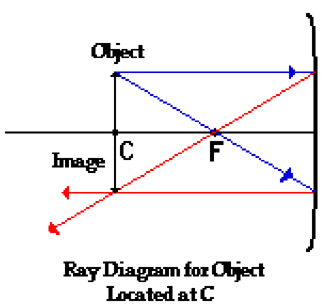


Figure 3.10a

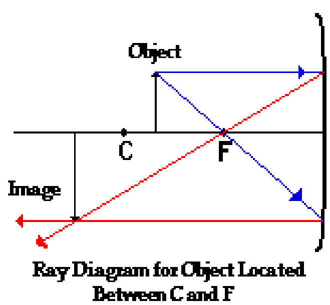


Figure 3.10b

It should be noted that the process of constructing a ray diagram is the same regardless of where the object is located. While the result of the ray diagram (image location, size, orientation, and type) is different, the same two rays are always drawn. The two rules of reflection are applied in order to determine the location where all reflected rays appear to diverge from (which for real images, is also the location where the reflected rays intersect).

In the three cases described above - the case of the object being located beyond C, the case of the object being located at C, and the case of the object being located between C and F -

light rays are converging to a point after reflecting off the mirror. In such cases, a **real image** is formed. As discussed previously, a real image is formed whenever reflected light passes through the image location. While plane mirrors always produce virtual images, concave mirrors are capable of producing both real and virtual images. As shown above, real images are produced when the object is located a distance greater than one focal length from the mirror. A **virtual image** is formed if the object is located less than one focal length from the concave mirror. To see why this is so, a ray diagram can be used.

Ray Diagram for the Formation of a Virtual Image

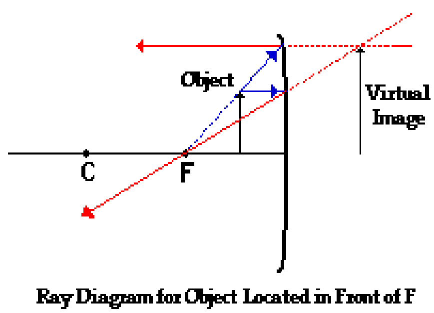
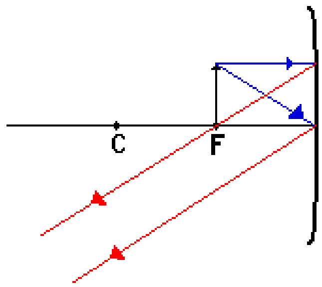


Figure 3.11: Object in front of F

A ray diagram for the case in which the object is located *in front of* the focal point is shown in the diagram at the right. Observe that in this case the light rays diverge after reflecting off the mirror. When light rays diverge after reflection, a virtual image is formed. As was done with plane mirrors, the image location can be found by tracing all reflected rays backwards until they intersect. For every observer, the reflected rays would seem to be diverging from this point. Thus, the point of intersection of the extended reflected rays is the image point. Since light does not actually pass through this point (light never travels behind the mirror), the image is referred to as a virtual image. Observe that when the object in located *in front of* the focal point, its image is an upright and enlarged image that is located on the other side of the mirror. In fact, one generalization that can be made about all virtual images produced by mirrors (both plane and curved) is that they are always upright and always located on the other side of the mirror.

Ray Diagram for an Object Located at the Focal Point

Thus far we have seen via ray diagrams that a real image is produced when an object is located more than one focal length from a concave mirror; and a virtual image is formed when an object is located less than one focal length from a concave mirror (i.e., *in front of F*). But what happens when the object is located at F? That is, what type of image is formed when the object is located exactly one focal length from a concave mirror? Of course a ray diagram is always one tool to help find the answer to such a question. However, when a ray diagram is used for this case, an immediate difficulty is encountered. The incident ray that begins from the top extremity of the object and passes through the focal point does not meet the mirror. Thus, a different incident ray must be used in order to determine the intersection point of all reflected rays. Any incident light ray would work as long as it meets up with the mirror. Recall that the only reason that we have used the two we have is that they can be conveniently and easily drawn. The diagram below shows two incident rays and their corresponding reflected rays.



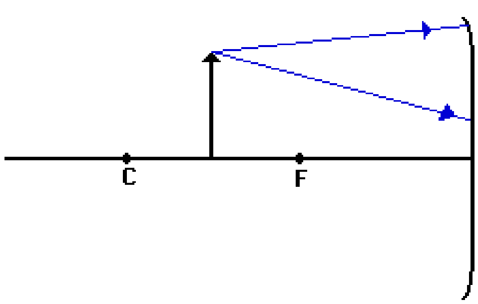
Ray Diagram for Object Located at F
(an image is not formed)

Figure 3.12 Object at F

For the case of the object located at the focal point (F), the light rays neither converge nor diverge after reflecting off the mirror. As shown in the diagram above, the reflected rays are traveling parallel to each other. Subsequently, the light rays will not converge on the object's side of the mirror to form a real image; nor can they be extended backwards on the opposite side of the mirror to intersect to form a virtual image. So how should the results of the ray diagram be interpreted? Simply there is no image. Surprisingly, when the object is located at the focal point, there is no location in space at which an observer can sight from which all the reflected rays appear to be diverging. An image is not formed when the object is located at the focal point of a concave mirror.

ACTIVITY 2

The diagram below shows two light rays emanating from the top of the object and incident towards the mirror. Describe how the reflected rays for these light rays can be drawn without actually using a protractor and the law of reflection.



3.3 Image Characteristics for Concave Mirrors

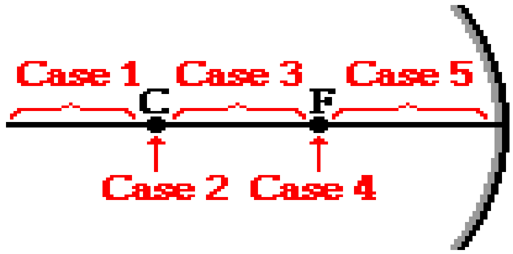
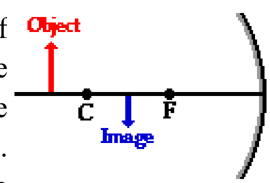


Figure 3.13: Five positions of object in front of concave mirror

The best means of summarizing this relationship between object location and image characteristics is to divide the possible object locations into five general areas or points:

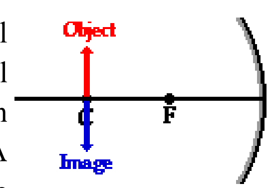
- Case 1: the object is located *beyond* the center of curvature (C)
- Case 2: the object is located at the center of curvature (C)
- Case 3: the object is located between the center of curvature (C) and the focal point (F)
- Case 4: the object is located at the focal point (F)
- Case 5: the object is located *in front of* the focal point (F)

Case 1: The object is located *beyond* C



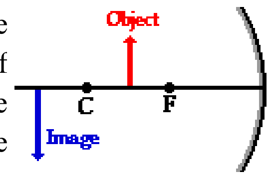
When the object is located at a location beyond the center of curvature, the image will always be located somewhere in between the center of curvature and the focal point. Regardless of exactly where the object is located, the image will be located in the specified region. In this case, the image will be an **inverted image**. That is to say, if the object is right side up, then the image is upside down. In this case, the image is **reduced in size**; in other words, the image dimensions are smaller than the object dimensions. If the object is a six-foot tall person, then the image is less than six feet tall. Earlier in Lesson 2, the term magnification was introduced; the **magnification** is the ratio of the height of the image to the height of the object. In this case, the absolute value of the magnification is less than 1. Finally, the image is a real image. Light rays actually converge at the image location. If a sheet of paper were placed at the image location, the actual replica of the object would appear projected upon the sheet of paper.

Case 2: The object is located at C



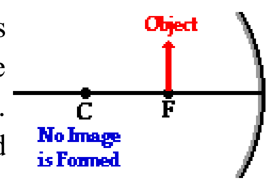
When the object is located at the center of curvature, the image will also be located at the center of curvature. In this case, the image will be inverted (i.e., a right side up object results in an upside-down image). The image dimensions are equal to the object dimensions. A six-foot tall person would have an image that is six feet tall; the absolute value of the magnification is equal to 1. Finally, the image is a real image. Light rays actually converge at the image location. As such, the image of the object could be projected upon a sheet of paper.

Case 3: The object is located between C and F



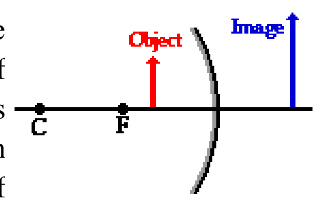
When the object is located *in front of* the center of curvature, the image will be located *beyond* the center of curvature. Regardless of exactly where the object is located between C and F, the image will be located somewhere *beyond* the center of curvature. In this case, the image will be inverted (i.e., a right side up object results in an upside-down image). The image dimensions are larger than the object dimensions. A six-foot tall person would have an image that is larger than six feet tall; the absolute value of the magnification is greater than 1. Finally, the image is a real image. Light rays actually converge at the image location. As such, the image of the object could be projected upon a sheet of paper.

Case 4: The object is located at F



When the object is located at the focal point, no image is formed. As discussed earlier in Lesson 3, light rays from the same point on the object will reflect off the mirror and neither converge nor diverge. After reflecting, the light rays are traveling parallel to each other and do not result in the formation of an image.

Case 5: The object is located *in front of* F



When the object is located at a location beyond the focal point, the image will always be located somewhere on the opposite side of the mirror. Regardless of exactly where in front of F the object is located, the image will always be located behind the mirror. In this case, the image will be an **upright image**. That is to say, if the object is right side up, then the image will also be right side up. In this case, the image is **magnified**; in other words, the image dimensions are greater than the object dimensions. A six-foot tall person would have an image that is larger than six feet tall; the magnification is greater than 1. Finally, the image is a virtual image. Light rays from the same point on the object reflect off the mirror and diverge upon reflection. For this reason, the image location can only be found by extending the reflected rays backwards beyond the mirror. The point of their intersection is the virtual image location. It would appear to any observer as though light from the object were diverging from this location. Any attempt to project such an image upon a sheet of paper would fail since light does not actually pass through the image location.

It might be noted from the above descriptions that there is a relationship between the object distance and object size and the image distance and image size. Starting from a large value, as the object distance decreases (i.e., the object is moved closer to the mirror), the image distance increases; meanwhile, the image height increases. At the center of curvature, the object distance equals the image distance and the object height equals the image height. As the object distance approaches one focal length, the image distance and image height approaches infinity. Finally, when the object distance is equal to exactly one focal length, there is no image. Then altering the object distance to values less than one focal length produces images that are upright, virtual and located on the opposite side of the mirror. Finally, if the object distance approaches 0, the image distance approaches 0 and the image height ultimately becomes equal to the object height. These patterns are depicted in the diagram below. Nine different object locations are drawn and labeled with a number; the corresponding image locations are drawn in blue and labeled with the identical number.

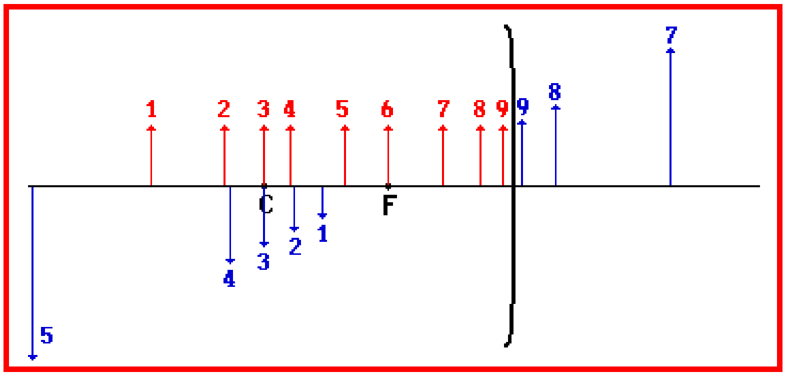


Figure 3.13 positions of 9 objects in concave mirror

Note that an object placed at the center of curvature results in the formation of a real, inverted image located at the same location and of the same size as the object.

ACTIVITY 3

1. Compare and contrast the images formed by concave and plane mirrors.

2. Identify the means by which you can use a concave and/or a plane mirror to form a real image.

3. Identify the means by which you can use a concave and/or a plane mirror to form a virtual image.

4. Identify the means by which you can use a concave and/or a plane mirror to produce an upright image.

5. Identify the means by which you can use a concave and/or a plane mirror to produce an inverted image.

6. Are all real images larger than the object?

7. The famous Chinese magician, Foo Ling Yu, conducts a classic magic trick utilizing a concave mirror with a focal length of 1.6 m. Foo Ling Yu is able to use the mirror in such a manner as to produce an image of a light bulb at the same location and of the same size as the actual light bulb itself. Use complete sentences to explain how Foo is able to accomplish this magic trick. Be specific about the light bulb location.

4.0 CONCLUSION

The curved mirror either concave or convex is part of a hollow sphere. When the sphere is silvered inside it is a concave mirror while it is convex if it is silvered outside. That is, a convex mirror reflect light from its outside whereas a concave mirror reflects light from its inside. Both or either the concave or convex mirror has center of curvature C, Principal Focus F, the principal axis and a pole.

Because the convex mirror diverges parallel rays of light, it is called a divergent mirror, whereas the concave mirror is called a convergent mirror, because it converges parallel rays of light. The image formed by either a convex mirror or a concave mirror can be determined using either the ray diagram or the mirror formula. For the same reason, the basic facts used are as follows:

i) a ray of light parallel to the axis of the mirror is reflected by the mirror through the principal focus;
ii) a ray of light directed to the center of curvature of the mirror is reflected back along the same path;
iii) a ray of light incident on the mirror through a Principal Focus is reflected parallel to the axis of the mirror.

In using the mirror equation the following sign convections are used:

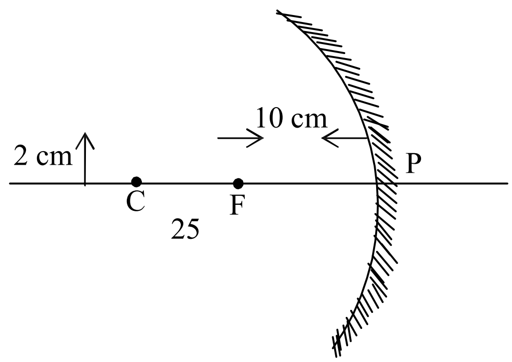
i) real objects and images have positive distances;
ii) virtual objects and images have negative distances;
iii) Concave mirrors have positive focal length and radii of curvature while convex mirrors have negative focal length and radii of curvature.

5.0 SUMMARY

- Curved mirrors, concave or convex, are part of hollow spheres.
- The reflecting surface of a concave mirror is inside while that of a convex mirror is outside.
- A concave mirror or convex mirror has a pole, center of curvature and the principal focus.
- The focal length of concave mirror is considered positive while that of the convex mirror is taken as negative.
- A concave mirror can form a real or a virtual image, depending on the location of the object. On the other hand, the convex mirror forms an erect and virtual image irrespective of where the object is located.
- A concave mirror can form either an enlarged or a diminished image depending on the position of the object.
- As a result, the convex mirrors have a wide field of view and always form an erect image. It is used as rear view mirrors in automobiles.

6.0 TUTOR-MARKED ASSIGNMENT

A pin 2 cm long is placed 25 cm away from the pole of a concave mirror of focal length 10 cm. Determine its magnification.



ANSWER TO ACTIVITY 1

1. **75 cm.** If the radius of curvature is 150 cm. then the focal length is 75 cm. The light will converge at the focal point, which is a distance of 75 cm from the mirror surface.
2. You will need to measure the distance from the vertex to the focal point. But first you must find the focal point. The trick involves focusing light from a distant source (the sun is ideal) upon a sheet of paper. Once you find the focal point, make your focal length measurement.

ANSWER TO ACTIVITY 2

These two incident rays will pass through the image point for the top of the object. In fact, any light rays emanating from the top of the object will pass through the image point. Thus, merely construct a ray diagram to determine the image location; use the two rules of reflection. Then draw the reflected rays for the two given incident rays through the same image point.

ANSWER TO ACTIVITY 3

1. Plane mirrors always produce virtual images which are upright and located behind the mirror; they are always the same size as the object. Concave mirrors can produce both real and virtual images; they can be upright (if virtual) or inverted (if real); they can be behind the mirror (if virtual) or in front of the mirror (if real); they can also be enlarged, reduced, or the same size as object.
2. Only a concave mirror can be used to produce a real image; and this only occurs if the object is located at a position of more than one focal length from the concave mirror. Plane mirrors never produce real images.
3. A plane mirror will always produce a virtual image. A concave mirror will only produce a virtual image if the object is located in front of the focal point.
4. A plane mirror will always produce an upright image. A concave mirror will only produce an upright image if the object is located in front of the focal point.
5. Only a concave mirror can be used to produce an inverted image; and this only occurs if the object is located at a position of more than one focal length from the concave mirror. Plane mirrors never produce inverted images.
6. No. Real images can be larger than the object, smaller than the object, or the same size as the object.
7. Foo Ling Yu has probably placed the object at the center of curvature - a distance of 3.2 meters from the mirror. When Foo does this, a real image is formed at the same location and of the same size.

7.0 REFERENCE/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 4 MIRROR EQUATION

CONTENTS

1.0 INTRODUCTION

In last unit you studied concave and convex mirrors and their various image positions. In this unit you will learn how to apply those rules to determine position of image of objects placed in imaginary positions basically by calculations.

2.0 OBJECTIVES

After studying this unit, you will be able to:

1. State the mirror formula
2. Apply the mirror formula to obtain either image distance or object distance or the focal length and solve problems involving a curved mirror
3. Define magnification and calculate magnification.

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.1 The Mirror Equation - Convex Mirrors

Ray diagrams can be used to determine the image location, size, orientation and type of image formed of objects when placed at a given location in front of a mirror. The use of these diagrams was demonstrated earlier in Lesson 3 andin Lesson 4. Ray diagrams provide useful information about object-image relationships, yet fail to provide the information in a quantitative form. While a ray diagram may help one determine the approximate location and size of the image, it will not provide numerical information about image distance and image size. To obtain this type of numerical information, it is necessary to use the **Mirror Equation** and the **Magnification Equation**. The mirror equation expresses the quantitative relationship between the object distance (d_o), the image distance (d_i), and the focal length (f). The equation is stated as follows:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

The magnification equation relates the ratio of the image distance and object distance to the ratio of the image height (h_i) and object height (h_o). The magnification equation is stated as follows:

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

These two equations can be combined to yield information about the image distance and image height if the object distance, object height, and focal length are known. Their use was demonstrated in Lesson 3 for concave mirrors and will be demonstrated here for convex mirrors. As a demonstration of the effectiveness of the Mirror equation and Magnification equation, consider the following example problem and its solution.

Example Problem 1

A 4.0-cm tall light bulb is placed a distance of 35.5 cm from a convex mirror having a focal length of -12.2 cm. Determine the image distance and the image size.

Like all problems in physics, begin by the identification of the known information.

$h_o = 4.0$ cm $d_o = 35.5$ cm $f = -12.2$ cm

Next identify the unknown quantities that you wish to solve for.

$d_i = ???$ $h_i = ???$

To determine the image distance (d_i), the mirror equation will have to be used. The following lines represent the solution to the image distance; substitutions and algebraic steps are shown.

$1/f = 1/do + 1/d_i$

$1/(-12.2 \text{ cm}) = 1/(35.5 \text{ cm}) + 1/d_i$

$-0.0820 \text{ cm}^{-1} = 0.0282 \text{ cm}^{-1} + 1/d_i$

$-0.110 \text{ cm}^{-1} = 1/d_i$

$$\mathbf{d_i = -9.08 \text{ cm}}$$

The numerical values in the solution above were rounded when written down, yet unrounded numbers were used in all calculations. The final answer is rounded to the third significant digit.

To determine the image height (h_i), the magnification equation is needed. Since three of the four quantities in the equation (disregarding the M) are known, the fourth quantity can be calculated. The solution is shown below.

$h_i/h_o = - d_i/d_o$

$h_i /(4.0 \text{ cm}) = - (-9.08 \text{ cm})/(35.5 \text{ cm})$

$h_i = - (4.0 \text{ cm}) \bullet (-9.08 \text{ cm})/(35.5 \text{ cm})$

$$\mathbf{h_i = 1.02 \text{ cm}}$$

The negative values for image distance indicate that the image is located behind the mirror. As is often the case in physics, a negative or positive sign in front of the numerical value for a physical quantity represents information about direction. In the case of the image distance, a negative value always indicates the existence of a virtual image located behind the mirror. In the case of the image height, a positive value indicates an upright image. Further information about the sign conventions for the variables in the Mirror Equation and the Magnification Equation can be found in Lesson 3.

From the calculations in this problem it can be concluded that if a 4.0-cm tall object is placed 35.5 cm from a convex mirror having a focal length of -12.2 cm, then the image will be upright, 1.02-cm tall and located 9.08 cm behind the mirror. The results of this calculation agree with the principles discussed earlier in this lesson. Convex mirrors always produce images that are upright, virtual, reduced in size, and located behind the mirror.

ACTIVITY 1

1. A convex mirror has a focal length of -10.8 cm. An object is placed 32.7 cm from the mirror's surface. Determine the image distance.

2. Determine the focal length of a convex mirror that produces an image that is 16.0 cm behind the mirror when the object is 28.5 cm from the mirror.

3. A 2.80-cm diameter coin is placed a distance of 25.0 cm from a convex mirror that has a focal length of -12.0 cm. Determine the image distance and the diameter of the image.

4. A focal point is located 20.0 cm from a convex mirror. An object is placed 12 cm from the mirror. Determine the image distance.

The Mirror Equation - Concave Mirrors

Ray diagrams can be used to determine the image location, size, orientation and type of image formed of objects when placed at a given location in front of a concave mirror. The use of these diagrams was demonstrated earlier IN unit 3. Ray diagrams provide useful information about object-image relationships, yet fail to provide the information in a quantitative form. While a ray diagram may help one determine the approximate location and size of the image, it will not provide numerical information about image distance and object size. To obtain this type of numerical information, it is necessary to use the **Mirror Equation** and the **Magnification Equation**. The mirror equation expresses the quantitative relationship between the object distance (d_o), the image distance (d_i), and the focal length (f). The equation is stated as follows:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

The magnification equation relates the ratio of the image distance and object distance to the ratio of the image height (h_i) and object height (h_o). The magnification equation is stated as follows:

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

These two equations can be combined to yield information about the image distance and image height if the object distance, object height, and focal length are known.

As a demonstration of the effectiveness of the mirror equation and magnification equation, consider the following example problem and its solution.

Example Problem 2

A 4.00-cm tall light bulb is placed a distance of 45.7 cm from a concave mirror having a focal length of 15.2 cm. Determine the image distance and the image size.

Like all problems in physics, begin by the identification of the known information.

$h_o = 4.0$ cm $\quad$ $d_o = 45.7$ cm $\quad$ $f = 15.2$ cm

Next identify the unknown quantities that you wish to solve for.

$d_i = ???$ $\quad$ $h_i = ???$

To determine the image distance, the mirror equation must be used. The following lines represent the solution to the image distance; substitutions and algebraic steps are shown.

$1/f = 1/do + 1/d_i$

$1/(15.2 \text{ cm}) = 1/(45.7 \text{ cm}) + 1/d_i$

$0.0658 \text{ cm}^{-1} = 0.0219 \text{ cm}^{-1} + 1/d_i$

$0.0439 \text{ cm}^{-1} = 1/d_i$

$$\mathbf{d_i = 22.8 \text{ cm}}$$

The numerical values in the solution above were rounded when written down, yet un-rounded numbers were used in all calculations. The final answer is rounded to the third significant digit.

To determine the image height, the magnification equation is needed. Since three of the four quantities in the equation (disregarding the M) are known, the fourth quantity can be calculated. The solution is shown below.

$h_i/h_o = - d_i/d_o$

$h_i /(4.0 \text{ cm}) = - (22.8 \text{ cm})/(45.7 \text{ cm})$

$h_i = - (4.0 \text{ cm}) \bullet (22.8 \text{ cm})/(45.7 \text{ cm})$

$$\mathbf{h_i = -1.99 \text{ cm}}$$

The negative values for image height indicate that the image is an inverted image. As is often the case in physics, a negative or positive sign in front of the numerical value for a physical quantity represents information about direction. In the case of the image height, a negative value always indicates an inverted image.

From the calculations in this problem it can be concluded that if a 4.00-cm tall object is placed 45.7 cm from a concave mirror having a focal length of 15.2 cm, then the image will be inverted, 1.99-cm tall and located 22.8 cm from the mirror. The results of this calculation agree with the principles discussed earlier in this lesson. In this case, the object is located *beyond* the center of curvature (which would be two focal lengths from the mirror), and the image is located between the center of curvature and the focal point. This falls into the category of Case 1 : The object is located beyond C.

Example Problem 3

A 4.0-cm tall light bulb is placed a distance of 8.3 cm from a concave mirror having a focal length of 15.2 cm. (NOTE: this is the same object and the same mirror, only this time the object is placed closer to the mirror.) Determine the image distance and the image size.

Again, begin by the identification of the known information.

h_o = 4.0 cm d_o = 8.3 cm f = 15.2 cm

Next identify the unknown quantities that you wish to solve for.

d_i = ??? h_i = ???

To determine the image distance, the mirror equation will have to be used. The following lines represent the solution to the image distance; substitutions and algebraic steps are shown.

$1/f = 1/do + 1/d_i$

$1/(15.2\ \text{cm}) = 1/(8.3\ \text{cm}) + 1/d_i$

$0.0658\ \text{cm}^{-1} = 0.120\ \text{cm}^{-1} + 1/d_i$

$-0.0547\ \text{cm}^{-1} = 1/d_i$

$$\mathbf{d_i = -18.3\ cm}$$

The numerical values in the solution above were rounded when written down, yet un-rounded numbers were used in all calculations. The final answer is rounded to the third significant digit.

To determine the image height, the magnification equation is needed. Since three of the four quantities in the equation (disregarding the M) are known, the fourth quantity can be calculated. The solution is shown below.

$h_i/h_o = - d_i/d_o$

$h_i /(4.0\ \text{cm}) = - (-18.2\ \text{cm})/(8.3\ \text{cm})$

$h_i = - (4.0\ \text{cm}) \bullet (-18.2\ \text{cm})/(8.3\ \text{cm})$

$$\mathbf{h_i = 8.8\ cm}$$

The negative value for image distance indicates that the image is a virtual image located behind the mirror. Again, a negative or positive sign in front of the numerical value for a physical quantity represents information about direction. In the case of the image distance, a negative value always means behind the mirror. Note also that the image height is a positive value, meaning an upright image. Any image that is upright and located behind the mirror is considered to be a virtual image.

From the calculations in the second example problem it can be concluded that if a 4.0-cm tall object is placed 8.3 cm from a concave mirror having a focal length of 15.2 cm, then the image will be magnified, upright, 8.8-cm tall and located 18.3 cm behind the mirror. The results of this calculation agree with the principles discussed earlier in this lesson. In this case, the object is located in front of the focal point (i.e., the object distance is less than the focal length), and the image is located behind the mirror. This falls into the category of Case 5: The object is located in front of F.

The +/- Sign Conventions

The sign conventions for the given quantities in the mirror equation and magnification equations are as follows:

- f is + if the mirror is a concave mirror
- f is - if the mirror is a convex mirror
- d_i is + if the image is a real image and located on the object's side of the mirror.
- d_i is - if the image is a virtual image and located behind the mirror.
- h_i is + if the image is an upright image (and therefore, also virtual)
- h_i is - if the image an inverted image (and therefore, also real)

Like many mathematical problems in physics, the skill is only acquired through much personal practice. Perhaps you would like to take some time to try the problems activity 2.

ACTIVITY 2

1. Determine the image distance and image height for a 5.00-cm tall object placed 45.0 cm from a concave mirror having a focal length of 15.0 cm.

2. Determine the image distance and image height for a 5.00-cm tall object placed 30.0 cm from a concave mirror having a focal length of 15.0 cm.

3. Determine the image distance and image height for a 5.00-cm tall object placed 20.0 cm from a concave mirror having a focal length of 15.0 cm.

4. Determine the image distance and image height for a 5.00-cm tall object placed 10.0 cm from a concave mirror having a focal length of 15.0 cm.

5. A magnified, inverted image is located a distance of 32.0 cm from a concave mirror with a focal length of 12.0 cm. Determine the object distance and tell whether the image is real or virtual.

6. An inverted image is magnified by 2 when the object is placed 22 cm in front of a concave mirror. Determine the image distance and the focal length of the mirror.

4.0 CONCLUSION

The image formed by either a convex mirror or a concave mirror can be determined using either the ray diagram or the mirror formula. For the same reason, the basic facts used are as follows:

i) a ray of light parallel to the axis of the mirror is reflected by the mirror through the principal focus;
ii) a ray of light directed to the center of curvature of the mirror is reflected back along the same path;
iii) a ray of light incident on the mirror through a Principal Focus is reflected parallel to the axis of the mirror.

In using the mirror equation the following sign convections are used:

i) real objects and images have positive distances;
ii) virtual objects and images have negative distances;
iii) Concave mirrors have positive focal length and radii of curvature while convex mirrors have negative focal length and radii of curvature.

5.0 SUMMARY

A concave mirror or convex mirror has a pole, center of curvature and the principal focus. The focal length of concave mirror is considered positive while that of the convex mirror is taken as negative. A concave mirror can form a real or a virtual image, depending on the location of the object. On the other hand, the convex mirror forms an erect and virtual image irrespective of where the object is located. Many examples and plenty problems in the activities were given because there is a belief that the more you practice, the more perfect you become in using mirror formula.

6.0 TUTOR-MARKED ASSIGNMENT

1. A diverging mirror of 50.0 cm focal length produces a virtual image of 25.0 cm from the mirror. How far from the mirror should the object be placed?
2. A man has a concave mirror with focal length of 40 cm. How far should the mirror be held from his face in order to give an image of two fold magnification?

ANSWER TO ACTIVITY 1

1. Answer: d_i = -8.1 cm. Use the equation 1 / f = 1 / d_o + 1 / d_i where f = -10.8 cm and d_o = + 32.7 cm. Substitute and solve for d_i.
2. Answer: **f = -36.6 cm.** Use the equation 1 / f = 1 / d_o + 1 / d_i where d_o = 28.5 cm and d_i = -16.0 cm (Careful: image distances for convex mirrors are always negative.). Substitute and solve for f.
3. Answer: **d_i = -8.1 cm** and **h_i = 0.909 cm.** Use the equation 1 / f = 1 / d_o + 1 / d_i where f = -12.0 cm and d_o = +25.0 cm. Substitute and solve for d_i. Then use h_i / h_o = - d_i / d_o where h_o = 2.80 cm, d_o = +25 cm and d_i = -8.1 cm. Substitute and solve for h_i.
4. Answer: **d_i = -7.5 cm.** Use the equation 1 / f = 1 / d_o + 1 / d_i where f = - 20.0 cm and d_o = +12.0 cm
5. (Careful: convex mirrors have focal lengths which are negative.). Substitute and solve for d_i.

ANSWER TO ACTIVITY 2

1. Answer: **d_i = 22.5 cm** and **h_i = -2.5 cm.** Use $1/f = 1/d_o + 1/d_i$ where f = 15 cm and d_o = 45 cm. Then use $h_i / h_o = - d_i / d_o$ where h_o = 5 cm, d_o = 45 cm, and d_i = 22.5 cm.
2. Answer: **d_i = 30.0 cm** and **h_i = -5.0 cm.** Use $1 / f = 1 / d_o + 1 / d_i$ where f =15 cm and d_o = 45 cm. Then use $h_i / h_o = - d_i / d_o$ where h_o = 5 cm, d_o = 45 cm, and d_i = 30.0 cm
3. Answer: **d_i = 60.0 cm** and **h_i = -15.0 cm.** Use $1 / f = 1 / d_o + 1/d_i$ where f =15 cm and d_o=45 cm. Then use $h_i / h_o = -d_i / d_o$ where h_o = 5 cm, d_o = 45 cm, and d_i = 60.0 cm
4. Answer: **d_i = -30.0 cm** and **h_i = +15.0 cm.** Use $1 / f = 1 / d_o + 1 / d_i$ where f = 15 cm and d_o = 10.0 cm. Then use $h_i / h_o = -d_i / d_o$ where h_o = 5 cm, d_o = 45 cm, and d_i = -30.0 cm
5. Answer: **d_i = 19.2 cm** and **Real.** Use the equation $1 / f = 1 / d_o + 1 / d_i$ where f = 12 cm and d_o = 32 cm. All inverted images are real images.
6. Answer: **d_i = 44 cm** and **f = 14.7 cm** and **Real.** Use the equation $1 / f = 1 / d_o + 1 / d_i$ where d_o = 22 cm and M = 2. If M=2 and the image is inverted, then the d_i must be +44 cm. (It isn't -44 cm since the negative sign would only correspond to an upright and virtual image.). Solve for f one you find d_i =+44 cm. You know that the image is real if it is described as being upright.

7.0 REFERENCE/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

UNIT 5 SPHERICAL ABERRATIONS

CONTENTS

1.0 INTRODUCTION

In the last Unit, you studied reflections at plane (flat) and curved surfaces. In this unit you will study spherical aberrations. You will study the concept of spherical aberration, their forms and how they could be corrected

2.0 OBJECTIVES

After studying this unit, you will be able to:

1. Explain the concept of aberration
2. Discuss chromatic and monochromatic types of aberration
3. List and discuss any six forms of aberration
4. Represent the forms diagrammatically and suggest how to correct them

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.1 The Concept Spherical Aberration

Aberration - a departure from the expected or proper course. (Webster's Dictionary). Spherical mirrors have an aberration. There is an intrinsic defect with any mirror that takes on the shape of a sphere. This defect prohibits the mirror from focusing all the incident light from the same location on an object to a precise point. The defect is most noticeable for light rays striking the outer edges of the mirror. Rays that strike the outer edges of the mirror fail to focus in the same precise location as light rays that strike the inner portions of the mirror. While light rays originating at the same location on an object reflect off the mirror and focus to a point, any light rays striking the edges of the mirror fail to focus at that same point. The result is that the images of objects as seen in spherical mirrors are often blurry.

Figure 5.1 shows six incident rays traveling parallel to the principal axis and reflecting off a concave mirror. The six corresponding reflected rays are also shown. In the diagram we can observe a departure from the expected or proper course; there is an aberration. The two incident rays that strike the outer edges (top and bottom) of the concave mirror fail to pass through the focal point. This is a *departure from the expected or proper course*.

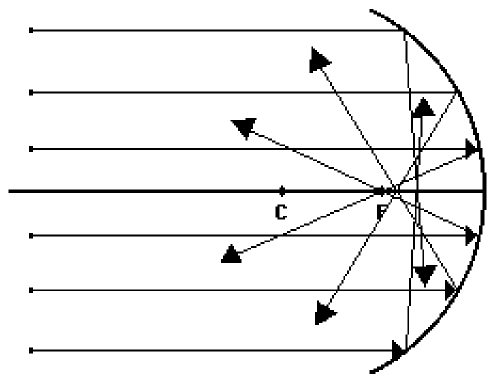


Figure 5.1 Rays from concave mirror not sharply focused

This problem is not limited to light that is incident upon the mirror and traveling parallel to the principal axis. Any incident ray that strikes the outer edges of the mirror is subject to this *departure from the expected or proper course*. A common Physics demonstration utilizes a large demonstration mirror and a candle. The image of the candle is first projected upon a screen and focused as closely as possible. While the image is certainly discernible, it is slightly blurry. Then a cover is placed over the outer edges of the large demonstration mirror. The result is that the image suddenly becomes more clear and focused. When the problematic portion of the mirror is covered so that it can no longer focus (or mis-focus) light, the image appears more focused.

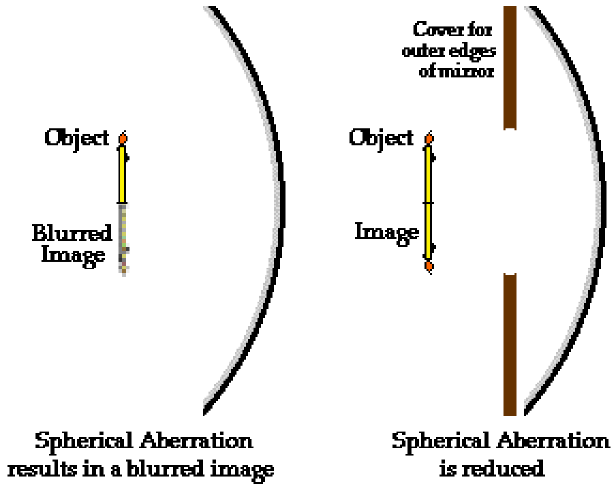


Figure 5.2a: Existence of spherical aberration Figure 5.2c: Reduced spherical aberration

Spherical aberration is most commonly corrected by use of a mirror with a different shape. Usually, a parabolic mirror is substituted for a spherical mirror. The outer edges of a parabolic mirror have a significantly different shape than that of a spherical mirror. Parabolic mirrors create sharp, clear images that lack the blurriness which is common to those images produced by spherical mirrors.

A lens is said to have an aberration if it does not produce a point image. In practice, lenses that are not spherical do not produce point images. Spherical aberration is the term used to describe the formation of an image by a spherical lens will not be perfect because rays striking regions near the perimeter of the lens will be brought to a slightly different focus than those striking the inner regions.

In general a point object will form an image that is a small circle of light. There are several ways of reducing this effect.

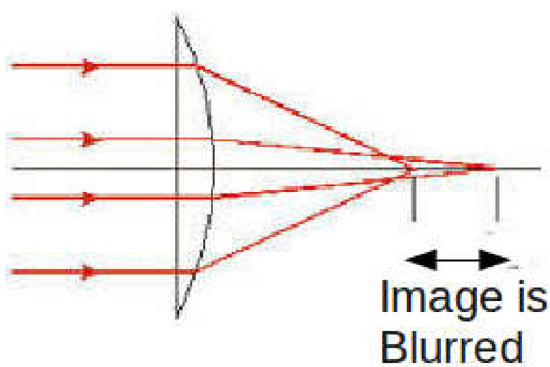


Figure 5.3 Blurred image due to spherical aberration

The shape of the lens could be change to sharpen the image. The lens would no longer be spherical. This only works for objects a certain distance from the lens.

The aperture can be reduced by use of a diaphragm as shown below.

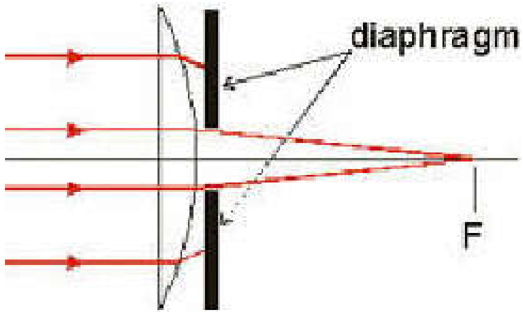


Figure 5.4: Correction of spherical aberration in a lens

Spherical Aberration in Lenses

For lenses made with spherical surfaces, rays which are parallel to the optic axis but at different distances from the optic axis fail to converge to the same point. For a single lens, spherical aberration can be minimized by bending the lens into its best form. For multiple lenses, spherical aberrations can be canceled by overcorrecting some elements. The use of symmetric doublets like the orthoscopic doublet greatly reduces spherical aberration

When the concept of principal focal length is used, the presumption is that all parallel rays focus at the same distance, which is of course true only if there are no aberrations. The use of the lens equation likewise presumes an ideal lens, and that equation is practically true only for the rays close to the optic axis, the so-called paraxial rays.

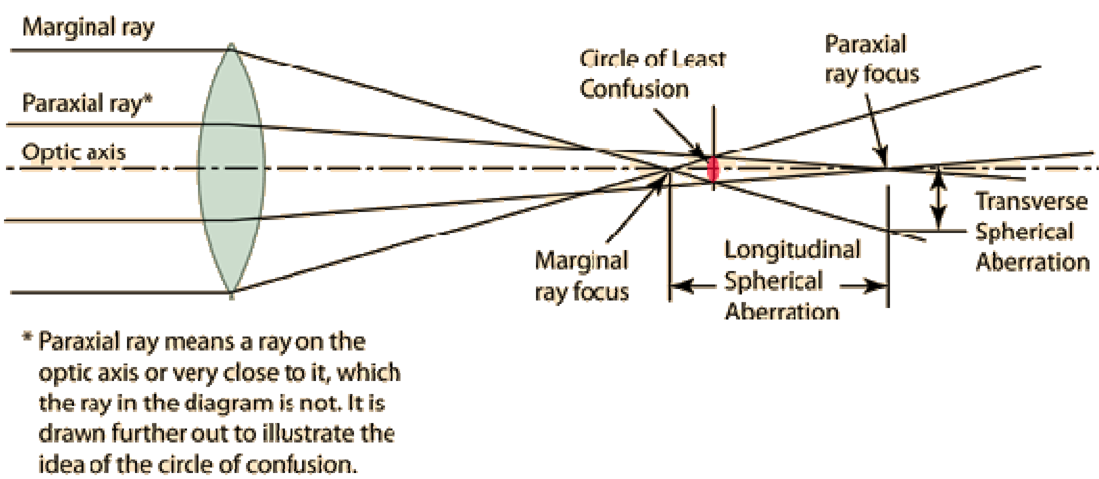


Figure 5.5 Forms of spherical aberration in lenses

For a lens with spherical aberration, the best approximation to use for the focal length is the distance at which the difference between the paraxial and marginal rays is the smallest. It is not perfect, but the departure from perfect focus forms what is called the "circle of least confusion". Spherical aberration is one of the reasons why a smaller aperture (larger f-number) on a camera lens will give a sharper image and greater depth of field since the difference between the paraxial and marginal rays is less

Monochromatic Aberrations

Aberrations are errors in an image that occur because of imperfections in the optical system. Another way of saying this is that aberrations result when the optical system misdirects some of the object's rays. Optical components can create errors in an image even if they are made of the best materials and have no defects. Some types of aberrations can occur when electromagnetic radiation of one wavelength is being imaged (monochromatic aberrations), and other types occur when electromagnetic radiation of two or more wavelengths is imaged (chromatic aberrations).

Monochromatic aberrations can be grouped into several different categories: **spherical, coma, astigmatism, field curvature, and distortion.** The idea of reference sphere is often used in discussions of aberrations. For all spheres, a ray drawn perpendicular to the sphere's surface will intersect the center of the sphere, no matter what spot on the surface is picked.

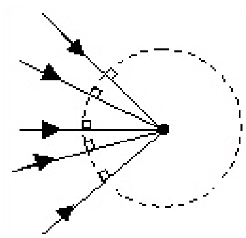


Figure 5.6: Reference sphere

Sphere with rays drawn perpendicular to the surface intersect at the center of the sphere.

A reference sphere isn't a physical structure; it's just a mathematical construct that the wavefront of the electromagnetic radiation is compared to. If the electromagnetic wavefront has the shape of the reference sphere, then the wavefront will come to a perfect focus at the center of the sphere. Remember that the definition of a ray specifies that rays are drawn perpendicular to the wavefront. All of the rays associated with a spherical wavefront will intersect at the center of the sphere. If the wavefront is not spherical, some of the rays will pass through the center of the sphere.

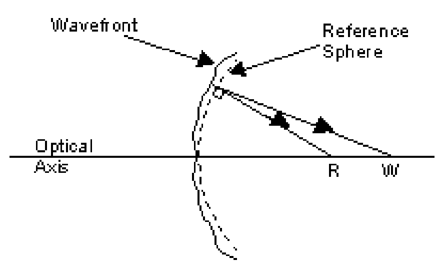


Figure 5.7: Wavefront and spherical aberration

Some rays on an aberrated wavefront focus to a different point, W, than do rays that are perpendicular to the reference sphere.

By comparing the wavefront of the electromagnetic radiation with the reference sphere, it is possible to determine what aberrations are present in an image and how severe they are.

Forms of Aberration

Spherical aberrations occur for lenses that have spherical surfaces. Rays passing through points on a lens farther away from an axis are refracted more than those closer to the axis. This results in a distribution of foci along the optical axis.

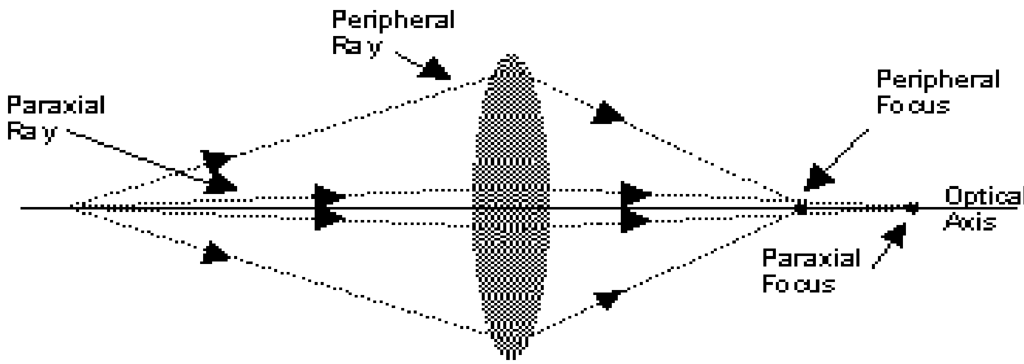


Figure 5.8: Spherical aberrations resulting in paraxial and peripheral rays having different foci.

Rays that make a small angle with the optical axis, and which travel close to the axis, are called paraxial rays. Peripheral rays interact with the edges of the components in an optical system. When a wavefront is spherically aberrated, peripheral rays focus closer to the lens than paraxial rays do. The difference between where these two types of rays come to a focus is a way to measure the severity of spherical aberration in a system.

It is possible to design optical components with a spherical surfaces that are free of spherical aberration. Gradient-index lenses, which have refractive indices that are highest at the center of the lens and gradually decrease closer to the edge of the lens, can also eliminate spherical aberration. However, optical components with spherical surfaces are much easier and cheaper to manufacture than those with a spherical surfaces or gradient-index characteristics. Because of this, most designers of optical systems use off-the-shelf components with spherical surfaces.

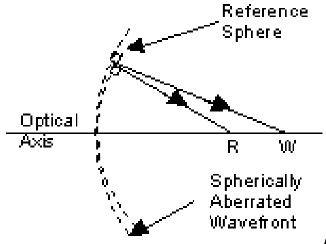


Figure 5.9: Spherically aberrated wavefront

A measure of the spherical aberration of the optical system is the physical distance between the foci of the reference sphere and of the peripheral rays of the aberrated wavefront (between R and W).

When multiple-lens systems are designed, optical designers use the interactions of all of the system components to minimize spherical, as well as other, aberrations. The under correction of one lens can be used to compensate for the overcorrection of another lens. If the optical system must contain only one spherical lens, the spherical aberration can be minimized if both lens surfaces contribute equally to the power of the lens. Making a lens with large radii of curvature will also help minimize spherical aberration.

Coma

Spherical aberrations describe where different points focus along the optical axis. The image of an object point that lies off of the optical axis will form a tear-drop shaped image. The flared tail of the image is usually directed away from the axis, but it can also be oriented towards the axis.

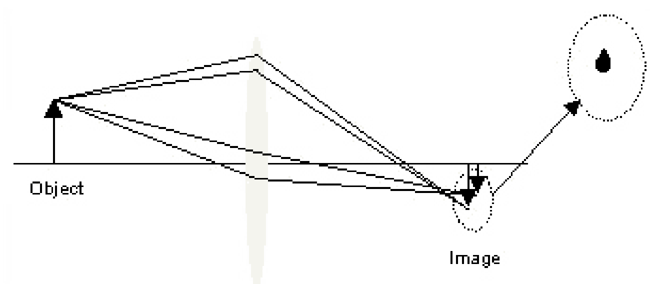


Figure 5.10: coma aberration

Coma blurs the image of an off-axis point into the shape of a teardrop.

When an object is imaged by a lens that suffers from coma, rays that pass through the periphery of the lens form a larger image than the rays that pass through the lens closer to the axis. Correcting for coma requires that the different images are made to overlap. In effect, the images formed by the paraxial and the peripheral rays need to experience different degrees of magnification.

Coma can be minimized by carefully specifying the radii of curvature of the two sides of a single lens, or by using a combination of optical elements. When an optical system has no spherical aberration or coma, it is called aplanatic.

Astigmatism

Rays that are emitted from an object point form a right circular cone as they travel towards a lens. When the object point is located off-axis, this cone of rays forms an ellipse on the surface of the lens. (If the cone of rays had been emitted from an on-axis object point, they would have formed a circle on the surface of the lens.) The tangential plane intersects the major axis of the ellipse, and it contains both the optical axis and the object point. The sagittal plan is oriented perpendicular to the tangential plane.

Because of the different ways in which they intersect the lens, the rays in the tangential plane and the rays in the sagittal plane effectively experience lenses different focal lengths. The effective lens that the rays in the tangential plane experience has a higher power. Because of this asymmetry, the rays in the tangential plane focus closer to the lens than the sagittal rays.

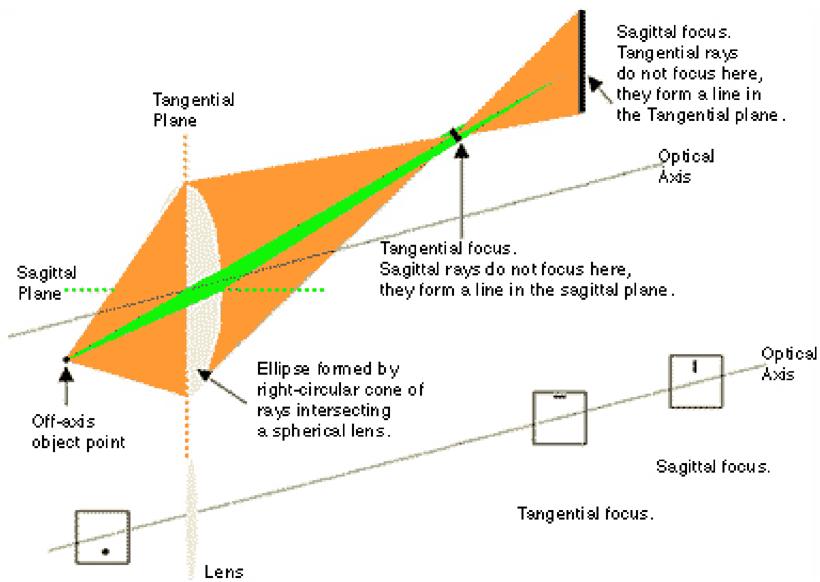


Figure 5.11: Astigmatism aberration

The top figure depicts how a system suffering from astigmatism focuses a cone of rays from an off-axis point onto the tangential and sagittal planes. The bottom figure places a viewing screen at the object point, the tangential focus, and sagittal focus.

The location of the image points for the tangential and sagittal rays coincide on the optical axis, and they diverge for points farther from the optical axis.

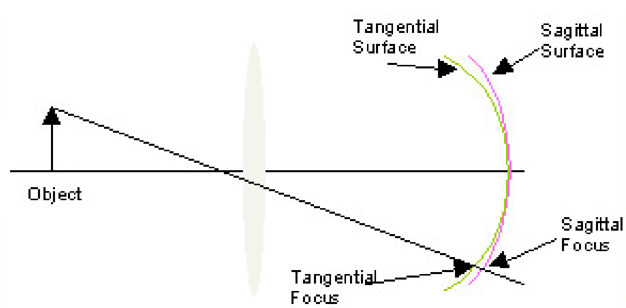


Figure 5.12: details on Astigmatism aberration

Astigmatism causes rays in the tangential plane focus on a different surface than rays in the sagittal plane.

Field Curvature

Field curvature (or curvature of field) is an aberration that is related to astigmatism, but it can exist in a system that does not suffer from astigmatism. In the case of field curvature, the object is imaged on a curved surface, rather than on a plane. The image is not blurred by this aberration; it is just projected onto a curved surface. This is a problem for cameras and slide projectors, because the image plane needs to be flat for these applications. The curved image field can be flattened by using a combination of lenses. If two lenses are used, their indices of refraction (n1 and n2) and their focal lengths (f1 and f2) must meet the following condition:

n1f1 + n2f2 = 0.

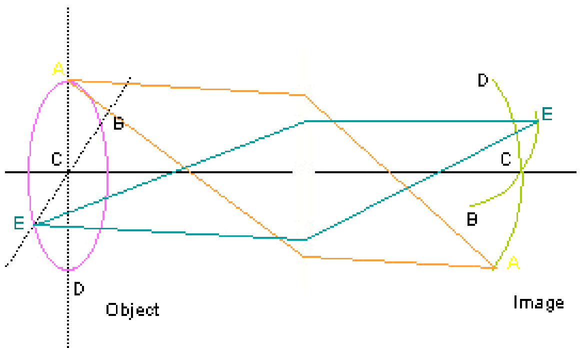


Figure 5.13: field curvature aberration

If a system exhibits field curvature, objects are imaged onto a curved image plane.

Distortion

Like field curvature, an image suffering from distortion is not blurred. Instead, the image points are displaced radially from the positions predicted when paraxial rays are traced through the optical system. The image points may be displaced either towards or away from the optical axis. This effect suggests that the various parts of the object experience different magnifications.

In pincushion distortion, the magnification increases along the indicated directions. An image of a square suffering from pincushion distortion would have drawn-out corners.

In barrel distortion, the magnification decreases along the indicted directions. An image of a square suffering from barrel distortion would be characterized by retraced corners.

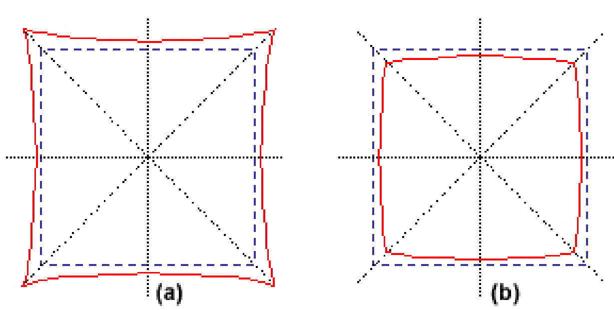


Figure 5.14: Distortion aberration

Pincushion distortion pinches and stretches the image of a square at the corners (a). Barrel distortion pushes the corners of an image of a square in towards the center (b).

Summary of Aberrations

Aberration	Character	Correction
1. Spherical aberration	Monochromatic, on- and off-axis, image blur	Bending, high index, aspherics, gradient index, doublet
2. Coma	Monochromatic, off-axis only, blur	Bending, spaced doublet with central stop
3. Oblique astigmatism	Monochromatic, off-axis blur	Spaced doublet with stop
4. Curvature of field	Monochromatic, off-axis	Spaced doublet
5. Distortion	Monochromatic, off-axis	Spaced doublet with stop
6. Chromatic aberration	Heterochromatic, on- and off-axis, blur	Contact doublet, spaced doublet

ACTIVITY 1

1. Differentiate between chromatic and monochromatic aberrations
2. With the aid of diagram explain the concept of spherical aberration

4.0 CONCLUSION

Spherical aberrations exist in two forms: chromatic and monochromatic types. The monochromatic type consists of spherical, coma, astigmatism, curvature of field and distortion. They can be corrected in most cases by adjustment.

5.0 SUMMARY

A lens is said to have an aberration if it does not produce a point image. In practice, lenses that are not spherical do not produce point images. Spherical aberration is the term used to describe the formation of an image by a spherical lens will not be perfect because rays striking regions near the perimeter of the lens will be brought to a slightly different focus than those striking the inner regions.

Spherical aberration is most commonly corrected by use of a mirror with a different shape. Usually, a parabolic mirror is substituted for a spherical mirror. The outer edges of a parabolic mirror have a significantly different shape than that of a spherical mirror. Parabolic mirrors create sharp, clear images that lack the blurriness which is common to those images produced by spherical mirrors.

6.0 TUTOR-MARKED ASSIGNMENT

1. Produce in a tabula form any six forms of aberration, their characteristics and how they could be corrected.
2. How are coma and astigmatism different as forms of aberration?

ANSWER TO ACTIVITY 1
Refer to the text for correct answers to questions 1 and 2.

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. NewYork: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Stone, B. (1998).*Geometrical optics class notes.* University of Rochester: Rochester, *NY: OPT 441*

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/aberrcon.html#c1

http://www.optics4kids.org/home/content/what-is-optics/refraction/aberrations/

MODULE 3 LENSES AND OPTICAL INSTRUMENTS

UNIT 1 RAY TRACING IN CONVERGING AND DIVERGING LENSES

CONTENT

1.0 INTRODUCTION

In the previous Unit, we have discussed that refraction at a curved surface gives rise to image formation. That is, if an object is placed in front of a curved refracting surface, the image of the object is formed.

In this Unit, you will study how images are formed by lenses (either converging or diverging) for various object positions. This unit will concentrate on using ray diagrams to determine the position of images formed by such lenses. As we have discussed about the refraction. The law of refraction is responsible to govern the behaviour of lens images.

2.0 OBJECTIVES

After studying this Unit, you will be able to:

- trace rays to locate the image formed by a convex lens for various objects distances
- trace the rays to locate the image formed by a concave lens for various object distances
- distinguish the differences between images formed by convex and concave lenses
- solve problems associated with images formed by convex and concave lenses using ray tracing.

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.0 MAIN CONTENT

3.1 Images Formed by a Convex (Converging) lens

In this section discussion is on the image formed by a convex (converging) lens. Here, we are going to look at how the image of an object is formed by a convex lens for the three different object position discussed below. The method is illustrated in Fig. 1.1 (a), 1.1 (b) and 1.1 (c).

In using the ray diagram to determine the position of the image of an object formed by a lens either (convex or concave), a set of rules, similar to rules that govern the reflection case, exist. These are as follows:

(i) a ray parallel to the principal axis incident on one side of the lens is refracted to the far side of the lens through the far focus as shown in Fig.1.1 (a).

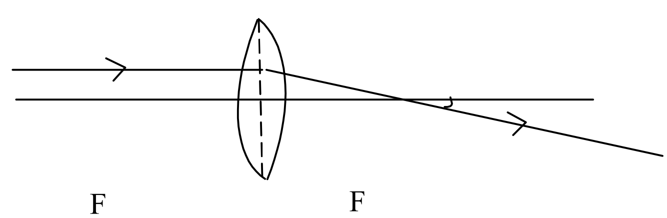


Fig. 1.1 (a): A ray parallel to the principal axis passes through the focus on the far side of the lens.

(ii) A ray passing through the near focus on one side emerges parallel to the principal axis on other side as shown in Fig 1.1 (b).

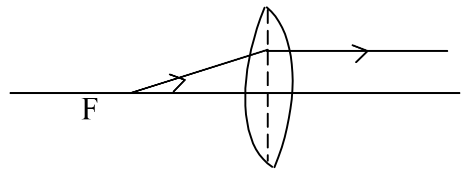


Fig. 1.1 (b): A ray coming through the near focus becomes parallel to the principal axis on the other side.

(iii) A ray incident along the optical centre of the lens goes through to the other side without any deviation as shown in Fig. 1.1 (c).

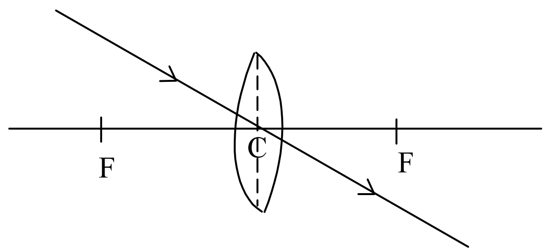


Fig. 1.1 (c): A ray incident along the optical centre of the line is undeviated and passes to the other side without any deviation.

As it will be seen in the case discussed below, the use of any two of three rays is sufficient to determine the location and magnitude of the image.

3.1.1 Object Placed at Distance Greater than 2f

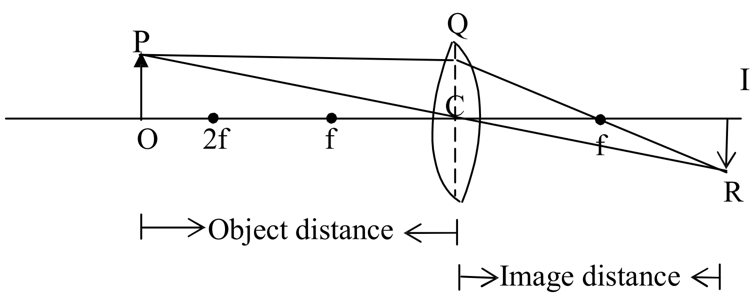


Fig. 1.2: Shows the image formed when the object placed at a distance greater than 2f.

Fig. 1.2 shows the ray diagram for the image formed by a convex lens of focal length f, in which object OP is placed at distance greater than 2f from the lens. Ray PQ which is parallel to the principal axis is refracted through the principal focus to give ray QR. Then the ray PC which is directed towards the optical center C of the lens through the lens undeviated to give ray CR. The two refracted rays QR and CR intersect at R to form the image IR. So, therefore IR gives the magnitude of the image and CI the image distance and OC is the object distance so the magnification M as earlier defined equal to

$$M = \frac{IR}{OP} = \frac{CI}{OC} = \frac{\text{Image distance}}{\text{Object distance}}$$

It can be seen from Fig. 1.2 that the image formed (IR) is real, inverted and magnified.

3.1.2 Object Placed at the Position 2f

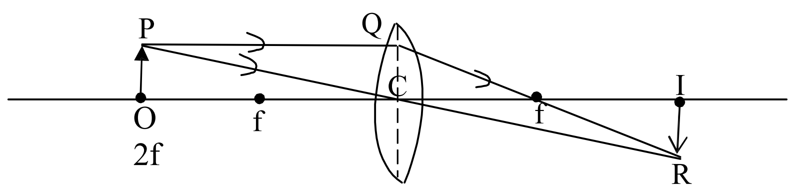


Fig. 1.3: A ray diagram for an object placed at 2f

Fig. 1.3 shows the ray diagram for the image formed by a convex lens of focal length f when the object distance is 2f. The two rays considered are similar to those in Fig. 1.2. It can be seen from Fig. 1.3 that the image formed is real, inverted, and of unit magnification. That is, the size of the image is same as that of the object.

3.1.3 Object is kept at Principal Focus

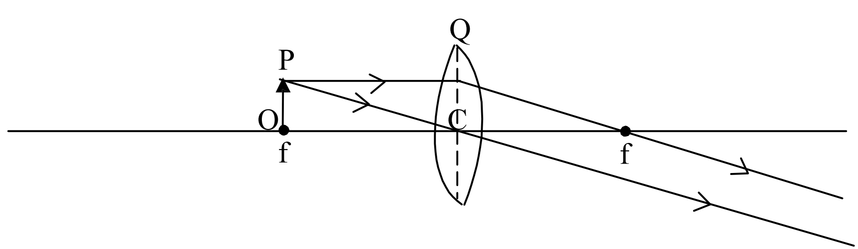


Fig. 1.4: A ray diagram for an object placed at f.

Fig. 1.4 shows the ray diagram for the image formed by a convex lens when the object is kept at focus which is at focal length f. Considering just the two rays either discussed above, ray PQ parallel to the principal axis is refracted through the far focus to give ray Qf. On the other hand ray PC goes through the optical centre of the lens undeviated on the other side. Thus, we have a set of parallel rays emerging on the other side of the lens. Since parallel rays (lines) only converge infinity, it applies that the image formed under this condition is at infinity. Thus, the image formed by a convex lens, when the object is placed at the principal focus, is at infinity.

3.1.4 Object kept between f and the Lens

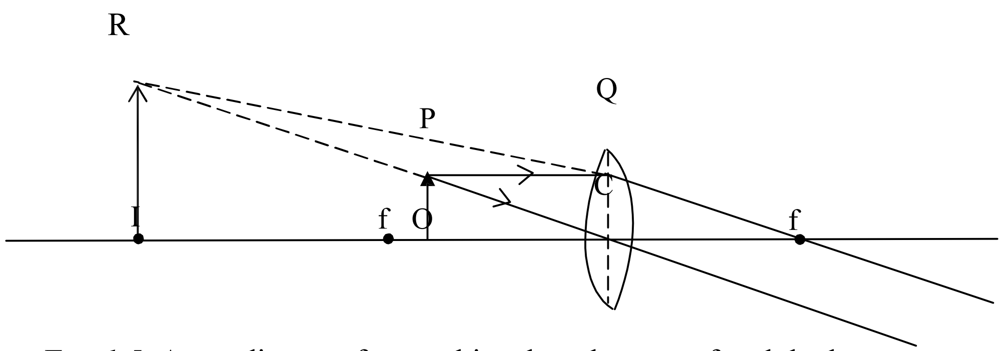


Fig. 1.5: A ray diagram for an object kept between f and the lens.

Fig. 1.5 shows a ray diagram for image formed by convex lens when the object distance is less than the focal length of the lens. Ray PQ is refracted to give ray of while ray PC, as usual, is undeflected. Consequently, the emerging, (refracted rays) diverge and appear to come from point R consequently given rise to image IR.

From Fig. 1.5 it can be seen that the image IR is virtual, erect and magnified.

3.2 Images Formed by Concave Lens

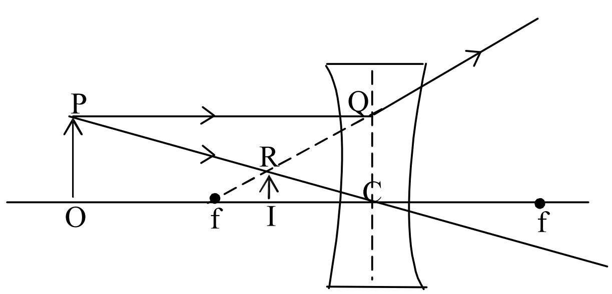


Fig. 1.6: Image formed by a concave lens.

Fig. 1.6 shows the ray diagram for the image formed by a concave (diverging) lens. As can be seen from this figure that a ray PQ, parallel to the axis, diverges at the other side of the lens after refraction to give ray QR, ray PC through the optical center of the lens passes through to the other side of the lens without any deviation. Hence, the image is formed by the intersection of the apparent source of the divergent ray (dotted line) and ray PC.

These two rays intersect at *R*. therefore, *IC* gives the image distance and *IR* gives the magnitude of the image. As before the magnification of the image can be written as

$$M = \frac{IR}{OP} = \frac{IC}{OC}$$

It can be observed from Fig. 5.6 that the image formed is imaginary, it is erect and it is diminished.
Also it has been found that irrespective of the position of the Object, the shape of image and type of the image formed are always the same.

Example 1.1

An object 3 cm tall is placed 30 cm in front of a convex lens of focal length 10 cm. Determine using a ray diagram

(i) magnification of the image
(ii) the image distance

Solution

Choose a suitable scale e.g. 1 cm = 5 cm

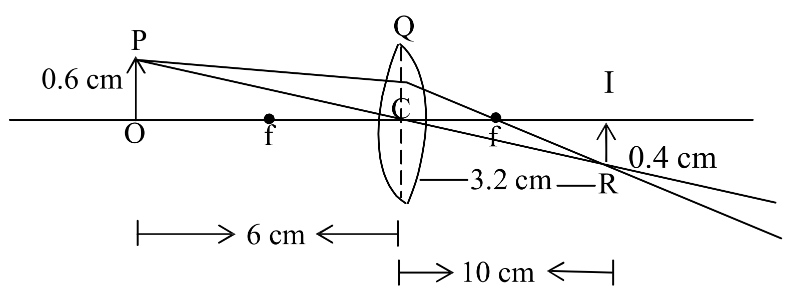


Fig. 1.7

The object distance OC = 30 cm= 6cm (in choosen unit) Object height OP= 3cm =0.6cm (in choosen unit)
Utilizing the above information, the resulting ray diagram is shown in Fig. 1.7. IR as indicated earlier is the magnitude of the image and
OC is the object distance.

So (i) Using the definition of magnification

$$M = \frac{IR}{OP}$$

Which means the image is diminished, it is reduced by half.

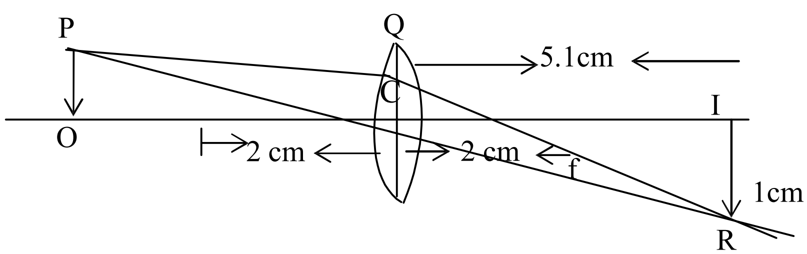
(ii) the image distance IC = (3x5) cm = 15 cm

Example 1.2

If the object in example 6.1 above is placed 15 cm away from lens, what will be

(i) the height of the new image formed?

(ii) the new image distance?



Solution

Refer Fig. 1.8

(i) The new height of the image = (1.0 x 5) cm = 5 cm

The new magnification = $\frac{5.0}{3.0}$ cm = 1.67 cm

(ii) The new image distance = (5 x 5) cm = 25 cm

So that there is a magnification 1.67 times which means the image is enlarged.

Fig. 1.8 IR = 1 cm = 5cm

IC = 5cm = 25 cm

ACTIVITY 1

1. Identify the means by which you can use a converging lens to form a real image.
2. Identify the means by which you can use a converging lens to form a virtual image.
3. A converging lens is sometimes used as a magnifying glass. Explain how this works; specifically, identify the general region where the object must be placed in order to produce the magnified effect.

4.0 CONCLUSION

The image formed by convex and concave lens can be determined by ray tracing for various object distance. For obtaining these images, the basic rules to be followed are;

(i) Rays parallel to the principal axis incident to the lens on one side of a convex lens are brought to a focus on the other side of the lens after refraction of the lens. For the concave lens, on the other hand, the rays are diverge from the same side as the incident parallel rays are appear to be brought to a focus on the far focus.

(ii) For a convex lens, rays emanating from focus on one side incident on the one side of the lens emerge parallel to the principal axis on other side. For a concave lens, such rays are reflected on the same side parallel to the principal focus.

(iii) Light rays directed to the optical centre of the lens (whether Convex or Concave) pass through the lens to the other side undeviated.

When the object distance for a convex lens is greater than 2f, the image formed is real, inverted and magnified.

When the object distance for a convex lens is equal to 2f, the image formed is real, inverted and of unit magnification.

When the object distance for a convex lens is at f, the image is formed at infinity.

When the object distance is less than f the image formed is virtual, erect and magnified.

Finally, the image formed by a concave lens is always virtual and erect.

5.0 SUMMARY

Ray tracing is an interesting technique to determine the images formed by concave and convex lenses. The three rules governing the rays are summarized as in Section 3.1 above. The characteristics of image formed by convex lens are as follows:

When object distance is greater than 2f then image formed is as follows:

(i) Real
(ii) Inverted
(iii) diminished.

The characteristics of image formed by convex lens when the object is kept at 2f are;

(i) Real
(ii) Inverted
(iii) It is of unit magnification

When the object is placed at the focal point of a convex lens, the image is formed at infinity.

The characteristics of image formed by a convex when the object distance is less than f is as follows:

(i) it is virtual
(ii) it is erect
(iii) and it is enlarged

The image formed by a concave lens irrespective of its object distance is always virtual and erect and upright. It may be diminished or enlarged.

6.0 TUTOR-MARKED ASSIGNMENT

1.a) A convex lens with focal length 15 cm is placed 45 cm away from the object 2.5 cm tall (a) Determine the position and the size of the image.

b) If the convex lens were a concave lens, what is the value of the magnitude of the image and the image distance?

ANSWER TO ACTIVITY 1

1. Only a converging lens can be used to produce a real image; and this only occurs if the object is located at a position of more than one focal length from the lens.
2. A converging lens will only produce a virtual image if the object is located in front of the focal point.
3. A converging lens produced a virtual image when the object is placed in front of the focal point. For such a position, the image is magnified and upright, thus allowing for easier viewing.

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.physicsclassroom.com/class/refrn/Lesson-5/Converging-Lenses-Object-Image-Relations

UNIT 2 LENS FORMULA

CONTENTS

1.0 INTRODUCTION

In Unit 1, we discussed how to obtain image distance and the magnification of the image by ray tracing formed by convex and concave lenses for different object distances. The same kind of information can be obtained using the lens formula.

This equation relates the focal length f to the object distance u and image distance v of a lens to the refractive index and the radii of curvature, r_1 and r_2 of the curved surface of the lens.

Also, there is another equation that relates the focal length of a lens to its refractive index and the radii of curvature of the lens, this law is known as the lens maker's law. You will know more about these laws while you study this unit.

2.0 OBJECTIVES

After studying this unit, you will be able to:

1. State the lens maker's law
2. Apply the lens maker's law in solving problems

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.

4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.0 MAIN CONTENT

3.1 The Lens Formula

It has been found that there is a mathematical relationship linking the focal length of a lens (f), the object distance from the lens (u) and image distance from the lens (v).

This relation is given as

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u} \qquad \text{.................................. (2.1)}$$

This Eq.(2.1) is the same as that for curved mirrors (concave or convex). Hence, if any two of these parameter f, u and v are known, Eq. (2.1) can be used to determine the third unknown parameter. Consequently, this equation can be used to derive the same pieces of information obtained in Unit 6 by ray tracing.

3.2 The Lens Makers' Equation

The best way to represent the focal length of a lens is by using the radius of curvature of the two faces (or surfaces).

Now in this section, we will derive an expression for the focal length f of a convex lens. Here, it is assumed that the ray falls on the flat surface of the lens and these surfaces at which a ray enters and leaves similar to the surfaces of a prism. So, we will use prism formula to determine the deviation d.

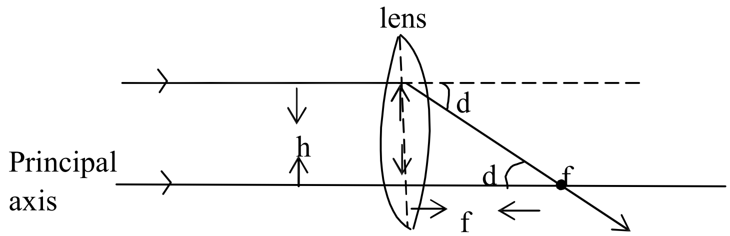


Fig. 2.1 (a): Convex lens

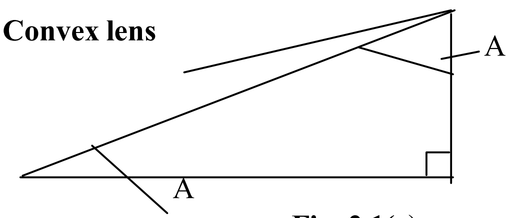


Fig. 2.1(c)

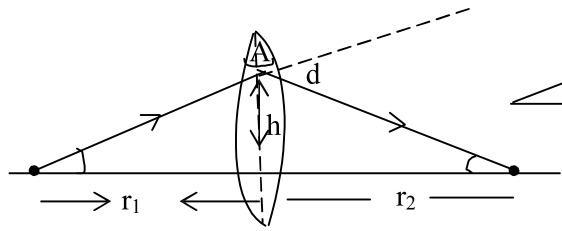


Fig. 2.1(b)

When a ray of light enters a prism, it deviates. Let d be the angle of deviation of light from a small angle prism. The small angle of the prism is A. Let n is the refractive index of the glass. Then the expression for the deviation of a ray passing through a prism can be written as

$$d = (n - 1) A \qquad \ldots\ldots(2.2)$$

But from Fig. 2.1 (a), it is observed that the light rays are parallel to the principal axis. To focus these light rays on the focal point f, each ray is deflected by an angle , then

$$d = \frac{h}{f} \qquad \ldots..(2.3)$$

(here the value of is small $<15^{o}$)
It means that all light rays do not hit lens too far from its centre. It is also known that a transparent material whose surface is spherical will deflect light rays according to Eq. 7.3 i.e. will make useful lens.

Combining Eq. (2.2) and (2.3), we get another expression,

$$\frac{h}{f} = (n - 1) A$$

Rewriting the above equation in another form, we get

$$\frac{1}{f} = (n - 1)\frac{A}{h} \qquad \text{…. (2.4)}$$

from Fig. 2.1 (b), it can be seen that r_1 and r_2 are the radius of curvature and

$$d = \frac{1}{r_1} + \frac{1}{r_2} \qquad \text{………………………………………… (2.5)}$$

(The sum of two interior opposite angles is equal to the exterior angle)

$$\text{and} \quad d = A \qquad \text{….. (2.6)}$$

Now substituting Eq. (2.6) in Eq. (2.5), and also substituting the values of

$$\frac{h}{r_1} \quad and \quad \frac{h}{r_2}$$

we get

$$\frac{h}{r_1} \ and \ \frac{h}{r_2} \ or \ \frac{A}{h} = \frac{1}{r_1} + \frac{1}{r_2}$$

Substituting Eq. (2.7) into Eq.(2.4), we get

$$\frac{1}{f} = (n - 1)\left[\frac{1}{r_1} + \frac{1}{r_2}\right] \qquad \text{……………………………… (2.8)}$$

So, now you can see a relation between focal length of a lens in terms of its refractive index and radius of curvature. Now, it can be seen from the Eq. (2.8) that to obtain a short focal length f the lens should have a small value of r_1 and r_2 and refractive index of the material should be high. The Eq. (2.8) is known as the lens maker's equation.

It can also be noted that the values of the radii of curvature of the two spherical surfaces, which a lens of required focal length should have, can be determined by using this formula. Then the two surfaces of glass can be given the calculated value of the radii of curvature. Hence, the lens so produced will possess the required focal length.

Example 7.1

A pin is placed 40 cm away from a convex lens of focal length 15 cm. Determine the magnification of the pin formed by the lens.

Solution

Focal length (f) of the lens = 15 cm
Object distance (u) = 40 cm
The image distance (v) is to be determined

Using Eq. (2.1)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

On rearranging the terms on either side, we get

$$\frac{1}{v} = \frac{1}{f} - \frac{1}{u}$$

Now substituting the given values

$$\frac{1}{v} - \frac{1}{15} - \frac{1}{40}$$

$$\frac{1}{v} = \frac{8-3}{120} = \frac{5}{120}$$

$$\frac{1}{v} = \frac{5}{120}$$

$$v = 24cm$$

But Magnification is defined as

Again substituting the values, we get

$$\frac{24}{40}$$

M = 0..6

Intensity and colour are the two properties of light. The colour of the light is related to the wavelength or frequency of the light. The intensity (brightness) of light is related to the square of the amplitude of the wave. The visible spectrum to which our eyes are sensitive lies in the range of 450 x 10^{-9}m to 750 x 10^{-9}m . Within this spectrum lie the different colours from violet to red. Light with wavelength shorter than 450 x 10^{-9}m is called ultraviolet and light with wavelength greater than 750 x 10^{-9}m is called infrared. It is to be noted that human eyes are not sensitive to ultraviolet and infrared.

In your physics course earlier, you come across with the prism. A prism is a triangle (wedge) shaped piece of transparent material make up of glass. So, what happens if while light from a source is passed through this prism? Let us discuss about it.

Example 2.2

When an object is placed 10 cm away from a lens it is found that the image formed 5 cm behind the object on the same side of the lens (i) Determine the focal length of the lens (ii) the magnification and (iii) type of image.

Solution

(i) The object distance, u = 10 cm
The image distance, v = - (10 + 5) cm
(since the image is on the same side as the side of the object.)

Because the image distance is negative, Eq. 2.1 becomes

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

That is on inserting the values of the parameters, we get

$$\frac{1}{f} = \frac{1}{10} + \frac{1}{15}$$

$$\frac{1}{f} = \frac{1}{30}$$

$f =$ 30 cm.

(b) Since the sign of f is positive, the lens concerned must be a convex one as by convention, a convex lens has positive focal length.

Magnification, $M = \frac{v}{u}$

But v is negative, therefore the magnification is

$$M = \frac{-15}{10}$$

M = - 1.5

Actually, the negative sign of the magnification, M and the image distance, v shows the image is virtual.

Example 2.3

The curved face of a plano- convex lens of refractive index 1.5 is placed in contact with a plane mirror. An object at a distance of 20 cm coincides with the image produced by the lens and reflects by the mirror. A film of liquid is now placed between the lens and the mirror and the coincident object and image are at 100 cm distance. Determine the refractive index of the liquid.

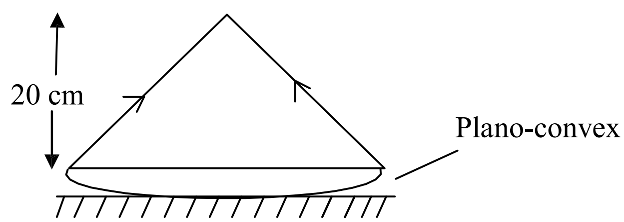


Fig. 2.6(a)

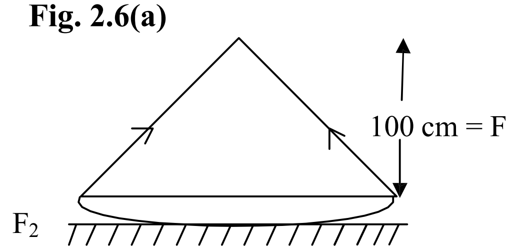


Fig. 2.6(b)

Solution

In Fig 2.6 (a), the rays reflected by mirror are parallel. Therefore, they would converge at the focus of the overlying lens after reflection. Similarly, when the space between the lens and the mirror is filled with liquid the reflected rays converge at the joint focus of Plano concave lens formed by the liquid and the existed Plano convex lens after refraction through these two lenses.

This implies that f_1 = 20 cm, f = 100 cm.
Relating the focus lens to their joint focal (f) is

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{f_2} = \frac{1}{f} - \frac{1}{f_1} = \frac{1}{100} - \frac{1}{20}$$

$$\frac{1}{f_2} = \frac{-4}{100}$$

$$\frac{1}{f_2} = \frac{-1}{25}$$

$$f_2 = -25cm$$

The negative sign indicates that the lens is a concave lens.

A Plano – concave has a negative focal length. Using the lens maker's equation for the Plano – convex lens, we have

$$\frac{1}{f} = (n-1)\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

hence, is the refractive index of the glass for the Plano-convex lens r_2 is infinity ($r_2 =\ 0\ $) because one of it's surfaces is flat.

$$\frac{1}{20} = (1.5-1)\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

$$\frac{1}{20} = (1.5-1)\left(since\frac{1}{r_2} = 0\right)$$

Therefore, r_1 = 10.0 cm

Using the lens maker's equation for the Plano-concave liquid lens, we have

$$\frac{1}{f} = (n-1)\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$$

When n= refractive index

f_2=-25cm

r_1=10cm

$$\frac{1}{25} = (n-1)\frac{1}{10}$$

= 0.6

ACTIVITY 1

1. A 4.00-cm tall light bulb is placed a distance of 45.7 cm from a double convex lens having a focal length of 15.2 cm. Determine the image distance and the image size.
2. A 4.00-cm tall light bulb is placed a distance of 35.5 cm from a diverging lens having a focal length of -12.2 cm. Determine the image distance and the image size.

4.0 CONCLUSION

The lens formula is $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

Thus, if any two of the three parameter f, v, and u are known, then third one can be easily computed using the above Equation. Therefore the information that can be obtained about the object for image distance through ray tracing can also be obtained by using this equation.

The lens maker's equation

$$\frac{1}{f_1}(n-1)\left[\frac{1}{r_1}+\frac{1}{r_2}\right]$$

This equation relates a focal length, refractive index and the radii of curvature of a given lens. Consequently, if we know any of the three parameters above we can always use the equation to determine the fourth one.

5.0 SUMMARY

The lens formula can be used to obtain the same information about the u or v or f as by a lens by ray tracing.

The lens maker's equation relates the focal length, the refractive index and the radii of curvature of a given lens.

6.0 TUTOR-MARKED ASSIGNMENT

1. Determine the radius of curvature of the convex surface of a plano convex lens if its focal length is 0.3m and the refractive index of the material of the lens is 1.5.

2. An object placed 45 cm away from a lens forms an image on a screen placed 90 cm on the other side of the lens.

(a) What type of lens it is?
(b) Determine the focal length of this lens.
(c) Calculate the size of the image if the size of the object is 15 cm.

ANSWER TO ACTIVITY 1

1. Like all problems in physics, begin by the identification of the unknown information.

h_o = 4.00 cm d_o = 45.7 cm f = 15.2 cm

Next identify the unknown quantities that you wish to solve for.

d_i = ??? h_i = ???

To determine the image distance, the lens equation must be used. The following lines represent the solution to the image distance; substitutions and algebraic steps are shown.

$1/f = 1/do + 1/d_i$
$1/(15.2\ cm) = 1/(45.7\ cm) + 1/d_i$

$0.0658\ cm^{-1} = 0.0219\ cm^{-1} + 1/d_i$

$0.0439\ cm^{-1} = 1/d_i$

$\mathbf{d_i = 22.8\ cm}$

The numerical values in the solution above were rounded when written down, yet unrounded numbers were used in all calculations. The final answer is rounded to the third significant digit.

To determine the image height, the magnification equation is needed. Since three of the four quantities in the equation (disregarding the M) are known, the fourth quantity can be calculated. The solution is shown below.

$h_i/h_o = - d_i/d_o$
$h_i /(4.00\ cm) = - (22.8\ cm)/(45.7\ cm)$

$h_i = - (4.00\ cm) \bullet (22.8\ cm)/(45.7\ cm)$

$\mathbf{h_i = -1.99\ cm}$

The negative values for image height indicate that the image is an inverted image. As is often the case in physics, a negative or positive sign in front of the numerical value for a physical quantity represents information about direction. In the case of the image height, a negative value always indicates an inverted image.

From the calculations in this problem it can be concluded that if a 4.00-cm tall object is placed 45.7 cm from a double convex lens having a focal length of 15.2 cm, then the image will be inverted, 1.99-cm tall and located 22.8 cm from the lens. The results of this calculation agree with the principles discussed earlier in this lesson. In this case, the object is located *beyond the* 2F point (which would be two focal lengths from the lens) and the image is located between the 2F point and the focal point. This falls into the category of Case 1: The object is located *beyond* 2F for a converging lens.

2. Like all problems in physics, begin by the identification of the unknown information.

   h_o = 4.00 cm d_o = 35.5 cm f = -12.2 cm

   Next identify the unknown quantities that you wish to solve for.

   d_i = ??? h_i = ???

To determine the image distance, the lens equation will have to be used. The following lines represent the solution to the image distance; substitutions and algebraic steps are shown.

$1/f = 1/do + 1/d_i$
$1/(-12.2 \text{ cm}) = 1/(35.5 \text{ cm}) + 1/d_i$
$-0.0820 \text{ cm}^{-1} = 0.0282 \text{ cm}^{-1} + 1/d_i$
$-0.110 \text{ cm}^{-1} = 1/d_i$

d_i = -9.08 cm

The numerical values in the solution above were rounded when written down, yet unrounded numbers were used in all calculations. The final answer is rounded to the third significant digit.

To determine the image height, the magnification equation is needed. Since three of the four quantities in the equation (disregarding the M) are known, the fourth quantity can be calculated. The solution is shown below.

$h_i/h_o = - d_i/d_o$
$h_i /(4.00 \text{ cm}) = - (-9.08 \text{ cm})/(35.5 \text{ cm})$

$h_i = - (4.00 \text{ cm}) * (-9.08 \text{ cm})/(35.5 \text{ cm})$

h_i = 1.02 cm

The negative values for image distance indicate that the image is located on the object's side of the lens. As mentioned, a negative or positive sign in front of the numerical value for a physical quantity represents information about direction. In the case of the image distance, a negative value always indicates the existence of a virtual image located on the object's side of the lens. In the case of the image height, a positive value indicates an upright image.

From the calculations in this problem it can be concluded that if a 4.00-cm tall object is placed 35.5 cm from a diverging lens having a focal length of 12.2 cm, then the image will be upright, 1.02-cm tall and located 9.08 cm from the lens on the object's side. The results of this calculation agree with the principles discussed earlier in this lesson. Diverging lenses always produce images that are upright, virtual, reduced in size, and located on the object's side of the lens.

7.0 REFERENCES/FURTHER READINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.physicsclassroom.com/class/refrn/Lesson-5/The-Mathematics-of-Lenses

UNIT 3 THE EYE

CONTENTS

1.0 INTRODUCTION

We have learnt about the lenses in earlier units. You can recall that how different kind of images are formed when the object is kept at different positions. Eyeglasses are now a day commonly used to correct visual problems. We use lenses to solve the problem of nearsightedness, farsightedness or to magnify object. Now, you may ask logically what goes wrong with our vision. Why are we not able to see properly? To get the answer of these questions, first, it is vital to know about the human eye and its essential parts and their functions.

The eye is a natural optical instrument which an average person uses to see. It is analogous in every way to the camera. It has a lens, a shutter known as the iris and a retina which acts like film of a camera. The image of an object being viewed is formed on the retina in the same manner as the image is formed on the film of a camera.

In this unit you will study about essential parts of the eye as well as the defects of the eye and their corrections. The major defects of eye are far – sightedness, and short sightedness. They are corrected using appropriate convex or concave lenses which are usually worn in form of eye glasses (spectacles).

2.0 OBJECTIVES

After studying this unit, you will be able to:

- identify the various parts of the eye
- discuss the function(s) of each part of the eye define the power of a lens
- solve problems involving power of a lens
- discuss the major defects of the eye and their corrections.

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.0 MAIN CONTENT

3.1 The Human Eye

A vertical section of the human eye is shown in Fig. 3.1 below. As you can see, the eye has the following essential parts:

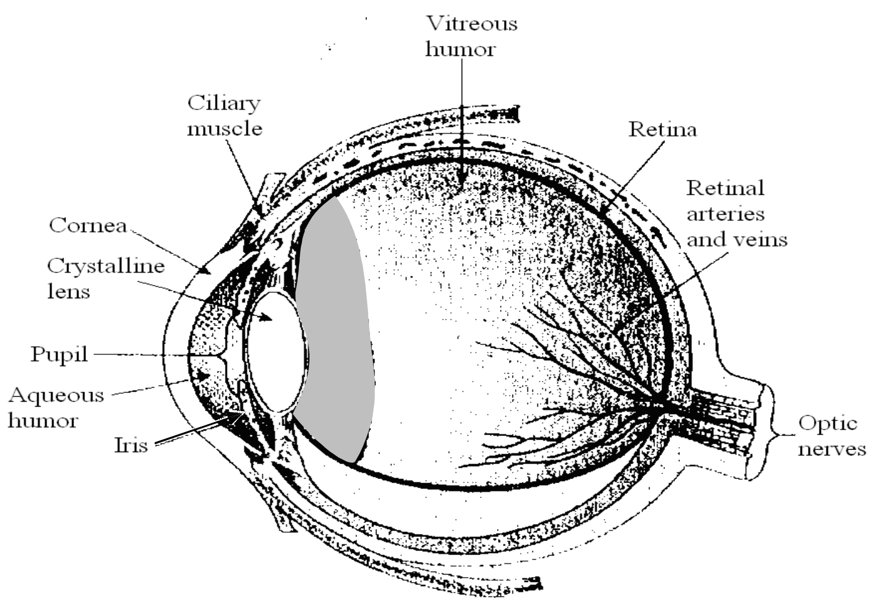


Fig. 3.1: Main parts of human eye

i) The cornea is the transparent part of the eye. The light which enters to the eye passes through it. It serves as a protective covering to the parts like pupil, crystalline lens etc. and also partly focuses light entering the eye.

ii) The iris which acts as a muscular diaphragm of variable size that controls the size of pupil. Its function is to regulate the amount of light entering to the eye. In low light conditions, it dilates the pupil and on the other hand, it contracts the pupil in high light conditions.

iii) The pupil is a circular aperture in the iris.

iv) The eye lens which is supported by the ciliary's muscles and its function is to focus light entering the eye onto the retina. The action of the ciliary's muscles alters the focal length of the lens by changing its shape.

v) The retina is the light sensitive portion at the back inside surface of the eye. The optic nerves of the brain begin at the retina from which they transmit massages to the brain. The most sensitive spot of the retina is known as the yellow spot and its least sensitive portion is the blind spot, which is where the optic nerve leaves the eye for the brain. An image is perceived. The retina in the eye works in the same way as the film in a camera. It is interesting to note that our brains interpret the object scene as right side up.

vi) Cornea is the curved membrane forming the front surface of the eye.

vii) The aqueous humor is the transparent liquid between the lens and the cornea.

viii) The vitreous humor is a jelly liquid between the lens and the rest of the eye ball.

The optical system of the eye consists of the cornea, the aqueous and vitreous humor and the lens. The rod and cones known as receptors, when stimulated by light, send signals to the brain through optic nerves and where an image is perceived. They form an ideal and inverted image of an external object on the retina. The retina transmits the impression created on it by this image through the optic nerve to the brain. The brain then interprets the inverted image as being vertical in reality.

The focal length of the eye lens is not constant. The shape of the lens is altered by the action of the ciliary muscles to obtain a convex lens of appropriate focal length required to focus the object viewed (far or near) on the retina. The ability of the lens to focus on near and far objects is known as **accommodation.**

You may have come across with Optometrists and Ophthalmologists in regard of eyeglass or contact lenses. It is important to know that they use inverse of the focal length to determine the strength of the eyeglass or lens. This inverse of the focal length is called power, which we will discuss in the next section 3.2.

3.2 Power of a Lens

The power of a lens is measured by opticians in a unit known as a diopter. A **diopter** is the reciprocal of the focal length.

diopters = 1/(focal length)
A lens system with a focal length of 1.8 cm (0.018 m) is a 56-diopter lens. A lens system with a focal length of 1.68 cm is a 60-diopter lens. A healthy eye is able to bring both distant objects and nearby objects into focus without the need for corrective lenses. That is, the healthy eye is able to assume both a small and a large focal length; it would have the ability to view objects with a large variation in distance. The maximum variation in the power of the eye is called the **Power of Accommodation**. If an eye has the ability to assume a focal length of 1.80 cm (56 diopters) to view objects many miles away as well as the ability to assume a 1.68 cm focal length to view an object 0.25 meters away (60 diopters), then its Power of Accommodation would be measured as 4 diopters (60 diopters - 56 diopters).

The healthy eye of a young adult has a Power of Accommodation of approximately 4 diopters. As a person grows older, the Power of Accommodation typically decreases as a person becomes less able to view nearby objects. This failure to view nearby objects leads to the need for corrective lenses. In the next two sections of Lesson 6, we will discuss the two most common defects of the eye - nearsightedness and farsightedness.

Where P is the power of the lens and f is the focal length. The power of a lens is measured in diopter (D). For example, when the focal length is 1m, the power of the lens is 1D.

$$P = \frac{1}{f} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Hence the power of a lens in diopters is given by the expression

$$P = \frac{100}{f(cm)}$$

Here the focal length is taken in centimeters.

The power of a converging lens is positive while that of a diverging lens is negative because their focal lengths are positive and negative respectively,

Example 3.1

Determine the focal length of a lens with power +2.5 diopters.

Solution

$$P = \frac{100}{f} \quad 2.5$$

$$\Rightarrow f = \frac{100}{2.5}$$

$$= 40\,cm$$

Example 3.2

Determine the power of a concave lens with focal length 20 cm.

Solution

$$\frac{100}{f} = \frac{100}{20},$$

$\Rightarrow f \;=\; 5 \;\; cm$

Many of us encountered with the visual problems like nearsightedness and farsightedness. Most of us use glasses at some point of time in our life. In section 3.1, various parts of eyes and their functions were discussed. It was mentioned that focused image of an object is observed on the retina. But sometimes the image of the object is not formed on the retina because the lens in the eye does not focus the light rays properly on to the retina. Hence, we are not able to see properly or there is some defect observed in the eye. Now in the next section, let us discuss about the eye defects and also learn how these defects can be corrected?

3.3 Eye Defects and their Corrections

The closest distance a normal eye can see an object clearly (without accommodation) is called "the **near point** or the least distant of distinct vision". The near point is the closest distance for which the lens can accommodate to focus light on the retina. This distance is equal to 25 cm for a normal eye. This distance increases with the age. It is mentioned in the literature that it is about 50 cm at age 40 and to 500 cm or greater at age 60.

The farthest distance a normal eye can see an object is called the **far point** and is at infinity for a normal eye. Therefore, a person with normal eye can see very distant objects like moon.

3.3.1 Farsightedness (hyperopia)

In farsightedness (or hyperopia), a person can usually see far away objects clearly but not nearby objects. The light rays do not converged by the eye on the retina but focuses behind the retina. Hence, the image formed by the lens in the eye fall behind the retina (see Fig. 3.2(a).

Correction: In order to correct this defect, a convex lens needs to be placed before the eye. It help in converging furthermore the incoming rays before they enter the eye, so that by the time the lens in the eye converges them, they would exactly fall on the retina (see Fig. 3.2 b).

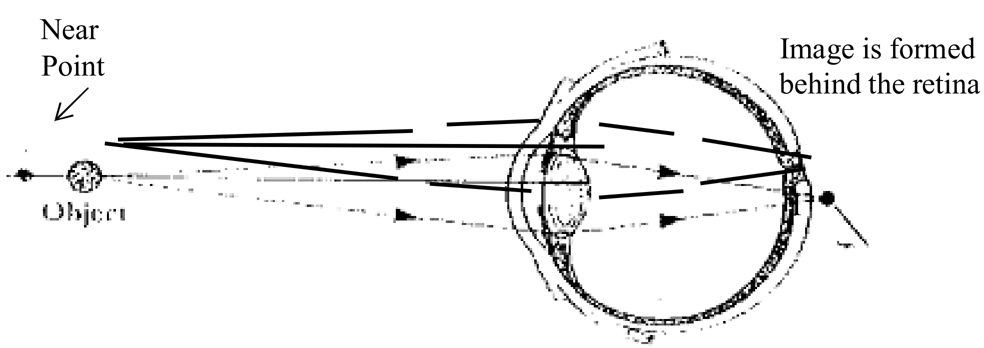


Fig. 3.2 (a): Farsightedness

Convex lens

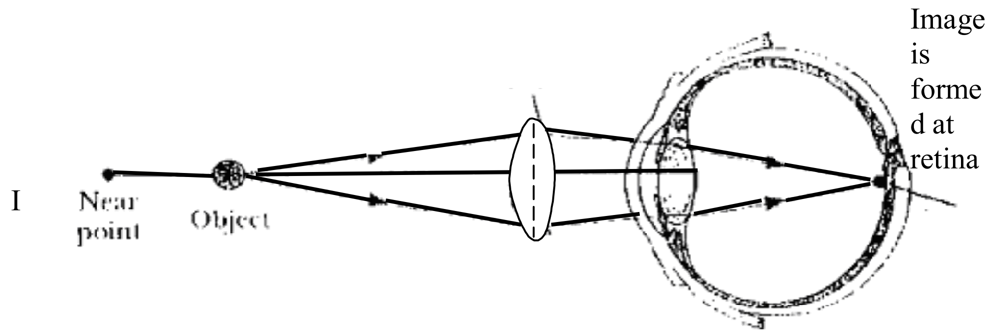


Fig. 3.2 (b): Use of convex lens to correct farsightedness.

3.3.2 Nearsightedness (or myopia)

When a person cannot see clearly or focus to the retina objects at the far point but can focus on the nearby objects, then the person is said to be suffering from nearsightedness (or myopia). Usually this problem arises with the people who do a lot of reading. Fig. 3.3 (a) shows that for nearsighted person, rays from a distance objects get focused before getting to the retina.

Correction

The type of defect can be corrected by using a concave lens placed before the eye (see Fig. 3.3 (b). It can be seen in Fig. 3.3 (b) that the concave lens diverge the rays from distant object before getting to the cornea and thereby enabling the natural lens of the eye to focus the rays on the retina.

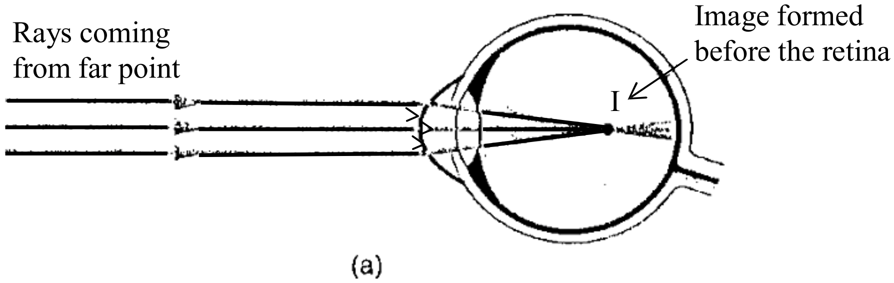
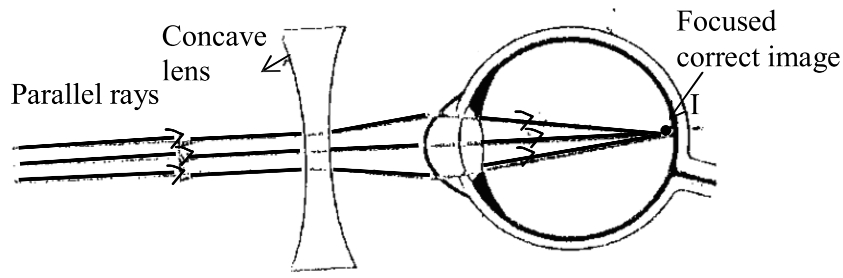


Fig. 3.3 (a): Nearsightedness



(b)

Fig. 3.3 (b): Use of a concave lens to correct nearsightedness.

ACTIVITY 1

A man cannot see clearly objects beyond 100 cm from his eye. Calculate the power of the lens he needs to see distant object clearly.

4.0 CONCLUSION

The eye is similar to the camera in many ways. It has a lens, a shutter (iris) and a film (retina). Its mode of image formation is very similar to that of a camera in all respect. The image formed on the retina is always inverted just like the image formed by a camera on a film. The only difference is that the human brain interprets the image and also the lens of the eye is usually adjustable to enable it focus on far or near objects. The ability of the lens to adjust it so if for the purpose is known as accommodation.

The power of a lens is usually expressed as the inverse of the focal length. Its unit is diopter. The diopter is represented by D.

The two defects of eye are farsightedness (or hyperopia) and nearsightedness (or myopia). In farsightedness, the person is able to see distinctively far away objects but not nearly objects. In this case, light from near objects are focused behind the retina and a convex lens is used to correct these defect.

In the case of nearsightedness, the eye is able to see distinctively objects that are near but not those far off. In this case, rays from distant object are focused by the lens before the retina. A concave lens is used to correct this defect.

ANSWER TO ACTIVITY 1

Since the man cannot see beyond 100 cm, it implies that he is shortsighted and would need a diverging lens for correction.

For him to see the object at infinity, the lens must assure his object distance to be infinity and image distance at 100 cm, because the object to man appear to be at 100 cm away.

u =

v = - 100 cm (negative sign because lens is concave)

f = ?

Using the lens formula, we can insert the values

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{100}$$

f = - 100 cm

Therefore, power

$$= \frac{100}{100} \text{ dioptres}$$

= - 1.0 dioptres

= - 1.0 D

5.0 SUMMARY

The eye is similar in form and in operation to the camera.
The image formed by the eye is inverted just like the one formed on the camera film but it's interpreted as being correct by the brain.

Also the shape and consequently the focal length of the normal eye are variable and can therefore focus on near or far object when required. The ability of the eye to do these is known as accommodation.

One of the major defects of the eye is farsightedness. This defect occurs when light from far objects are focused behind the retina. The defect is corrected by introduction of a convex lens before the eye.

Another major defect of the eye is nearsightedness. This defect occurs when light from distant object are focused before the retina. The defect is corrected by the introduction of a concave lens before the eye.

6.0 TUTOR-MARKED ASSIGNMENT

1. (a) Calculate the focal length of a lens of power 2.0 D.

(b) Explain, how a normal eye produces a sharp image?

2. What are the two defects of vision? How they can be corrected? Explain with diagram.

ANSWER TO ACTIVITY 1

Since the man cannot see beyond 100 cm, it implies that he is shortsighted and would need a diverging lens for correction.

For him to see the object at infinity, the lens must assure his object distance to be infinity and image distance at 100 cm, because the object to man appear to be at 100 cm away.

u =

v = - 100 cm (negative sign because lens is concave)

f = ?

Using the lens formula, we can insert the values

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$

$$\frac{1}{f} = \frac{1}{100}$$

f = - 100 cm

Therefore, power

$$= \frac{100}{100} \text{ dioptres}$$

= - 1.0 dioptres

= - 1.0 D

7.0 REFERENCES/FURTHER REDINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.physicsclassroom.com/Class/refrn/u14l6c.cfm#accomdn

UNIT 4 OPTICAL INSTRUMENTS

CONTENTS

1.0 INTRODUCTION

In earlier units, we have studied about reflection and refractions and see how the rays are reflected and refracted. Then, we learnt about lenses and studied that these lenses can be used to converge or diverges the rays from distant objects. These lenses focus light and produce a sharp image. In the last unit, we discussed that how lenses are used to correct the defects of vision. Now the question arises: can we make use of these lenses further? Yes, we can, in the form of microscope, telescope, which you may have come-across in your earlier school physics curriculum. In this unit, we will study about the further use of lenses in optical instruments like microscope and telescope. You will also learn that how the combination of lenses form these optical instruments.

The microscope or a magnifier is used to see very tiny objects or to magnify the size of the objects which cannot be seen by naked eyes whereas telescope is used to view the distant object such as planets or other Astronomical objects.

The invention of these instruments (i.e. microscope and telescope etc.) has made a great impact on our life. So, now we will study in detail about these two optical instruments and also look how they operate.

2.0 OBJECTIVES

After studying this unit you will be able to:

- define visual angle and angle of magnification distinguish between microscope and the telescope explain how the microscope function
- explain how the astronomical telescope functions in

normal adjustment
- explain how the astronomical telescope functions when its image is formed at near point.

How to Study this Unit:

a. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
b. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
c. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
d. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.0 MAIN CONTENT

3.1 The Microscope

Before discussing the details of simple microscope, first we briefly discuss about visual angle subtended by an object at the eye. This is because in the optical instruments like telescopes and microscopes, we are concerned with the visual angle.

An object NM is placed at some distance from the eye as shown in Fig. 4.1. This object is subtended an angle at the eye.

The length of the image q formed by the eye is proportional to angle subtended at the eye by the object. This angle is called the visual angle.

Using the relation

$$\text{Angle} = \frac{\text{Arc}}{\text{Radius}}$$

q = p

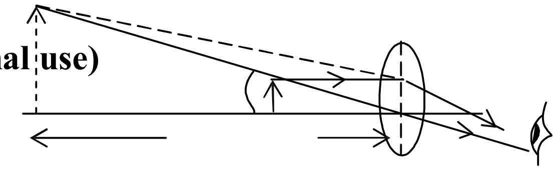
q is directly proportional to (as p is constant)

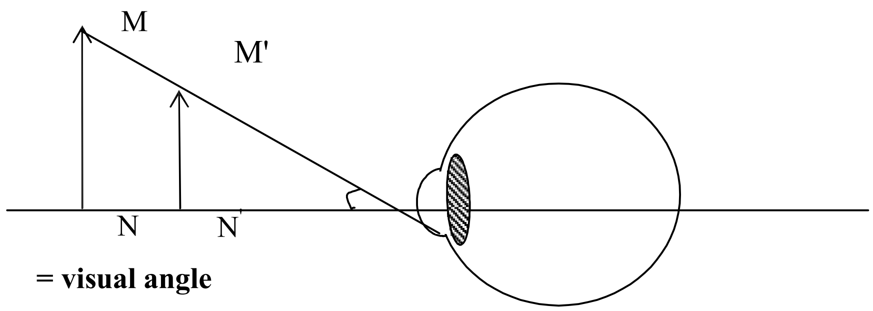
This shows that the visual angle is directly proportioned to apparent size of the object.

Optical Instruments such as telescopes and microscopes are designed to increase the visual angle. The resultant effect of this is that the image of the object formed on the retina becomes much bigger than it is when these instruments are not used to view them i.e., image formed on the retina when these instruments are used become much magnified than when they are not used.

Now in our next subsections you will learn about simple microscope and compound microscope. First, we will discuss about the simple microscope in normal use and then simple microscope with image infinity.

3.1.1 Simple Microscope (in normal use)

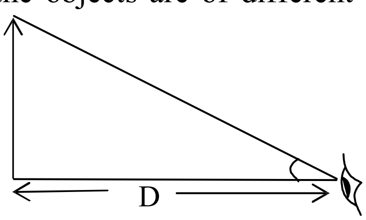




= visual angle

Fig. 4.2: Object move from N to N'

Fig. 4.2 shows that two objects (of different size) MN and M'N' are subtending the same visual angle at the eye, therefore appears to be of equal size. But in actual, the objects are of different sizes and object MN is bigger in size.



eye

α'

D

Fig. 4.3: (a) The visual angle without microscope

Fig. 4.3 (b): ' is the visual angle subtended after using the microscope

A simple microscope in normal use means that the image is formed at the near point as shown in Fig. 4.3 above. Here h is the length of the object viewed at near point (it means at D). The visual angles subtended are α (in radian) and α (in radian). The α is the increased angle when the simple microscope is used to view the object given in Fig 4.3 (b).

As you can see, magnified image is obtained which is erect and the distance of image is equal to D.

The angular magnification is maximum when the image is at the near point of the eye.

The angular magnification in terms of visual angle is

$$M = \frac{\alpha}{\alpha} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(4.1)$$

Now, the values of α and α can be obtained from Fig. 4.3 (a) and (b).

$\alpha = h/_D$ and $\alpha = {}^{h}/_D$

Therefore Eq. 4.1 becomes

$$M = \frac{h/_D}{h/_D} = \frac{h}{h}\left(\frac{v}{u}\right) \qquad \text{...........(4.2)}$$

Here u is the object distance and v is the image distance.

Since you know that

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

On multiplying the above Eq. by v and on rearranging the terms, we get

$$\frac{v}{u} - \frac{v}{f} = 1$$

Or

$$\frac{v}{u} = \left(\frac{D}{f} - 1\right)$$

$$\frac{h}{h} = \frac{v}{u} = \left(\frac{D}{f} - 1\right) \qquad \text{...........(4.3)}$$

Substituting Eq. (4.3) in Eq. (4.2), we get the expression

$$M = \left(\frac{D}{f} \quad 1\right)$$

Numerically, magnification can be written as

$$M = \left(\frac{D}{f} - 1\right) \qquad \text{...} \quad (4.4)$$

Eq. (4.4) gives the angular magnification of a microscope in normal use and the negative sign is an indication that the final image is virtual. Further, it can be seen that for higher angular magnification, a lens of short focal length is needed. You know that the eye has the tendency to focus on an image formed anywhere between the near point and infinity by a simple microscope. So, now you will study another case, when the image is formed at infinity.

3.1.2 Simple Microscope (with Image at Infinity)

You know that the eye has the tendency to focus on an image formed anywhere between the near point and infinity by a simple microscope.

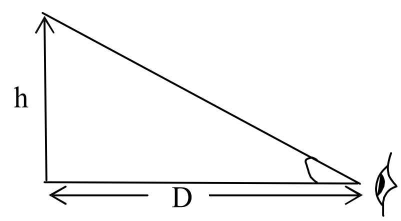
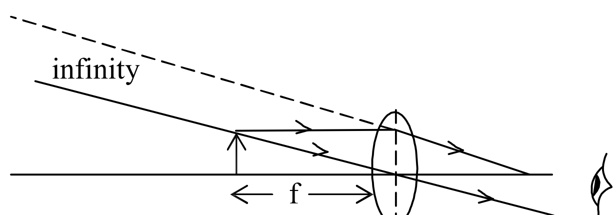


Fig. 4.4 (a)

A simple microscope is an instrument which is used to see very small objects. As discussed in the earlier sub-section (section 3.1.1) that, when it is in normal use, the image is formed at D (least distance of distinct vision)



(b)

Fig. 4.4: (a) Visual angle when the object is placed at the distance D, (b) Visual angle formed when the object is placed near the focal point.

A simple microscope with the image formed at infinity means that the eye must be accommodated to bring the image to infinity as shown in Fig. 4.4 above.

Where f is the focal length of a lens. The magnification M can be defined as

$$M = \frac{h/f}{h/d} = \frac{D}{f} M = \frac{D}{f} \qquad \ldots\ldots (4.5)$$

Eq. (4.5) gives an angular magnification with a microscope having a single lens. Magnification can be further increased by using one or two additional lenses. So, now we will discuss about the compound microscope.

3.1.3 Compound Microscope

A simple microscope in normal adjustment has its magnification numerically as

$$M = \frac{D}{f} - 1$$

A decrease in f implies an increase in angular magnification. But in practice, it is difficult to obtain a very small f. Therefore two lenses can be used to increase angular magnification. This two lens microscope is known as the compound microscope as shown in Fig. 4.4 below.

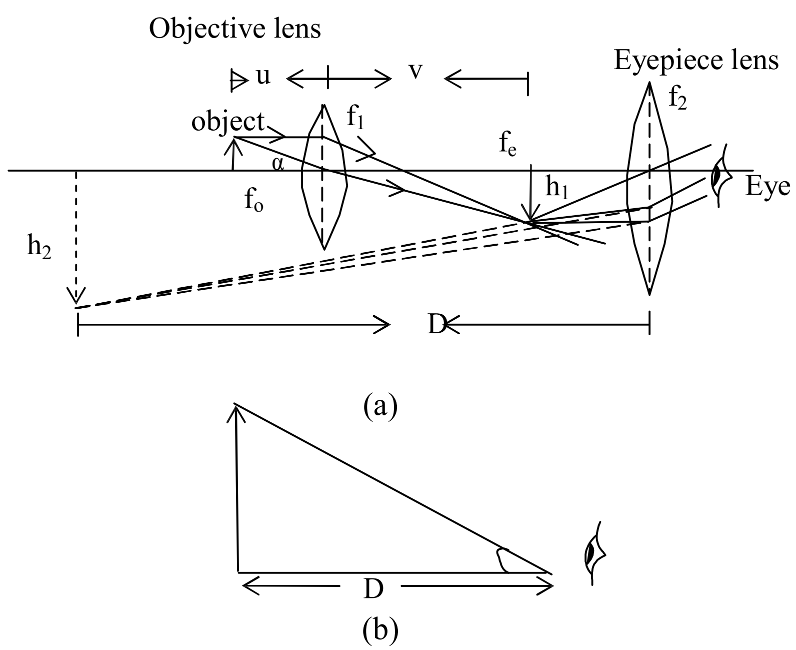


Fig. 4.4 (a) Compound microscope (b) compound microscope in normal use

A compound Microscope in normal use means that the final image is formed at the near point. The details of the image formation are discussed below.

The compound microscope essentially consists of two convex lenses of focal length f_1 and f_2 in which one of the lenses (of focal length f_1) is the objective lens and the second lens (of focal length $f_{2)}$ is the eye piece. The objective lens is placed near the object being viewed while the eyepiece is the lens near the eye as shown in Fig. 4.4(a). f_o is the focus of the objective lens and the f_e is the focus of the eyepiece. h_1 is the height of the image formed by the objective and finally we get image h_2 by an eyepiece.

The lenses are arranged such that their separation is less than $f_1 + f_2$. As such the image of the object formed by the objective lens is located from the second lens at a distance less than the focal length of the second lens. Thus the image of the first image formed by the second lens must be virtual and magnified. Consequently the final image formed is several times larger than the object to the observer.

Now, the formula of angular magnification for compound microscope is given by

$$M = \left(\frac{D}{f_2} - 1\right)\left(\frac{v}{f_1} - 1\right) \quad \text{..........} (4.6)$$

Therefore, from Eq. (4.6), it can be noted that M is large for small f_1 and f_2. It means that if the focal lengths of the objective lens and eyepiece lens are both small, angular magnification will be high.
After discussing about microscope, now you will learn about the telescopes in the next subsection. Telescopes are the instruments used to see distant objects or heavenly bodies like stars, planets etc.

3.1.4 Telescope

The angular magnification of a Telescope is defined as the ratio:

$$M = \frac{\alpha}{\alpha_1} \quad \text{..........} (4.7)$$

Where α is the visual angle subtended by the distance object at the unaided eye and α is the angle subtended at the eye by its final image when telescope is used.

There are different types of telescope one of which is the astronomical telescope. Now, we will discuss in the succeeding section first the Astronomical Telescope in normal Adjustment.

3.1.5 The Astronomical Telescope in Normal Adjustment

Astronomical telescope like compound microscope, consist of two lenses: objective and eyepiece. The objective is of large focal length and eyepiece is of short focal length closer to the image. f_1 is the focal length of the objective while f_2 is the focal length of eyepiece.

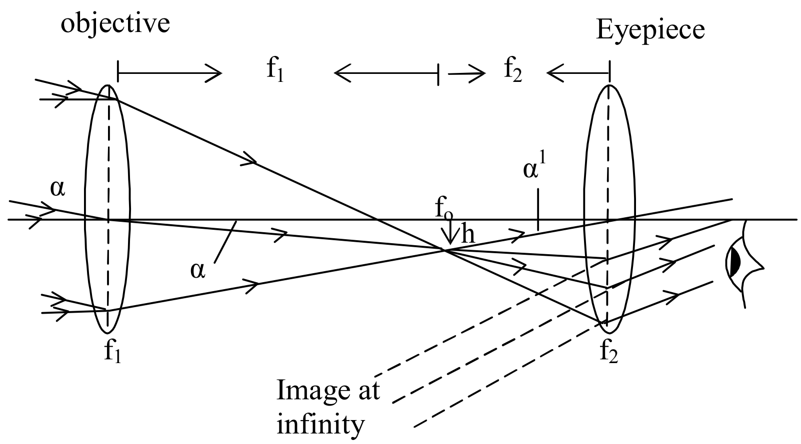


Fig. 4.5: An Astronomical Telescope in normal adjustment.

The parallel rays are collected by the objective lens O and an image h is formed. Final image is formed at infinity (∞).

In normal adjustment the two foci f_1 and f_2 coincides and it therefore implies that the distance between the two lenses is $f_1 + f_2$.

If α is the angle subtended by the unaided eye and α is the angle subtended by the aided eye. Then, since α and α are small the angular magnification of the telescope is

$$M = \frac{h/f_2}{h/f_1} = \frac{D}{f} \qquad \text{.......... (4.8)}$$

$$M = \frac{f_1}{f_2}$$

So from Eq. 4.8, the angular magnification is the ratio of focal length of objective to the focal length of eyepiece. For high angular magnification the eyepiece should have a small focal length and objective should have high focal length.

ACTIVITY 1

An astronomical telescope consists an objective of focal length 100 cm and an eyepiece of focal length 4 cm. Calculate the angular magnification of the telescope and also determine the distance between the two lenses.

4.0 CONCLUSION

The visual angle of an object dictates the size of the image on the retina. To increase the image size of an object therefore required increasing the visual angle of the object. This is usually done by means of optical instruments such as the microscope and the telescope.

Angular magnification is defined as,

$$M = \frac{\alpha}{\alpha_1}$$

where α is the angle subtended by the unaided eye and α is the angle subtended by the aided eye.

The Compound microscope consists of an object lens and an eyepiece of focal length f_1 and f_2 respectively. These lenses are separated at a distance slightly less than $f_1 + f_2$. The image formed by the objective lens serves as the objects for the eyepiece. As the object distance for the eyepiece is less than f_2, the image formed by the eye piece is virtual and enlarged and this is the image of the object seen by the eye. Consequently, the image is magnified.

Also the telescope essentially consists of two convex lenses of focal length f_1 and f_2. At normal adjustment the distance between two lenses is $f_1 + f_2$ and the image formed is at infinity. Thus the astronomical telescope is useful for viewing objects at infinity such as the moon and stars.

5.0 SUMMARY

magnitude of the image of an object formed on the retina is The determined by its visual angle.

Angular magnification is achieved by using optical instruments such as the microscope and telescope.

The microscope has an objective lens and an eye-piece, the two lenses are arranged in such a way that their separation is slightly less than $f_1 + f_2$.

In the microscope the image formed by the objective lens falls between the second lens and its near principal focus. As such, the final image formed by the eyepiece is virtual and enlarged. Consequently, the final image seen by the eye is much more magnified than the object.

The telescope also consists of an objective lens and eyepiece of focal lengths f_1 and f_2 respectively. At normal adjustment, the separation of the two lenses equals $f_1 + f_2$ and the final image formed is at infinity. Hence such telescope is good for viewing very distant (astronomical) objects at infinity.

6.0 TUTOR-MARKED ASSIGNMENT

Calculate the angular magnification of a magnifying glass of focal length 7 cm. Also, determine the object distance.

ANSWER TO ACTIVITY 1

Given $f_1 = 100$ cm , $f_2 = 4$ cm
The angular magnification of the telescope is (Refer Eq. 4.8)

$$M = \frac{f_1}{f_2}$$

$$= \frac{100}{4}$$

$$= 25\,cm$$

The distance between the two lenses = $f_1 + f_2$
= 100 + 4 = 104 cm

7.0 REFERENCES/FURTHER REDINGS

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

IGNOU (2005). *Electricity and Magnetism Physics* PHE-07, New Delhi, India.

Nelkon, M., & Parker, P. (1979). *Advance level physics*. London: Heinemann.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

MODULE 4 : SOUND WAVES AND APPLICATIONS

UNIT 1 NATURE OF SOUND WAVES

CONTENTS

1.0 INTRODUCTION

In earlier units, we have studied about light waves but in this unit we shall study about sound waves and particularly the standing wave; what is standing wave pattern and necessary problems related to standing wave..

2.0 OBJECTIVES

After studying this unit you will be able to:

1. define sound wave
2. explain nature of standing wave
3. obtain mathematical expression for first, second, third and fourth harmonics of standing wave.
4. Solve some problems on standing wave

How to Study this Unit:

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more

3.0 MAIN CONTENT

3.1 TRAVELING AND STANDING WAVES

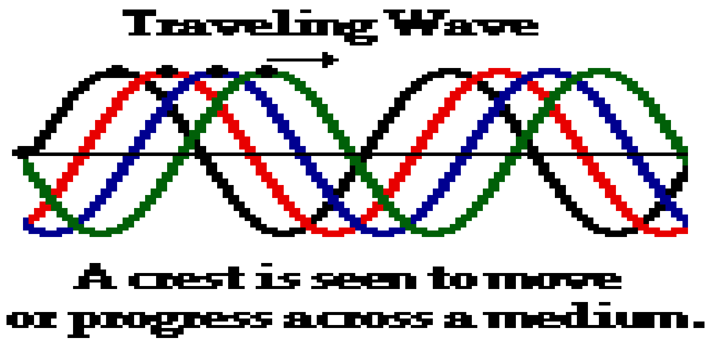


Figure 1.1 Travelling wave

A mechanical wave is a disturbance that is created by a vibrating object and subsequently travels through a medium from one location to another, transporting energy as it moves. The mechanism by which a mechanical wave propagates itself through a medium involves particle interaction; one particle applies a push or pull on its adjacent neighbor, causing a displacement of that neighbor from the equilibrium or rest position. As a wave is observed traveling through a medium, a crest is seen moving along from particle to particle. This crest is followed by a trough that is in turn followed by the next crest. In fact, one would observe a distinct wave pattern (in the form of a sine wave) traveling through the medium. This sine wave pattern continues to move in uninterrupted fashion until it encounters another wave along the medium or until it encounters a boundary with another medium. This type of wave pattern that is seen traveling through a medium is sometimes referred to as a **traveling wave**.

Traveling waves are observed when a wave is not confined to a given space along the medium. The most commonly observed traveling wave is an ocean wave. If a wave is introduced into an elastic cord with its ends held 3 meters apart, it becomes confined in a small region. Such a wave has only 3 meters along which to travel. The wave will quickly reach the end of the cord, reflect and travel back in the opposite direction. Any reflected portion of the wave will then interfere with the portion of the wave incident towards the fixed end. This interference produces a new shape in the medium that seldom resembles the shape of a sine wave. Subsequently, a traveling wave (a repeating pattern that is observed to move through a medium in uninterrupted fashion) is not observed in the cord. Indeed there are traveling waves in the cord; it is just that they are not easily detectable because of their interference with each other. In such instances, rather than observing the pure shape of a sine wave pattern, a rather irregular and non-repeating pattern is produced in the cord that tends to change appearance over time. This irregular looking shape is the result of the interference of an incident sine wave pattern with a reflected sine wave pattern in a rather non-sequenced and untimely manner. Both the incident and reflected wave patterns continue their motion through the medium, meeting up with one another at different locations in different ways. For

example, the middle of the cord might experience a crest meeting a *half crest*; then moments later, a crest meeting a *quarter trough*; then moments later, a *three-quarters crest* meeting a *one-fifth trough*, etc. This interference leads to a very irregular and non-repeating motion of the medium.
The appearance of an actual wave pattern is difficult to detect amidst the irregular motions of the individual particles.

3.1.1 What is a Standing Wave Pattern?

It is however possible to have a wave confined to a given space in a medium and still produce a regular wave pattern that is readily discernible amidst the motion of the medium. For instance, if an elastic rope is held end-to-end and vibrated at just the right frequency, a wave pattern would be produced that assumes the shape of a sine wave and is seen to change over time. The wave pattern is only produced when one end of the rope is vibrated at just the right frequency. When the proper frequency is used, the interference of the incident wave and the reflected wave occur in such a manner that there are specific points along the medium that appear to be standing still. Because the observed wave pattern is characterized by points that appear to be standing still, the pattern is often called a **standing wave pattern**. There are other points along the medium whose displacement changes over time, but in a regular manner. These points vibrate back and forth from a positive displacement to a negative displacement; the vibrations occur at regular time intervals such that the motion of the medium is regular and repeating. A pattern is readily observable.

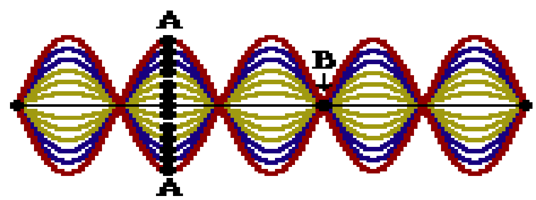


Figure 1.2 Standing wave pattern

Figure 1.2 depicts a standing wave pattern in a medium. A snapshot of the medium over time is depicted using various colours. Note that point A on the medium moves from a maximum positive to a maximum negative displacement over time. The diagram only shows one-half cycle of the motion of the standing wave pattern. The motion would continue and persist, with point A returning to the same maximum positive displacement and then continuing its back-and-forth vibration between the up to the down position. Note that point B on the medium is a point that never moves. Point B is a point of no displacement. Such points are known as **nodes** and will be discussed in more detail later in this lesson. The standing wave pattern that is shown at the right is just one of many different patterns that could be produced within the rope. Other patterns will be discussed later in the lesson.

3.1.2 Mathematics of Standing Waves

Standing wave patterns are wave patterns produced in a medium when two waves of identical frequencies interfere in such a manner to produce points along the medium that always appear to be standing still. Such standing wave patterns are produced within the medium when it is vibrated at certain frequencies. Each frequency is associated with a different standing wave pattern. These frequencies and their associated wave patterns are referred to as harmonics. A careful study of the standing wave patterns reveal a clear mathematical relationship between the wavelength of the wave that produces the pattern and the length of the medium in which the pattern is displayed. Furthermore, there is a predictability about this mathematical relationship that allows one to generalize and deduce a statement concerning this relationship. To illustrate, consider the first harmonic standing wave pattern for a vibrating rope as shown Figure 1.3.

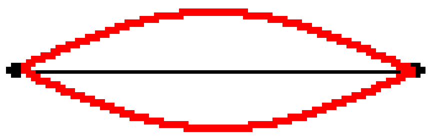


Figure 1.3: First harmonic of a standing wave

Analyzing the First Harmonic Pattern

The pattern for the first harmonic reveals a single antinode in the middle of the rope. This antinode position along the rope vibrates up and down from a maximum upward displacement from rest to a maximum downward displacement as shown. The vibration of the rope in this manner creates the appearance of a **loop** within the string. A complete wave in a pattern could be described as starting at the rest position, rising upward to a peak displacement, returning back down to a rest position, then descending to a peak downward displacement and finally returning back to the rest position. As shown in Figure 1.4, one complete wave in a standing wave pattern consists of two *loops*. Thus, one loop is equivalent to one-half of a wavelength.

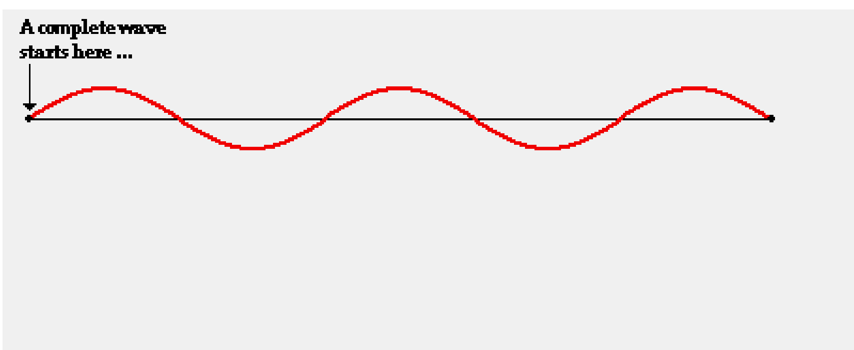


Figure 1.4 Complete wave

In comparing the standing wave pattern for the first harmonic with its single loop to the diagram of a complete wave, it is evident that there is only one-half of a wave stretching across the length of the string. That is, the length of the string is equal to one-half the length of a wave. Put in the form of an equation:

1st Harmonic: $L = \frac{1}{2}\lambda$

Analyzing the Second Harmonic Pattern

Now consider the string being vibrated with a frequency that establishes the standing wave pattern for the second harmonic.

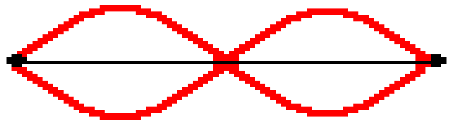


Figure 1.5 Second harmonic pattern

The second harmonic pattern consists of two anti-nodes. Thus, there are two loops within the length of the string. Since each loop is equivalent to one-half a wavelength, the length of the string is equal to two-halves of a wavelength. Put in the form of an equation:

2nd Harmonic: $L = \frac{2}{2}\lambda$

The same reasoning pattern can be applied to the case of the string being vibrated with a frequency that establishes the standing wave pattern for the third harmonic.

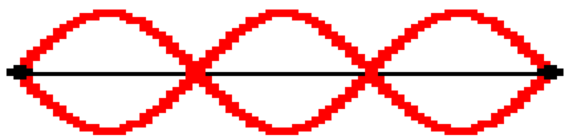


Figure 1.7 third harmonic pattern

Analyzing the Third Harmonic Pattern

The third harmonic pattern consists of three anti-nodes. Thus, there are three loops within the length of the string. Since each loop is equivalent to one-half a wavelength, the length of the string is equal to three-halves of a wavelength. Put in the form of an equation:

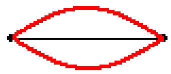
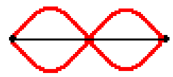
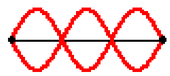
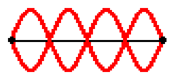
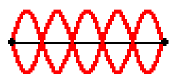
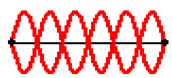
3rd Harmonic: $L = \frac{3}{2}\lambda$

When inspecting the standing wave patterns and the length-wavelength relationships for the first three harmonics, a clear pattern emerges. The number of antinodes in the pattern is equal to the **harmonic number** of that pattern. The first harmonic has one antinode; the second harmonic has two antinodes; and the third harmonic has three antinodes. Thus, it can be generalized that the **n**th harmonic has **n** antinodes where **n** is an integer representing the harmonic number. Furthermore, one notices that there are **n** halves wavelengths present within the length of the string. Put in the form of an equation:

n th Harmonic: $L = \frac{n}{2}\lambda$

Summarizing the Mathematical Relationships

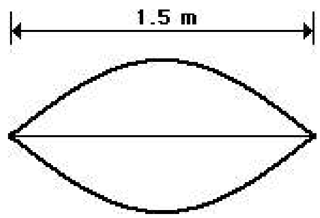
This information is summarized in the table below.

Harmonic	**Pattern**	**No. of Loops**	**Length-Wavelength Relationship**
1st		1	**L** = 1 / 2 • **λ**
2nd		2	**L** = 2 / 2 • **λ**
3rd		3	**L** = 3 / 2 • **λ**
4th		4	**L** = 4 / 2 • **λ**
5th		5	**L** = 5 / 2 • **λ**
6th		6	**L** = 6 / 2 • **λ**
Nth	--	N	**L** = n / 2 • **λ**

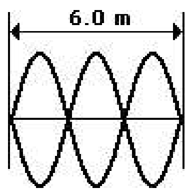
For standing wave patterns, there is a clear mathematical relationship between the length of a string and the wavelength of the wave that creates the pattern. The mathematical relationship simply emerges from the inspection of the pattern and the understanding that each loop in the pattern is equivalent to one-half of a wavelength. The general equation that describes this length-wavelength relationship for any harmonic is:

n th Harmonic: $L = \frac{n}{2}\lambda$

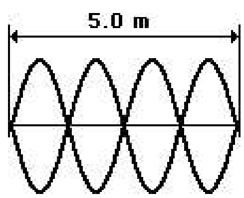
ACTIVITY 1

1. Suppose that a string is 1.2 meters long and vibrates in the first, second and third harmonic standing wave patterns. Determine the wavelength of the waves for each of the three patterns.
2. 

The string at the right is 1.5 meters long and is vibrating as the first harmonic. The string vibrates up and down with 33 complete vibrational cycles in 10 seconds. Determine the frequency, period, wavelength and speed for this wave.

3. 

The string at the right is 6.0 meters long and is vibrating as the third harmonic. The string vibrates up and down with 45 complete vibrational cycles in 10 seconds. Determine the frequency, period, wavelength and speed for this wave.

4. 

The string at the right is 5.0 meters long and is vibrating as the fourth harmonic. The string vibrates up and down with 48 complete vibrational cycles in 20 seconds. Determine the frequency, period, wavelength and speed for this wave.

4.0 CONCLUSION

A mechanical wave is a disturbance that is created by a vibrating object and subsequently travels through a medium from one location to another, transporting energy as it moves. This type of wave pattern that is seen traveling through a medium is sometimes referred to as a **traveling wave**. Because the observed wave pattern is characterized by points that appear to be standing still, the pattern is often called a **standing wave pattern**.

5.0 SUMMARY

Having discussed the nature of standing wave, its pattern and the mathematical relations, the first, second, third and nth harmonics are summarized thus:

1st Harmonic: $L = \frac{1}{2}\lambda$

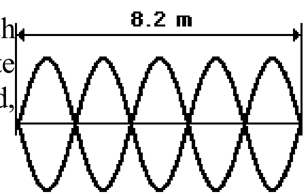
2nd Harmonic: $L = \frac{2}{2}\lambda$

3rd Harmonic: $L = \frac{3}{2}\lambda$

n th Harmonic: $L = \frac{n}{2}\lambda$

6.0 TUTOR-MARKED ASSIGNMENT

The string at the right is 8.2 meters long and is vibrating as the fifth harmonic. The string vibrates up and down with 21 complete vibrational cycles in 5 seconds. Determine the frequency, period, wavelength and speed for this wave.



ANSWER TO ACTIVITY 1

Question 1

First harmonic: **2.4 m** Second harmonic: **1.2 m** Third harmonic: **0.8 m**

For the first harmonic, the length of the string is equivalent to one-half of a wavelength. If the string is 1.2 meters long, then one-half of a wavelength is 1.2 meters long. The full wavelength is 2.4 meters long.

For the second harmonic, the length of the string is equivalent to a full wavelength. If the string is 1.2 meters long, then the wavelength is 1.2 meters long.

For the third harmonic, the length of the string is equivalent to three-halves of a wavelength. If the string is 1.2 meters long, then 1.5 wavelengths is 1.2 meters long. A single wavelength is less than 1.2 meters; it can be found by dividing 1.2 meters by 1.5. The wavelength of the third harmonic is 0.8 meters.

Question 2

Given: L = 1.5 m

33 cycles in 10 seconds

The frequency refers to how often a point on the medium undergoes back-and-forth vibrations; it is measured as the number of cycles per unit of time. In this case, it is

f = (33 cycles) / (10 seconds) = **3.3 Hz**

The period is the reciprocal of the frequency.

T = 1 / (3.3 Hz) = **0.303 seconds**

The wavelength of the wave is related to the length of the rope. For the first harmonic as pictured in this problem, the length of the rope is equivalent to one-half of a wavelength. That is, L = 0.5 • W where W is the wavelength. Rearranging the equation and substituting leads to the following results:

W = 2 • L = 2 • (1.5 m) = **3.0 m**

The speed of a wave can be calculated from its wavelength and frequency using the wave equation:

v = f • W = (3.3 Hz) • (3. 0 m) = **9.9 m/s**

Question 3

Given: L = 6.0 m

45 cycles in 10 seconds

The frequency refers to how often a point on the medium undergoes back-and-forth vibrations; it is measured as the number of cycles per unit of time. In this case, it is

f = (45 cycles) / (10 seconds) = **4.5 Hz**

The period is the reciprocal of the frequency.

T = 1 / (4.5 Hz) = **0.222 seconds**

The wavelength of the wave is related to the length of the rope. For the third harmonic as pictured in this problem, the length of the rope is equivalent to three-halves of a wavelength. That is, L = 1.5 • W where W is the wavelength. Rearranging the equation and substituting leads to the following results:

W = (2 / 3) • L = (2 / 3) • (6.0 m) = **4.0 m**

The speed of a wave can be calculated from its wavelength and frequency using the wave equation:

v = f • W = (4.5 Hz) • (4. 0 m) = **18 m/s**

Question 4

Given: L = 5.0 m

48 cycles in 20 seconds

The frequency refers to how often a point on the medium undergoes back-and-forth vibrations; it is measured as the number of cycles per unit of time. In this case, it is

f = (48 cycles) / (20 seconds) = **2.4 Hz**

The period is the reciprocal of the frequency.

T = 1 / (2.4 Hz) = **0.417 seconds**

The wavelength of the wave is related to the length of the rope. For the fourth harmonic as pictured in this problem, the length of the rope is equivalent to two full wavelengths. That is, L = 2 • W where W is the wavelength. Rearranging the equation and substituting leads to the following results:

W = 0.5 • L = 0.5 • (5.0 m) = **2.5 m**

The speed of a wave can be calculated from its wavelength and frequency using the wave equation:

v = f • W = (2.4 Hz) • (2.5 m) = **6.0 m/s**

7.0 REFERENCES

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.genesis.net.au/~ajs/projects/medical_physics/ultrasound/

http://www.wikiradiography.com/page/Basic+Ultrasound+Physics

http://en.wikipedia.org/wiki/Echo_sounding

http://www.physicsclassroom.com/class/sound/Lesson-5/Resonance

UNIT 2 MECHANICAL AND LONGITUDINAL WAVE NATURE OF SOUND

TABLE OF CONTENTS

1.0 INTRODUCTION

Sound is a form of wave and its transmission is unique. In the last unit we examined the nature of standing wave and its mathematical expressions. In this unit we shall examine standing wave as both mechanical and longitudinal waves. It is envisaged that such study will make learners conversant with the nature of sound as mechanical and longitudinal waves and therefore develop interest in the study of sound wave and its applications.

2.0 OBJECTIVES

It is expected that after going through this unit, you will be able to:

1. Explain sound as a mechanical wave
2. Explain sound as a longitudinal wave
3. Discuss the nature of standing wave on a string

.3.1 Sound as a Mechanical Wave

Sound and music are parts of our everyday sensory experience. Just as humans have eyes for the detection of light and colour, so we are equipped with ears for the detection of sound. We seldom take the time to ponder the characteristics and behaviours of sound and the mechanisms by which sounds are produced, propagated, and detected. The basis for an understanding of sound, music and hearing is the physics of waves. Sound is a wave that is created by vibrating objects and propagated through a medium from one location to another.

A wave can be described as a disturbance that travels through a medium, transporting energy from one location to another location. The medium is simply the material through which the disturbance is moving; it can be thought of as a series of interacting particles. The example of a slinky wave is often used to illustrate the nature of a wave. A disturbance is typically created within the slinky by the back and forth movement of the first coil of the slinky. The first coil becomes disturbed and begins to push or pull on the second coil. This push or pull on the second coil will displace the second coil from its equilibrium position. As the second coil becomes displaced, it begins to push or pull on the third coil; the push or pull on the third coil displaces it from its equilibrium position. As the third coil becomes displaced, it begins to push or pull on the fourth coil. This process continues in consecutive fashion, with each individual *particle* acting to displace the adjacent particle. Subsequently the disturbance travels through the slinky. As the disturbance moves from coil to coil, the energy that was originally introduced into the first coil is transported along the medium from one location to another.

A sound wave is similar in nature to a slinky wave for a variety of reasons. First, there is a medium that carries the disturbance from one location to another. Typically, this medium is air, though it could be any material such as water or steel. The medium is simply a series of interconnected and interacting particles. Second, there is an original source of the wave, some vibrating object capable of disturbing the first particle of the medium. The disturbance could be created by the vibrating vocal cords of a person, the vibrating string and soundboard of a guitar or violin, the vibrating tines of a tuning fork, or the vibrating diaphragm of a radio speaker. Third, the sound wave is transported from one location to another by means of particle-to-particle interaction. If the sound wave is moving through air, then as one air particle is displaced from its equilibrium position, it exerts a push or pull on its nearest neighbors, causing them to be displaced from their equilibrium position. This particle interaction continues throughout the entire medium, with each particle interacting and causing a disturbance of its nearest neighbors. Since a sound wave is a disturbance that is transported through a medium via the mechanism of particle-to-particle interaction, a sound wave is characterized as a mechanical wave.

3.1.1 Production and Propagation of Sound Waves

The creation and propagation of sound waves are often demonstrated in class through the use of a tuning fork. A tuning fork is a metal object consisting of two tines capable of vibrating if struck by a rubber hammer or mallet. As the tines of the tuning forks vibrate back and forth, they begin to disturb surrounding air molecules. These disturbances are passed on to adjacent air molecules by the mechanism of particle interaction. The motion of the disturbance, originating at the tines of the tuning fork and traveling through the medium (in this case, air) is what is referred to as a sound wave. The generation and propagation of a sound wave is demonstrated in the animation below.

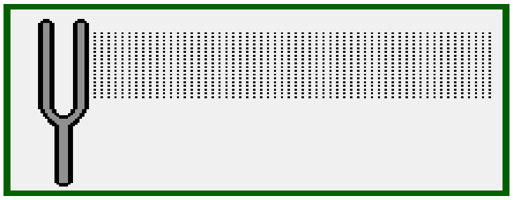


Figure 2. 1: Production Sound using Tuning Fork

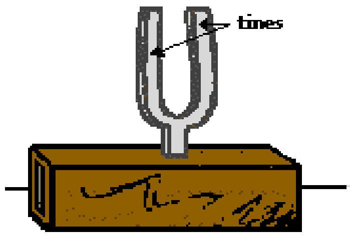


Figure 2.1: Propagation of Sound

Many Physics demonstration tuning forks are mounted on a sound box. In such instances, the vibrating tuning fork, being *connected* to the sound box, sets the sound box into vibrational motion. In turn, the sound box, being *connected* to the air inside of it, sets the air inside of the sound box into vibrational motion. As the tines of the tuning fork, the structure of the sound box, and the air inside of the sound box begin vibrating at the same frequency, a louder sound is produced. In fact, the more particles that can be made to vibrate, the louder or more amplified the sound. This concept is often demonstrated by the placement of a vibrating tuning fork against the glass panel of an overhead projector or on the wooden door of a cabinet. The vibrating tuning fork sets the glass panel or wood door into vibrational motion and results in an amplified sound.

We know that a tuning fork is vibrating because we hear the sound that is produced by its vibration. Nonetheless, we do not actually visibly detect any vibrations of the tines. This is because the tines are vibrating at a very high frequency. If the tuning fork that is being used corresponds to middle C on the piano keyboard, then the tines are vibrating at a frequency of 256 Hertz; that is, 256 vibrations per second. We are unable to visibly detect vibrations of such high frequency. A common physics demonstration involves *slowing down* the vibrations by through the use of a strobe light. If the strobe light puts out a flash of light at a frequency of 512 Hz (two times the frequency of the tuning fork), then the tuning fork can be observed to be moving in a back and forth motion. With the room darkened, the strobe would allow us to view the position of the tines two times during their vibrational cycle. Thus we would see the tines when they are displaced far to the left and again when they are displaced far to the right. This would be convincing proof that the tines of the tuning fork are indeed vibrating to produce sound.

Electromagnetic waves are waves that have an electric and magnetic nature and are capable of traveling through a vacuum. Electromagnetic waves do not require a medium in order to transport their energy. Mechanical waves are waves that require a medium in order to transport their energy from one location to another. Because mechanical waves rely on particle interaction in order to transport their energy, they cannot travel through regions of space that are void of particles. That is, mechanical waves cannot travel through a vacuum. This feature of mechanical waves is often demonstrated in a Physics class. A ringing bell is placed in a jar and air inside the jar is evacuated. Once air is removed from the jar, the sound of the ringing bell can no longer be heard. The clapper is seen striking the bell; but the sound that it produces cannot be heard because there are no particles inside of the jar to transport the disturbance through the vacuum. Sound is a mechanical wave and cannot travel through a vacuum.

3.2 Sound as a Longitudinal Wave

Sound is a mechanical wave that is created by a vibrating object. The vibrations of the object set particles in the surrounding medium in vibrational motion, thus transporting energy through the medium. For a sound wave traveling through air, the vibrations of the particles are best described as **longitudinal**. Longitudinal waves are waves in which the motion of the individual particles of the medium is in a direction that is parallel to the direction of energy transport. A longitudinal wave can be created in a slinky if the slinky is stretched out in a horizontal direction and the first coils of the slinky are vibrated horizontally. In such a case, each individual coil of the medium is set into vibrational motion in directions parallel to the direction that the energy is transported.

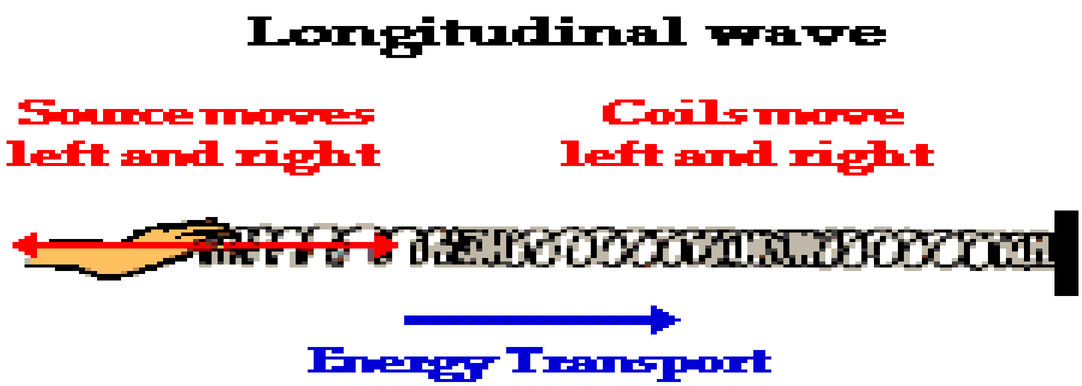


Figure 2.3: Longitudinal wave

Sound waves in air (and any fluid medium) are longitudinal waves because particles of the medium through which the sound is transported vibrate parallel to the direction that the sound wave moves. A vibrating string can create longitudinal waves as depicted in the animation below. As the vibrating string moves in the *forward* direction, it begins to push upon surrounding air molecules, moving them to the right towards their nearest neighbor. This causes the air molecules to the right of the string to be compressed into a small region of space. As the vibrating string moves in the reverse direction (leftward), it lowers the pressure of the air immediately to its right, thus causing air molecules to move back leftward. The lower pressure to the right of the string causes air molecules in that region immediately to the right of the string to expand into a large region of space. The back and forth vibration of the string causes individual air molecules (or a layer of air molecules) in the region immediately to the right of the string to continually vibrate back and forth horizontally. The molecules move rightward as the string moves rightward and then leftward as the string moves leftward. These back and forth vibrations are imparted to adjacent neighbors by particle-to-particle interaction. Other surrounding particles begin to move rightward and leftward, thus sending a wave to the right. Since air molecules (the particles of the medium) are moving in a direction that is parallel to the direction that the wave moves, the sound wave is referred to as a longitudinal wave. The result of such longitudinal vibrations is the creation of **compressions** and **rarefactions** within the air.

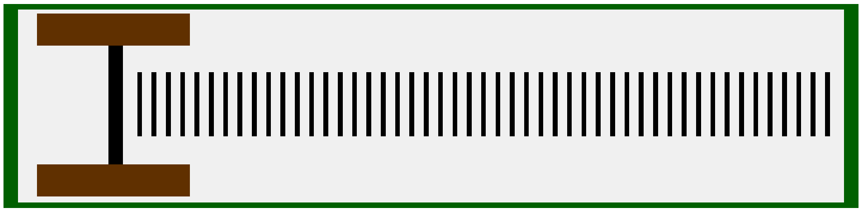


Figure 2.4: Movement or vibrating of wave particle

Regardless of the source of the sound wave - whether it is a vibrating string or the vibrating tines of a tuning fork - sound waves traveling through air are longitudinal waves. And the essential characteristic of a longitudinal wave that distinguishes it from other types of waves is that the particles of the medium move in a direction parallel to the direction of energy transport.

3.3 Standing Waves on a String

A standing wave pattern is a pattern which results from the interference of two or more waves along the same medium. All standing wave patterns are characterized by positions along the medium which are standing still. Such positions are referred to as nodal positions or *nodes*. Nodes occur at locations where two waves interfere such that one wave is displaced upward the same amount that a second wave is displaced downward. This form of interference is known as destructive interference and leads to a point of "no displacement." A node is a point of no displacement. Standing wave patterns are also characterized by antinodal positions - positions along the medium that vibrate back and forth between a maximum upward displacement to a maximum *downward* displacement. Antinodes are located at positions along the medium where the two interfering waves are always undergoing constructive interference. Standing wave patterns are always characterized by an alternating pattern of nodes and antinodes.

There are a variety of patterns which could be produced by vibrations within a string, slinky, or rope. Each pattern corresponds to vibrations which occur at a particular frequency and is known as a *harmonic*. The lowest possible frequency at which a string could vibrate to form a standing wave pattern is known as the fundamental frequency or the first harmonic. The second lowest frequency at which a string could vibrate is known as the second harmonic; the third lowest frequency is known as the third harmonic; and so on. An animation of a string vibrating with the fourth harmonic is shown below.

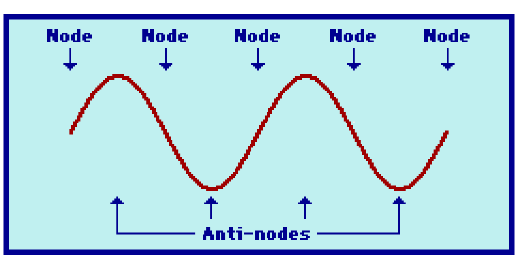


Figure 2.5: Standing wave characteristics

The frequency associated with each harmonic is dependent upon the speed at which waves move through the medium and the wavelength of the medium. The speed at which waves move through a medium is dependent upon the properties of the medium (tension of the string, thickness of the string, material composition of the string, etc.). The wavelength of the harmonic is dependent upon the length of the string and the harmonic number (first, second, third, etc.). Variations in either the properties of the medium or the length of the medium will result in variations in the frequency at which the string will vibrate.

ACTIVITY 1

Using tuning fork for example, explain how sound is produced and propagated

4.0 CONCLUSION

The study of sound as both mechanical and longitudinal waves has pointed clearly how energy and particle around the vibrating bodies are involved. In particular, the nature of standing wave on a string has helped to reveal the characteristics of a moving wave in terms of nodes, antinodes, and so on. Its pressure nature and applications will be examined in subsequent units.

5.0 SUMMARY

Sound is a wave that is created by vibrating objects and propagated through a medium from one location to another. A wave can be described as a disturbance that travels through a medium, transporting energy from one location to another location. The medium is simply the material through which the disturbance is moving; it can be thought of as a series of interacting particles. Mechanical waves are waves that require a medium in order to transport their energy from one location to another. Because mechanical waves rely on particle interaction in order to transport their energy, they cannot travel through regions of space that are void of particles.

. For a sound wave traveling through air, the vibrations of the particles are best described as **longitudinal**. Longitudinal waves are waves in which the motion of the individual particles of the medium is in a direction that is parallel to the direction of energy transport.

A standing wave pattern is a pattern which results from the interference of two or more waves along the same medium. All standing wave patterns are characterized by positions along the medium which are standing still. Such positions are referred to as nodal positions or *nodes*.

6.0 TUTOR-MARKED ASSIGNMENT

1. using a standing wave on a string as example, explain how nodes, antinodes and energy are associated.
2. Point out clearly the distinguishing attributes of sound as both mechanical and longitudinal waves.

ANSWER TO ACTIVITY 1

Refer to 3.1.1 for answer to activity 1

7.0 REFERENCES

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.genesis.net.au/~ajs/projects/medical_physics/ultrasound/

http://www.wikiradiography.com/page/Basic+Ultrasound+Physics

http://en.wikipedia.org/wiki/Echo_sounding

http://www.physicsclassroom.com/class/sound/Lesson-5/Resonance

UNIT 3: SOUND AS PRESSURE WAVE

TABLE OF CONTENTS

3.0 INTRODUCTION

Sound is not only a mechanical and longitudinal waves but it is also a pressure wave. As pressure wave, it possesses some unique characteristics such as compressions and rarefactions as well as interference. Their study and applications will enhance understanding and every day use of sound related waves.

4.0 OBJECTIVES

It is expected that after going through this unit, you will be able to:

1. Explain sound as pressure wave
2. Define compression and rarefaction
3. Explain how electromagnetic wave can be propagated
4. Explain wave interference

3.1 Sound is a Pressure Wave

Sound is a mechanical wave that results from the back and forth vibration of the particles of the medium through which the sound wave is moving. If a sound wave is moving from left to right through air, then particles of air will be displaced both rightward and leftward as the energy of the sound wave passes through it. The motion of the particles is parallel (and anti-parallel) to the direction of the energy transport. This is what characterizes sound waves in air as longitudinal waves.

3.1.1 Compressions and Rarefactions

A vibrating tuning fork is capable of creating such a longitudinal wave. As the tines of the fork vibrate back and forth, they push on neighboring air particles. The forward motion of a tine pushes air molecules horizontally to the right and the backward retraction of the tine creates a low-pressure area allowing the air particles to move back to the left.

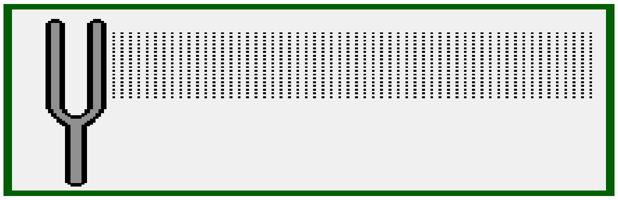


Figure 3.1 Compressions and Rarefactions

Because of the longitudinal motion of the air particles, there are regions in the air where the air particles are compressed together and other regions where the air particles are spread apart. These regions are known as **compressions** and **rarefactions** respectively. The compressions are regions of high air pressure while the rarefactions are regions of low air pressure. The diagram below depicts a sound wave created by a tuning fork and propagated through the air in an open tube. The compressions and rarefactions are labeled.

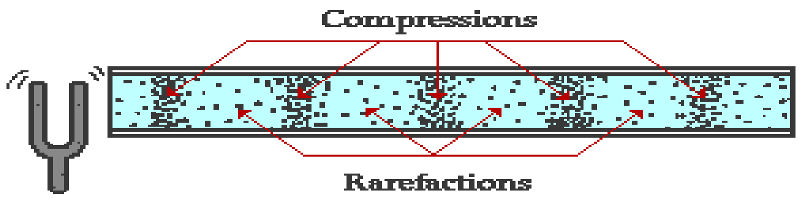


Figure 3.2: Compressions and Rarefactions

The **wavelength** of a wave is merely the distance that a disturbance travels along the medium in one complete wave cycle. Since a wave repeats its pattern once every wave cycle, the wavelength is sometimes referred to as the length of the repeating patterns - the length of one complete wave. For a transverse wave, this length is commonly measured from one wave crest to the next adjacent wave crest or from one wave trough to the next adjacent wave trough. Since a longitudinal wave does not contain crests and troughs, its wavelength must be measured differently. A longitudinal wave consists of a repeating pattern of compressions and rarefactions.

Thus, the wavelength is commonly measured as the distance from one compression to the next adjacent compression or the distance from one rarefaction to the next adjacent rarefaction.

3.1.2 What is a Pressure Wave?

Since a sound wave consists of a repeating pattern of high-pressure and low-pressure regions moving through a medium, it is sometimes referred to as a **pressure wave**. If a detector, whether it is the human ear or a man-made instrument, were used to detect a sound wave, it would detect fluctuations in pressure as the sound wave impinges upon the detecting device. At one instant in time, the detector would detect a high pressure; this would correspond to the arrival of a compression at the detector site. At the next instant in time, the detector might detect normal pressure. And then finally a low pressure would be detected, corresponding to the arrival of a rarefaction at the detector site. The fluctuations in pressure as detected by the detector occur at periodic and regular time intervals. In fact, a plot of pressure versus time would appear as a sine curve. The peak points of the sine curve correspond to compressions; the low points correspond to rarefactions; and the "zero points" correspond to the pressure that the air would have if there were no disturbance moving through it.

Sound waves traveling through air are indeed longitudinal waves with compressions and rarefactions. As sound passes through air (or any fluid medium), the particles of air do not vibrate in a transverse manner. Do not be misled - sound waves traveling through air are longitudinal waves.

ACTIVITY 1

1. A sound wave is a pressure wave; regions of high (compressions) and low pressure (rarefactions) are established as the result of the vibrations of the sound source. These compressions and rarefactions result because sound

a. is more dense than air and thus has more inertia, causing the bunching up of sound.

b. waves have a speed that is dependent only upon the properties of the medium.

c. is like all waves; it is able to bend into the regions of space behind obstacles.

d. is able to reflect off fixed ends and interfere with incident waves

e. vibrates longitudinally; the longitudinal movement of air produces pressure fluctuations.

3.2 Propagation of an Electromagnetic Wave

Electromagnetic waves are waves which can travel through the vacuum of outer space. Mechanical waves, unlike electromagnetic waves, require the presence of a material medium in order to transport their energy from one location to another. Sound waves are examples of mechanical waves while light waves are examples of electromagnetic waves.

Electromagnetic waves are created by the vibration of an electric charge. This vibration creates a wave which has both an electric and a magnetic component. An electromagnetic wave transports its energy through a vacuum at a speed of 3.00×10^8 m/s (a speed value commonly represented by the symbol **c**). The propagation of an electromagnetic wave through a material medium occurs at a net speed which is less than 3.00×10^8 m/s.

The mechanism of energy transport through a medium involves the absorption and reemission of the wave energy by the atoms of the material. When an electromagnetic wave impinges upon the atoms of a material, the energy of that wave is absorbed. The absorption of energy causes the electrons within the atoms to undergo vibrations. After a short period of vibrational motion, the vibrating electrons create a new electromagnetic wave with the same frequency as the first electromagnetic wave. While these vibrations occur for only a very short time, they delay the motion of the wave through the medium. Once the energy of the electromagnetic wave is reemitted by an atom, it travels through a small region of space between atoms. Once it reaches the next atom, the electromagnetic wave is absorbed, transformed into electron vibrations and then reemitted as an electromagnetic wave. While the electromagnetic wave will travel at a speed of c (3×10^8m/s) through the vacuum of interatomic space, the absorption and reemission process causes the net speed of the electromagnetic wave to be less than c. This is observed in Figure 3.4

.

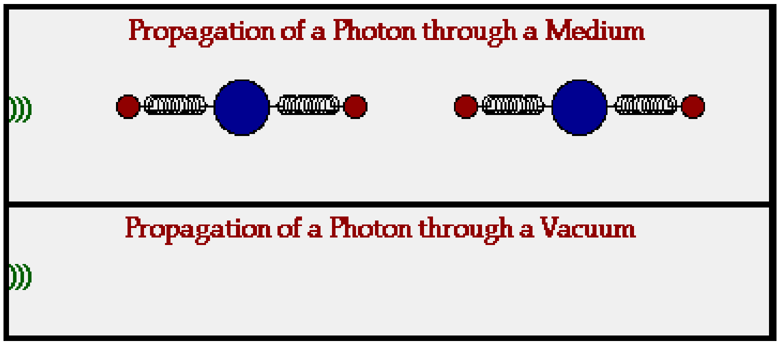


Figure 3.3: Electromagnetic wave

The actual speed of an electromagnetic wave through a material medium is dependent upon the optical density of that medium. Different materials cause a different amount of delay due to the absorption and reemission process. Furthermore, different materials have their atoms more closely packed and thus the amount of distance between atoms is less. These two factors are dependent upon the nature of the material through which the electromagnetic wave is traveling. As a result, the speed of an electromagnetic wave is dependent upon the material through which it is traveling.

3.3 Wave Interference

Wave interference is the phenomenon that occurs when two waves meet while traveling along the same medium. The interference of waves causes the medium to take on a shape that results from the net effect of the two individual waves upon the particles of the medium. If two upward displaced pulses having the same shape meet up with one another while traveling in opposite directions along a medium, the medium will take on the shape of an upward displaced pulse with twice the amplitude of the two interfering pulses. This type of interference is known as **constructive interference**. If an upward displaced pulse and a downward displaced pulse having the same shape meet up with one another while traveling in opposite directions along a medium, the two pulses will cancel each other's effect upon the displacement of the medium and the medium will assume the equilibrium position. This type of interference is known as **destructive interference**. The diagrams below show two waves - one is blue and the other is red - interfering in such a way to produce a resultant shape in a medium; the resultant is shown in green. In two cases (on the left and in the middle), constructive interference occurs and in the third case (on the far right, destructive interference occurs.

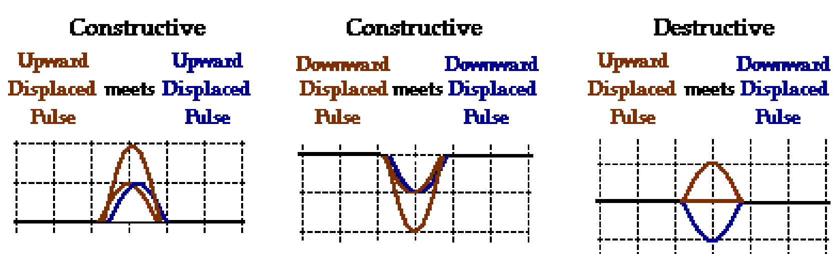


Figure 3.4: Constructive and destructive Interference

But how can sound waves that do not possess upward and downward displacements interfere constructively and destructively? Sound is a pressure wave that consists of compressions and rarefactions. As a compression passes through a section of a medium, it tends to pull particles together into a small region of space, thus creating a high-pressure region. And as a rarefaction passes through a section of a medium, it tends to push particles apart, thus creating a low-pressure region. The interference of sound waves causes the particles of the medium to behave in a manner that reflects the net effect of the two individual waves upon the particles. For example, if a compression (high pressure) of one wave meets up with a compression (high pressure) of a second wave at the same location in the medium, then the net effect is that that particular location will experience an even greater pressure. This is a form of constructive interference. If two rarefactions (two low-pressure disturbances) from two different sound waves meet up at the same location, then the net effect is that that particular location will experience an even lower pressure. This is also an example of constructive interference. Now if a particular location along the medium repeatedly experiences the interference of two compressions followed up by the interference of two rarefactions, then the two sound waves will continually *reinforce* each other and produce a very loud sound. The loudness of the sound is the result of the particles at that location of the medium undergoing oscillations from very high to very low pressures. As mentioned in a previous unit, locations along the medium where constructive interference continually occurs are known as **anti-nodes**. The animation below shows two sound waves interfering constructively in order to produce very large oscillations in pressure at a variety of anti-nodal locations. Note that compressions are labeled with a C and rarefactions are labeled with an R.

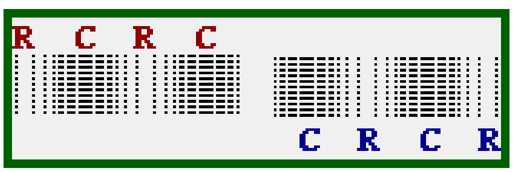


Figure 3.5 Compression and Rarefaction

Now if two sound waves interfere at a given location in such a way that the compression of one wave meets up with the rarefaction of a second wave, destructive interference results. The net effect of a compression (which pushes particles together) and a rarefaction (which pulls particles apart) upon the particles in a given region of the medium is to not even cause a displacement of the particles. The tendency of the compression to push particles together is canceled by the tendency of the rarefactions to pull particles apart; the particles would remain at their rest position as though there wasn't even a disturbance passing through them. This is a form of destructive interference. Now if a particular location along the medium repeatedly experiences the interference of a compression and rarefaction followed up by the interference of a rarefaction and a compression, then the two sound waves will continually *cancel* each other and no sound is heard. The absence of sound is the result of the particles remaining at rest and behaving as though there were no disturbance passing through it. Amazingly, in a situation such as this, two sound waves would combine to produce no sound. As mentioned in a previous unit, locations along the medium where destructive interference continually occurs are known as **nodes**.

4.0 CONCLUSION

Understanding of sound as pressure wave has great application in sound production, instruments and their maintenance. This unit has shown the relevance of sound and its propagation with reference to air pressure around it. This unit has introduced the learners to the basics of further studies in sound as you may come across in the next unit.

5.0 SUMMARY

Since a sound wave consists of a repeating pattern of high-pressure and low-pressure regions moving through a medium, it is sometimes referred to as a **pressure wave**. Because of the longitudinal motion of the air particles, there are regions in the air where the air particles are compressed together and other regions where the air particles are spread apart. These regions are known as **compressions** and **rarefactions** respectively. The compressions are regions of high air pressure while the rarefactions are regions of low air pressure.

The **wavelength** of a wave is merely the distance that a disturbance travels along the medium in one complete wave cycle. Since a wave repeats its pattern once every wave cycle, the wavelength is sometimes referred to as the length of the repeating patterns - the length of one complete wave.

Wave interference is the phenomenon that occurs when two waves meet while traveling along the same medium. The interference of waves causes the medium to take on a shape that results from the net effect of the two individual waves upon the particles of the medium. We have constructive and destructive interference patterns.

6.0 TUTOR-MARKED ASSIGNMENT

1. Discuss the concept of interference. What is the difference between constructive and destructive interference?

2. Using your knowledge of compressions and rarefactions, explain how sound is a pressure wave.

ANSWER TO ACTIVITY 1

Answer: **E**

Since the particles of the medium vibrate in a longitudinal fashion, compressions and rarefactions are created.

7.0 REFERENCES

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.genesis.net.au/~ajs/projects/medical_physics/ultrasound/

http://www.wikiradiography.com/page/Basic+Ultrasound+Physics

http://en.wikipedia.org/wiki/Echo_sounding

http://www.physicsclassroom.com/class/sound/Lesson-5/Resonance

UNIT 4: SOUND CHARACTERISTICS

TABLE OF CONTENTS

1. INTRODUCTION

Sound is a form of wave and its transmission is unique. Its study is wide; especially music. In such study, we have some terminology that should be understood clearly. They include: interference, beats and intervals and beat frequency. A clear understanding of these terms will open your door of successful and enjoyable study of this unit.

2. OBJECTIVES

It is expected that after going through this unit, you will be able to:

1. Explain sound interference
2. Define beats, intervals and beat frequencies
3. Identify and explain factors that affect natural frequency of musical instruments

3.1 Sound Interference

A popular Physics demonstration involves the interference of two sound waves from two speakers. The speakers are set approximately 1-meter apart and produced identical tones. The two sound waves traveled through the air in front of the speakers, spreading our through the room in spherical fashion. A snapshot in time of the appearance of these waves is shown in the diagram below. In the diagram, the compressions of a wavefront are represented by a thick line and the rarefactions are represented by thin lines. These two waves interfere in such a manner as to produce locations of some loud sounds and other locations of no sound. Of course the loud sounds are heard at locations where compressions meet compressions or rarefactions meet rarefactions and the "no sound" locations appear wherever the compressions of one of the waves meet the rarefactions of the other wave. If you were to plug one ear and turn the other ear towards the place of the speakers and then slowly walk across the room parallel to the plane of the speakers, then you would encounter an amazing phenomenon. You would alternatively hear loud sounds as you approached anti-nodal locations and virtually no sound as you approached nodal locations. (As would commonly be observed, the nodal locations are not true nodal locations due to reflections of sound waves off the walls. These reflections tend to fill the entire room with reflected sound. Even though the sound waves that reach the nodal locations directly from the speakers destructively interfere, other waves reflecting off the walls tend to reach that same location to produce a pressure disturbance.)

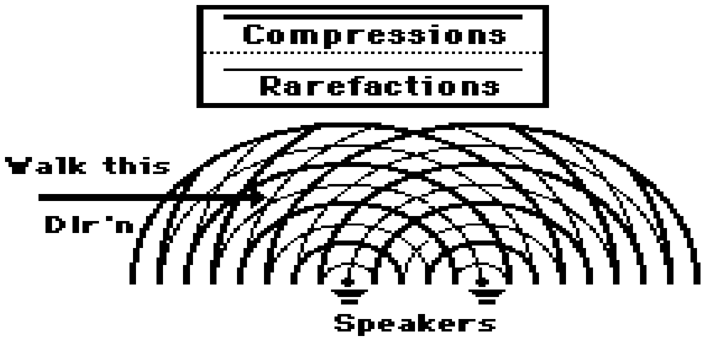


Figure 4.1 Wave Interference

Destructive interference of sound waves becomes an important issue in the design of concert halls and auditoriums. The rooms must be designed in such as way as to reduce the amount of destructive interference. Interference can occur as the result of sound from two speakers meeting at the same location as well as the result of sound from a speaker meeting with sound reflected off the walls and ceilings. If the sound arrives at a given location such that compressions meet rarefactions, then destructive interference will occur resulting in a reduction in the loudness of the sound at that location. One means of reducing the severity of destructive interference is by the design of walls, ceilings, and baffles that serve to absorb sound rather than reflect it.

The destructive interference of sound waves can also be used advantageously in **noise reduction systems**. Earphones have been produced that can be used by factory and construction workers to reduce the noise levels on their jobs. Such earphones capture sound from the environment and use computer technology to produce a second sound wave that one-half cycle *out of phase*. The combination of these two sound waves within the headset will result in destructive interference and thus reduce a worker's exposure to loud noise.

3.2 Beats and Intervals

Interference of sound waves has widespread applications in the world of music. Music seldom consists of sound waves of a single frequency played continuously. Few music enthusiasts would be impressed by an orchestra that played music consisting of the note with a pure tone played by all instruments in the orchestra. Hearing a sound wave of 256 Hz (middle C) would become rather monotonous (both literally and figuratively). Rather, instruments are known to produce overtones when played resulting in a sound that consists of a multiple of frequencies. Such instruments are described as being rich in tone color. And even the best choirs will *earn their money* when two singers sing two notes (i.e., produce two sound waves) that are an octave apart. Music is a mixture of sound waves that typically have whole number ratios between the frequencies associated with their notes. In fact, the major distinction between music and noise is that noise consists of a mixture of frequencies whose mathematical relationship to one another is not readily discernible. On the other hand, music consists of a mixture of frequencies that have a clear mathematical relationship between them. While it may be true that "one person's music is another person's noise" (e.g., your music might be thought of by your parents as being noise), a physical analysis of musical sounds reveals a mixture of sound waves that are mathematically related.

To demonstrate this nature of music, let's consider one of the simplest mixtures of two different sound waves - two sound waves with a 2:1 frequency ratio. This combination of waves is known as an octave. A simple sinusoidal plot of the wave pattern for two such waves is shown below. Note that the red wave has two times the frequency of the blue wave. Also observe that the interference of these two waves produces a resultant (in green) that has a periodic and repeating pattern. One might say that two sound waves that have a clear whole number ratio between their frequencies interfere to produce a wave with a regular and repeating pattern. The result is music.

Another simple example of two sound waves with a clear mathematical relationship between frequencies is shown below. Note that the red wave has three-halves the frequency of the blue wave. In the music world, such waves are said to be a fifth apart and represent a popular musical interval. Observe once more that the interference of these two waves produces a resultant (in green) that has a periodic and repeating pattern. It should be said again: two sound waves that have a clear whole number ratio between their frequencies interfere to produce a wave with a regular and repeating pattern; the result is music.

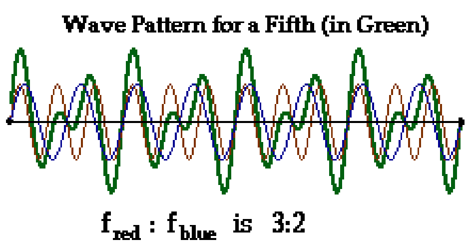


Figure 4.2 Wave Pattern

Finally, the diagram below illustrates the wave pattern produced by two dissonant or displeasing sounds. The diagram shows two waves interfering, but this time there is no *simple* mathematical relationship between their frequencies (in computer terms, one has a wavelength of 37 and the other has a wavelength 20 pixels). Observe (look carefully) that the pattern of the resultant is neither periodic nor repeating (at least not in the short sample of time that is shown). The message is clear: if two sound waves that have no simple mathematical relationship between their frequencies interfere to produce a wave, the result will be an irregular and non-repeating pattern. This tends to be displeasing to the ear.

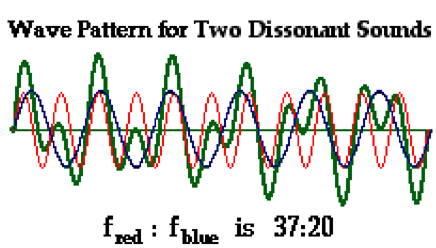


Figure 4.3 Wave Pattern for Two Dissonant sounds

A final application of physics to the world of music pertains to the topic of beats. **Beats** are the periodic and repeating fluctuations heard in the intensity of a sound when two sound waves of very similar frequencies interfere with one another. The diagram below illustrates the wave interference pattern resulting from two waves (drawn in red and blue) with very similar frequencies. A beat pattern is characterized by a wave whose amplitude is changing at a regular rate. Observe that the beat pattern (drawn in green) repeatedly oscillates from zero amplitude to a large amplitude, back to zero amplitude throughout the pattern. Points of constructive interference (C.I.) and destructive interference (D.I.) are labeled on the diagram. When constructive interference occurs between two crests or two troughs, a loud sound is heard. This corresponds to a peak on the beat pattern (drawn in green). When destructive interference between a crest and a trough occurs, no sound is heard; this corresponds to a point of no displacement on the beat pattern. Since there is a clear relationship between the amplitude and the loudness, this beat pattern would be consistent with a wave that varies in volume at a regular rate.

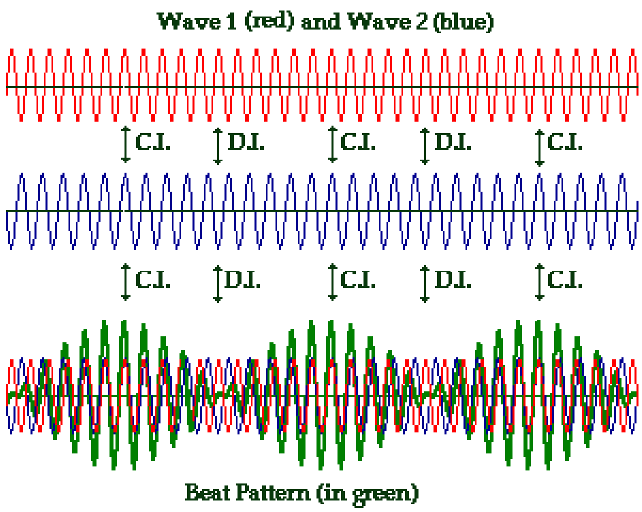


Figure 4.4 Beats Pattern

3.3 Beat Frequency

The **beat frequency** refers to the rate at which the volume is heard to be oscillating from high to low volume. For example, if two complete cycles of high and low volumes are heard every second, the beat frequency is 2 Hz. The beat frequency is always equal to the difference in frequency of the two notes that interfere to produce the beats. So if two sound waves with frequencies of 256 Hz and 254 Hz are played simultaneously, a beat frequency of 2 Hz will be detected. A common physics demonstration involves producing beats using two tuning forks with very similar frequencies. If a tine on one of two identical tuning forks is wrapped with a rubber band, then that tuning forks frequency will be lowered. If both tuning forks are vibrated together, then they produce sounds with slightly different frequencies. These sounds will interfere to produce detectable beats. The human ear is capable of detecting beats with frequencies of 7 Hz and below.

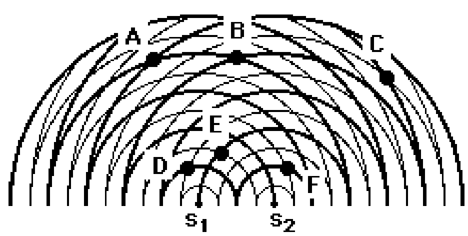
A piano tuner frequently utilizes the phenomenon of beats to tune a piano string. She will pluck the string and tap a tuning fork at the same time. If the two sound sources - the piano string and the tuning fork - produce detectable beats then their frequencies are not identical.

She will then adjust the tension of the piano string and repeat the process until the beats can no longer be heard. As the piano string becomes more in tune with the tuning fork, the beat frequency will be reduced and approach 0 Hz. When beats are no longer heard, the piano string is tuned to the tuning fork; that is, they play the same frequency. The process allows a piano tuner to match the strings' frequency to the frequency of a standardized set of tuning forks.

Important Note: Many of the diagrams in this unit represent a sound wave by a sine wave. Such a wave more closely resembles a transverse wave and may mislead people into thinking that sound is a transverse wave. Sound is not a transverse wave, but rather a longitudinal wave. Nonetheless, the variations in pressure with time take on the pattern of a sine wave and thus a sine wave is often used to represent the pressure-time features of a sound wave.

ACTIVITY 1

Two speakers are arranged so that sound waves with the same frequency are produced and radiate through the room. An interference pattern is created (as represented in the diagram at the right). The thick lines in the diagram represent wave crests and the thin lines represent wave troughs. *Use the diagram to answer the next two questions.*



1. At which of the labeled point(s) would constructive interference occur?

a. B only

b. A, B, and C

c. D, E, and F

d. A and B

2. How many of the six labeled points represent anti-nodes?

a. 1 b. 2 c. 3 d. 4 e. 6

3. A tuning fork with a frequency of 440 Hz is *played* simultaneously with a fork with a frequency of 437 Hz. How many beats will be heard over a period of 10 seconds?

4. Why don't we hear beats when different keys on the piano are played at the same time?

3.4 Factors Affecting the Natural Frequency

The actual frequency at which an object will vibrate at is determined by a variety of factors. Each of these factors will either affect the wavelength or the speed of the object. Since

frequency = speed/wavelength an alteration in either speed or wavelength will result in an alteration of the natural frequency. The role of a musician is to control these variables in order to produce a given frequency from the instrument that is being played. Consider a guitar as an example. There are six strings, each having a different linear density (the wider strings are more dense on a per meter basis), a different tension (which is controllable by the guitarist), and a different length (also controllable by the guitarist). The speed at which waves move through the strings is dependent upon the properties of the medium - in this case the tightness (tension) of the string and the linear density of the strings. Changes in these properties would affect the natural frequency of the particular string. The vibrating portion of a particular string can be *shortened* by pressing the string against one of the frets on the neck of the guitar. This modification in the length of the string would affect the wavelength of the wave and in turn the natural frequency at which a particular string vibrates at. Controlling the speed and the wavelength in this manner allows a guitarist to control the natural frequencies of the vibrating object (a string) and thus produce the intended musical sounds. The same principles can be applied to any string instrument - whether it is the harp, harpsichord, violin or guitar.

As another example, consider the trombone with its long cylindrical tube that is bent upon itself twice and ends in a flared end. The trombone is an example of a wind instrument. The *tube* of any wind instrument acts as a container for a vibrating air column. The air inside the tube will be set into vibration by a vibrating reed or the vibrations of a musician's lips against a mouthpiece. While the speed of sound waves within the air column is not alterable by the musician (they can only be altered by changes in room temperature), the length of the air column is. For a trombone, the length is altered by pushing the tube outward away from the mouthpiece to lengthen it or pulling it in to shorten it. This causes the length of the air column to be changed, and subsequently changes the wavelength of the waves it produces. And of course, a change in wavelength will result in a change in the frequency. So the natural frequency of a wind instrument such as the trombone is dependent upon the length of the air column of the instrument. The same principles can be applied to any similar instrument (tuba, flute, wind chime, organ pipe, clarinet, or pop bottle) whose sound is produced by vibrations of air within a *tube*.

The natural frequency of a trombone can be modified by changing the length of the air column inside the metal tube.

Figure 4.5: Example of a Natural Frequency from Trombone

There were a variety of classroom demonstrations (some of which are fun and some of which are corny) that illustrate the idea of natural frequencies and their modification. A pop bottle can be partly filled with water, leaving a volume of air inside that is capable of vibrating. When a person blows over the top of the bottle, the air inside is set into vibrational motion; turbulence above the lip of the bottle creates disturbances within the bottle. These vibrations result in a sound wave that is audible to students. Of course, the frequency can be modified by altering the volume of the air column (adding or removing water), which changes the wavelength and in turn the frequency. The principle is similar to the frequency-wavelength relation of air columns; a smaller volume of air inside the bottle means a shorter wavelength and a higher frequency.

A toilet paper roll orchestra can be created from different lengths of toilet paper rolls (or wrapping paper rolls). The rolls will vibrate with different frequencies when struck against a student's head. A properly selected set of rolls will result in the production of sounds that are capable of a Tony Award rendition of "Mary Had a Little Lamb."

Like a violin bowstring being pulled across a violin string, the finger sticks to the glass molecules, pulling them apart at a given point until the tension becomes so great. The finger then slips off the glass and subsequently finds another microscopic surface to *stick* to; the finger pulls the molecules at that surface, slips and then sticks at another location. This process of stick-slip friction occurring at a high frequency is sufficient to set the molecules of the glass into vibration at its natural frequency. Perhaps you have seen a pendulum bob vibrating back and forth about its equilibrium position. While a pendulum does not produce a sound when it oscillates, it does illustrate an important principle. A pendulum consisting of a longer string vibrates with a longer period and thus a lower frequency. Once more, there is an inverse relationship between the length of the vibrating object and the natural frequency at which the object vibrates. This very relationship carries over to any vibrating instrument - whether it is a guitar string, a xylophone, a pop bottle instrument, or a kettledrum.

All objects have a natural frequency or set of frequencies at which they vibrate when struck, plucked, strummed or somehow disturbed. The actual frequency is dependent upon the properties of the material the object is made of (this affects the speed of the wave) and the length of the material (this affects the wavelength of the wave). It is the goal of musicians to find instruments that possess the ability to vibrate with sets of frequencies that are musically sounding (i.e., mathematically related by simple whole number ratios) and to vary the lengths and (if possible) properties to create the desired sounds.

4.0 CONCLUSION

This unit discussed sound characteristics such as interference, beats and intervals, beat frequency and the destructive and constructive interference can both be used to advantage. It is an introductory study to music proper and therefore understanding of this unit could mean a lot in sound studies and applications and your progress in the next and last unit in this course.

5.0 SUMMARY

Sound is not a transverse wave, but rather a longitudinal wave. Nonetheless, the variations in pressure with time take on the pattern of a sine wave and thus a sine wave is often used to represent the pressure-time features of a sound wave.

The destructive interference of sound waves can also be used advantageously in **noise reduction systems.**

. **Beats** are the periodic and repeating fluctuations heard in the intensity of a sound when two sound waves of very similar frequencies interfere with one another. A beat pattern is characterized by a wave whose amplitude is changing at a regular rate. When constructive interference occurs between two crests or two troughs, a loud sound is heard. This corresponds to a peak on the beat pattern (drawn in green). When destructive interference between a crest and a trough occurs, no sound is heard; this corresponds to a point of no displacement on the beat pattern.

All objects have a natural frequency or set of frequencies at which they vibrate when struck, plucked, strummed or somehow disturbed. The actual frequency is dependent upon the properties of the material the object is made of (this affects the speed of the wave) and the length of the material (this affects the wavelength of the wave). It is the goal of musicians to find instruments that possess the ability to vibrate with sets of frequencies that are musically sounding (i.e., mathematically related by simple whole number ratios) and to vary the lengths and (if possible) properties to create the desired sounds.

6.0 TUTOR-MARKED ASSIGNMENT

1. Two musical notes that have a frequency ratio of 2:1 are said to be separated by an octave. A musical note that is separated by an octave from middle C (256 Hz) has a frequency of _____.

a. 128 Hz b. 254 Hz c. 258 Hz

d. 345 Hz e. none of these

3. Explain the basic difference between beat and interference.
4. With relevant examples explain factors that affect natural frequency of musical instruments.

ANSWER TO ACTIVITY 1

1. Answer: **D.** Both points A and B are on locations where a crest meets a crest.
2. Answer: **B.** Only points A and B are antinodes; the other points are points where crests and troughs meet.
3. Answer: **30 beats.** The beat frequency will be 3 Hz; thus in 10 seconds, there should be 30 beats.
4. Our ears can only detect beats if the two interfering sound waves have a difference in frequency of 7 Hz or less. No two keys on the piano are that similar in frequency.

7.0 REFERENCES

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.genesis.net.au/~ajs/projects/medical_physics/ultrasound/

http://www.wikiradiography.com/page/Basic+Ultrasound+Physics

http://en.wikipedia.org/wiki/Echo_sounding

http://www.physicsclassroom.com/class/sound/Lesson-5/Resonance

UNIT 5: SOUND AND ITS APPLICATIONS

TABLE OF CONTENTS

5.0 INTRODUCTION

Sound is a form of wave and its transmission is unique. Its study is wide; its daily and industrial applications are numerous. As we study sound and its applications, mention must be made of echoes and its advantages and disadvantages; echo sounding, ultrasound, properties of ultrasound, acoustic impedance and general applications of ultrasound. It is envisaged that such study will make learners conversant with life applications of what is learnt at the class room level and develop interest in areas to specialize in further studies.

6.0 OBJECTIVES

It is expected that after going through this unit, you will be able to:

Explain the relationship between sound and echo

1. List advantages and disadvantages of echoes
2. Explain what is mean by echo sounding
3. Define echo ranging
4. Discuss what is mean by ultrasound
5. Outline ultrasound and its properties
6. Discuss acoustic impedance interaction of ultrasound with tissues
7. Outline applications of ultrasound
8. Explain what is mean by a room acoustics.
9. Discuss the concept of resonance

3.0 MAIN CONTENTS

3.1 Echoes

Like all waves, sound waves can be reflected. Sound waves suffer reflection from the large obstacles. As a result of reflection of sound wave from a large obstacle, the sound is heard which is named as an echo. Ordinarily echo is not heard as the reflected sound gets merged with the original sound. Certain conditions have to be satisfied to hear an echo distinctly (as a separate sound).

The sensation of any sound persists in our ear for about 0.1 seconds. This is known as the persistence of hearing. If the echo is heard within this time interval, the original sound and its echo cannot be distinguished. So the most important condition for hearing an echo is that the reflected sound should reach the ear only after a lapse of at least 0.1 second after the original sound dies off. As the speed of sound is 340 m/s, the distance travelled by sound in 0.1 second is 34 m. This is twice the minimum distance between a source of sound and the reflector. So, if the obstacle is at a distance of 17 m at least, the reflected sound or the echo is heard after 0.1 second, distinctly.

Further, for reflection of any wave to take place, the size of the reflector should be large compared to the wavelength of the sound, which for ordinary sound is of the order of 1 metre. A large building, a mountain side, large rock formation etc. are good reflectors of sound for producing an echo. Also, for the reflected sound to be heard, it must have enough intensity or loudness. Moreover, if the echo is to be distinguished from the original sound the two should not mix or overlap. For this, the original sound should be of very short duration, like a clap or shout.

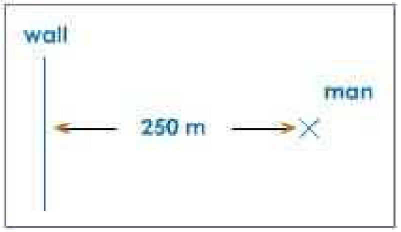
So, following conditions could be listed for formation of echo:

- The size of the obstacle/reflector must be large compared to the wavelength of the incident sound (for reflection of sound to take place).
- The distance between the source of sound and the reflector should be at least 17 m (so that the echo is heard distinctly after the original sound is over).
- The intensity or loudness of the sound should be sufficient for the reflected sound reaching the ear to be audible. The original sound should be of short duration.

Example 1:

A man stands 250 m from a wall and hears the echo of his gunshot after 1.5 s. Calculate the velocity of sound under the circumstances.

Suggested answer:



Distance travelled by sound = 2 x 250 m

= 500 m

Velocity of sound = $\frac{\text{distance travelled}}{\text{time taken}}$

$= \frac{500}{1.5}$

= 333.3 m/s

ACTIVITY 1

A man stands at a certain distance from a wall and sets up a simple pendulum which makes three vibrations per second. He claps his hands and hears the echo exactly after five complete vibrations of the pendulum. If the velocity of sound in air is 350 m/s, calculate the distance between the man and the wall.

ACTIVITY 2

Identify any three conditions that are necessary foe echo to take place.

3.2 Advantages and Disadvantages of Echoes

Echoes can be useful or a nuisance. In a concert hall, echoes can ruin a performance if the walls and ceiling are not properly designed. If the walls are too hard, or too flat, they make good reflecting surfaces for the sound waves.

Echoes can be used to give vital information. A sonar (Sonar stands for sound navigation ranging) device sends out high frequency sound waves from a ship to find out how close the vessel is to the sea bed. An ultrasound scanner, particularly known for giving images of the unborn baby, works in roughly the same way.

Bats use echoes to navigate as they fly in the night. This works on the same principle as sonar and ultrasound scanner. The bat sends out tiny, high pitched squeaks, which bounce off the objects in the bat's flight path. The echoes reach the bat, which then adjusts its course to avoid the obstacles. Many bats have very large ears to catch as much of the reflected sound as possible.

When animals such as bats and dolphins use echoes, it is called echo location. They use it to find their way around or to locate prey. Echo location describes the way of how some animals detect the size and position of objects around them.

Figure 5.1 Bat flying

At night, bats use echolocation to guide them in flight. They send out tiny 'clicks', which bounce off objects and return to the bat. It builds up a 'sound' picture of its surroundings

3.3 Echo Sounding

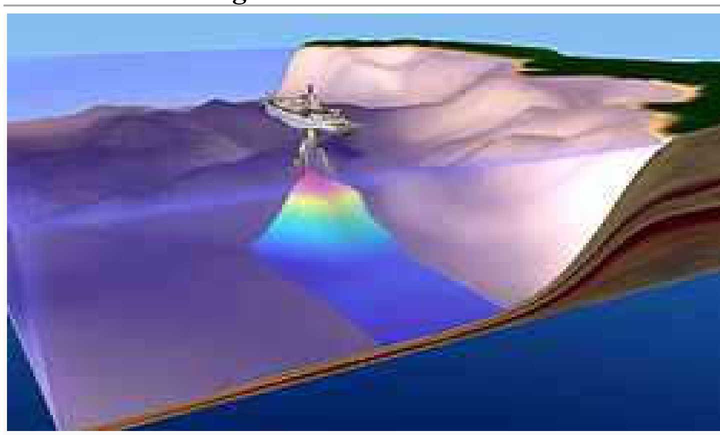


Figure 5.2: Illustration of echo sounding using a multibeam echosounder.

Echo sounding is a type of SONAR used to determine the depth of water by transmitting sound pulses into water. The time interval between emission and return of a pulse is recorded, which is used to determine the depth of water along with the speed of sound in water at the time. This information is then typically used for navigation purposes or in order to obtain depths for charting purposes. Echo sounding can also refer to hydroacoustic "echo sounders" defined as active sound in water (sonar) used to study fish. Hydroacoustic assessments have traditionally employed mobile surveys from boats to evaluate fish biomass and spatial distributions. Conversely, fixed-location techniques use stationary transducers to monitor passing fish.

The word *sounding* is used for all types of depth measurements, including those that don't use sound, and is unrelated in origin to the word *sound* in the sense of noise or tones. Echo sounding is a more rapid method of measuring depth than the previous technique of lowering a sounding line until it touched bottom.

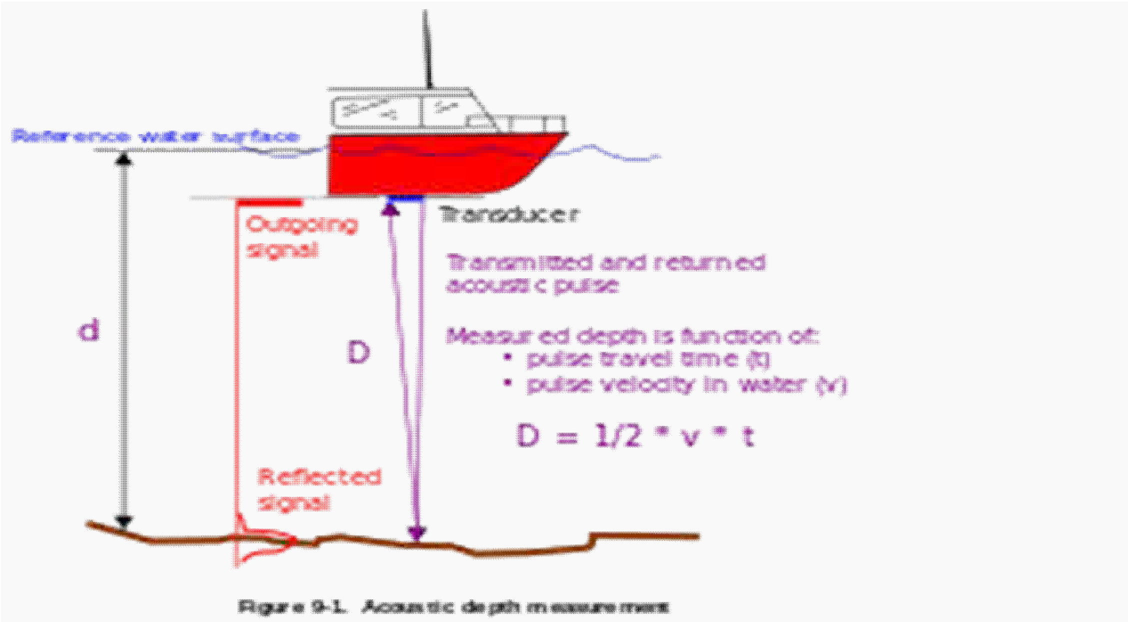


Figure 5.3: Diagram showing the basic principle of echo sounding

Distance is measured by multiplying half the time from the signal's outgoing pulse to its return by the speed of sound in the water, which is approximately 1.5 kilometres per second. For precise applications of echo sounding, such as hydrography, the speed of sound must also be measured typically by deploying a sound velocity probe into the water. Echo sounding is effectively a special purpose application of sonar used to locate the bottom. Since a traditional pre-SI unit of water depth was the fathom, an instrument used for determining water depth is sometimes called a *fathometer*.

Most charted ocean depths use an average or standard sound speed. Where greater accuracy is required average and even seasonal standards may be applied to ocean regions. For high accuracy depths, usually restricted to special purpose or scientific surveys, a sensor may be lowered to measure the temperature, pressure and salinity. These factors are used to calculate the actual sound speed in the local water column. This latter technique is regularly used by US Office of Coast Survey for navigational surveys of US coastal waters. As well as an aid to navigation (most larger vessels will have at least a simple depth sounder), echo sounding is commonly used for fishing. Variations in elevation often represent places where fish congregate. Schools of fish will also register. A fish finder is an echo sounding device used by both recreational and commercial fishers.

In areas where detailed bathymetry is required, a precise echo sounder may be used for the work of hydrography. There are many considerations when evaluating such a system, not limited to the vertical accuracy, resolution, acoustic beamwidth of the transmit/receive beam and the acoustic frequency of the transducer.

Figure 5.4: An example of a precision dual frequency echo sounder, the Teledyne Odom MkIII

The majority of hydrographic echo sounders are dual frequency, meaning that a low frequency pulse (typically around 24 kHz) can be transmitted at the same time as a high frequency pulse (typically around 200 kHz). As the two frequencies are discrete, the two return signals do not typically interfere with each other. There are many advantages of dual frequency echo sounding, including the ability to identify a vegetation layer or a layer of soft mud on top of a layer of rock.

Most hydrographic operations use a 200 kHz transducer, which is suitable for inshore work up to 100 metres in depth. Deeper water requires a lower frequency transducer as the acoustic signal of lower frequencies is less susceptible to attenuation in the water column. Commonly used frequencies for deep water sounding are 33 kHz and 24 kHz.

The beamwidth of the transducer is also a consideration for the hydrographer, as to obtain the best resolution of the data gathered a narrow beamwidth is preferable. This is especially important when sounding in deep water, as the resulting footprint of the acoustic pulse can be very large once it reaches a distant sea floor.

In addition to the single beam echo sounder, there are echo sounders that are capable of receiving many return "pings". These systems are detailed further in the section called multibeam echo sounder.

The required precision and accuracy of the hydrographic echo sounder is defined by the requirements of the International Hydrographic Organization (IHO) for surveys that are to be undertaken to IHO standards. These values are contained within IHO publication.

In order to meet these standards, the surveyor must consider not only the vertical and horizontal accuracy of the echo sounder and transducer, but the survey system as a whole. A motion sensor may be used, specifically the heave component (in single beam echo sounding) to reduce soundings for the motion of the vessel experienced on the water's surface. Once all of the uncertainties of each sensor are established, the hydrographer will create an uncertainty budget to determine whether the survey system meets the requirements laid down by IHO.

Different hydrographic organisations will have their own set of field procedures and manuals to guide their surveyors to meet the required standards.

3.4 Echo Ranging

Echo ranging is a technique used to determine the distance of an object from the transducer.
This technique relies on reflection.
The echo ranging equation is $z=ct$.
z = distance c = speed of ultrasound in tissue t = time

A sound beam is transmitted into a medium and is reflected back from an object. The elapsed time between the transmitted pulse and the received echo is converted into the total distance traveled. (The z value is only half of the distance traveled away and back to the transducer.)

Sound speed (c) is equal to one over the square root of the density times compressibility. OR c= square root compressibility / square root density. This indicates that if the density of a material is increased the speed of sound in that material will also be increased. Sound travels faster in media that are denser than air because of reduced compressibility. The velocity of ultrasound remains constant for a particular medium. Where c=frequency X wavelength. Indicating that for a constant velocity as the frequency is increased the wavelength is reduced. Ultrasonic waves are reflected at boundaries where there is a difference in acoustic impedance (Z) of the materials on each side of the boundary. This difference in Z is commonly referred to as the impedance mismatch. The greater the impedance mismatch, the greater the percentage of energy that will be reflected at the interface or boundary between one medium and another. The fraction of the incident wave intensity that is refracted can be derived because particle velocity and local particle pressures must be continuous across the boundary. When the acoustic impedances of the materials on both sides of the boundary are known, the fraction of the incident wave intensity that is reflected can be calculated with the equation below. The value produced is known as the reflection coefficient. Multiplying the reflection coefficient by 100 yields the amount of energy reflected as a percentage of the original energy. Reflection - sound is reflected at an interface regardless of the thickness of the material from which it is reflected. The reflection coefficient:

$$R = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

The equations for transmission and reflection of ultrasound intensity are independent of frequency for specula reflection. Therefore changing the transducer frequency does not alter the fraction of intensity transmitted/reflected at an interface. Transmission coefficient - %T = 4 Z2 Z1 / (Z2 + Z1) squared.

Nature of ultrasound

Ultrasound uses high frequency (above 20 kHz) mechanical vibrations or pressure waves that the human ear cannot detect. Typical diagnostic sonographic scanners operate in the frequency range of 2 to 18 megahertz, hundreds of times greater than the limit of human hearing. The choice of frequency is a trade-off between spatial resolution of the image and imaging depth: lower frequencies produce less resolution but image deeper into the body.

Superficial structures such as muscles, tendons, testes, breast and the neonatal brain are imaged at a higher frequency (7-18 MHz), which provides better axial and lateral resolution. Deeper structures such as liver and kidney are imaged at a lower frequency 1-6 MHz with lower axial and lateral resolution but greater penetration.

Wave parameters

PERIOD: time taken for one particle in the medium through which the wave travel, to make one complete oscillation (cycle)about its rest position, in response to the wave

FREQUENCY: the number of oscillations per second of the particle in the medium responding to the wave passing through it

WAVELENGTH: the distance between 2 consecutive, identical positions in the pressure wave

VELOCITY: the speed of propagation of a sound wave, determined by a combination af the density and compressibility of the medium through which it is propagating

PHASE: the stage at which a wave is within a cycle

AMPLITUDE: a measure of the degree of change within a medium, caused by the passage of a sound wave and relates to the severity of the disturbance POWER: rate of flow of energy through a given are

INTENSITY: the power per unit area

Properties of interfaces
An interface is the junction of two media with different acoustic properties.
It does not matter which impedance is the larger or smaller for two materials composing an interface; the difference between them squared is the same. This means that the same amount of reflection occurs at an interface going from high impedance to low impedance or vice versa

3.5 Ultrasound

Ultrasound refers to a cyclic sound pressure in the form of waves that has a frequency greater than the upper limit of human hearing. The highest human beings can detect is 20 thousand cycles per second (20,000 Hz).

3.5.1 Ultrasound and Properties

Defining Ultrasound: Ultrasound is ***high frequency mechanical vibrations*** or pressure waves above a frequency the human ear can hear.
Ultrasound uses sound waves:
Audible 20Hz and 20 000Hz.
I**nfra sound** Below 20Hz
Ultrasound Above 20 000Hz

Ultrasound uses a pulse-echo technique of imaging the body.
Pulses transmitted into patient and give rise to echoes when they encounter interfaces/reflectors.
These interfaces/reflectors are caused by variations in the **"acoustic impedance"** between different tissues.
Echo signals are amplified electronically and displayed on a monitor using shades of grey (from black to white),**stronger reflectors = brighter shades of grey and appear white in an image**. Those with **no echoes will appear black**, such as a full bladder.

Tissues with ***multiple interfaces*** are termed **echogenic**,
solid organs: spleen, liver and kidneys.

Structures with ***no internal interfaces*** are **hypoechoic** and return no echoes.
characteristics of liquid filled structures such as gall bladder and the urinary bladder.

Properties of an ultrasound wave

- Frequency higher than 20 000Hz (20kHz)
- Propagation of sound waves longitudinal, the mechanical displacement being in the

same direction as propagation.

- A medium is needed for sound waves to go through, no medium = no sound waves
- This propagation is whereby the particles of the medium which the sound is going through oscillate (move) back and forth from their original rest positions, in the same line as the wave. This is also known as Simple Harmonic Motion
- The motion of the these particles is caused by 2 factors; the pressure of the wave (which forces them to move in the beginning) and the forces of the restoring molecules (also known as the elasticity of the medium)
- The sound waves are transmitted as an alternation series of compressions (zones of high pressure) and rarefactions (zones of low pressure).
- The physical disturbance can be shown in a diagram (the dot diagram), and the individual movement of each particle in the diagram is/ can be described mathematically by the wave equation
- Also the amount of particle movement is dependent on the pressure change associated with the wave, therefore the increased pressure change equals the increased particle movement, and therefore louder sound

3.5.2 Interactions of ultrasound with soft tissues

When an ultrasound wave passes through tissues

- **Attenuation:** Reduction in amplitude and intensity of wave
- **Refraction:** Change in direction & velocity of wave

Attenuation is the rate at which intensity wave diminishes with the depth it covers or its penetration.

Types:

- Reflection.
- Scattering
- Absorption

Affecting attenuation:

Frequency of wave - Higher the frequency, higher the attenuation and less penetration of the wave

Type of tissue the wave is traveling

Depth the wave travels - more distance wave has to travel the more energy is lost.

Reflection

When a sound wave is incident on a interface between two tissues, part of it is reflected back into the original medium. The amount of energy reflected back depends on impedance.

The greater the difference in impedance between the tissues forming the interface the greater the amount of energy that is reflected back.

Impedance is a property of a tissue defined as density of tissue and velocity of sound in that tissue.

Denoted as Z = d (kg/m3) X c (m/s2)

Reflection co-efficient (R) is the ratio of the intensity of the reflected wave to the incident wave.

$$R = \frac{(Z2 - Z1)^2}{(Z2+Z1)^2}$$

The greater R, the greater the degree of reflection. i.e. R for a soft tissue interface such as liver and kidney is 0.01, i.e. only 1% of the sound is reflected. Muscle/bone interface 40% is reflected and for a soft tissue/air interface 99% is reflected.

This is the basis of ultrasound as different organs in the body have different densities and acoustic impedance and this creates different reflectors. In some cases the acoustic impedance can be so great that all the sound waves energy can be reflected, this happens when sound comes in contact with bone and air. This is the reason why ultrasound is not used as a primary imaging modality for bone, digestive tract and lungs.

Absorption

Absorption is the main form of attenuation. Absorption happens as sound travels through soft tissue, the particles that transmit the waves vibrate and cause friction and a loss of sound energy occurs and heat is produced. In soft tissue sound intensity decreases exponentially with depth.

Diffuse Scatter

Diffuse scatter when an objects size relative to the wave length becomes smaller. Imagine placing a thin stick upright in large rippling puddle or lake shore. The waves striking the stick will barely change course. The same applies to sound when the wave length of the sound waves are much larger than the object they are striking little or no sound waves will be reflected back. Because of this no strong reflections are seen as the sound, if reflected at all does not go directly back to the transducer. An example of this can be seen as speculation of an ultrasound image.

3.6 Acoustic impedance interactions of ultrasound with tissue

Acoustic Impedance (Z) is a measure of the resistance to sound passing through a medium. Z= density X speed of sound. It is similar to electrical resistance.

Unit - kg/m2/s called rayl

Materials with a higher density have increased c which therefore means the acoustic impedance is higher. eg bone.

Gases have low acoustic impedance.

Impedance mis-match - a difference in acoustic impedances cause some portion of the sound to be reflected at the interface. Therefore we want the acoustic impedance of two different materials to be as similar as possible to ensure that the beam is reflected at a specula reflector.

Whenever a sound wave encounters a material with a different density (acoustical impedance), part of the sound wave is reflected back to the probe and is detected as an echo. The greater the difference between acoustic impedance, the larger the echo is. If the pulse hits gases or solids, the density difference is so great that most of the acoustic energy is reflected and it becomes impossible to see deeper.

Huygen's Principle

A source of sound acts as if it is composed of an infinite number of point sources of sound. The waves from these point sources combine with each other to form a wavefront which then determines the direction of wave travel. The intensity at any point within the beam is determined by the sum of the contributions from all the point sources. Interferences occur between the waves from the point sources, leading to variation in intensity within the beam.

Attenuation

As an ultrasound beam passes through a medium, it loses energy and therefore undergoes a reduction in amplitude and intensity... this loss of energy is determined by the characteristics of the medium. Attenuation is frequency dependent thus dictating the limit of penetration of an ultrasound beam at a given frequency, resulting in the resolution/penetration trade-off in diagnostic sonography. Five processes responsible for attenuation of an ultrasound beam... reflection/scattering/absorption/refraction/beam divergence

Intensity

Is a physical parameter that describes the amount of energy flowing through a unit cross-sectional area of a beam each second or the rate at which the wave transmits the energy over a small area

unit - watt per square cm or joule per second per square cm.

- it can be used to describe the loudness of sound, in dB.

Amplitude

Indicates the strength of the detected echo or the voltage induced in a crystal by a pressure wave.

Is a measure of the degree of change within a medium, caused by the passage of a sound wave, relates to the severity of the disturbance

Distance measurement

distance (z) is equal to the product of speed of sound times the time of travel. $z = ct$

Echo ranging is used to determine the distance travelled by the sound beam. If the velocity of the ultrasound in the medium and the elapsed time from the original transmitted pulse are known echo ranging principle states that the distance to an interface can be determined.

3.7 Applications of Ultrasound

3. 7 .1 Night Club Isolation

The Structure
Concrete walls, concrete floor, concrete ceiling with suspended ceiling tiles 18" down. While the client noticed less-than-ideal sound in the club, the main problem he wished to combat was the structure borne transmission of sound to the apartment upstairs.

Ceiling
Roll out 6" unfaced insulation over the top of the suspended ceiling grid, then roll out a layer of SheetBlok over the top of the insulation (or at least back each ceiling tile with SheetBlok). Alternately, roll out 12" of insulation over the top of the suspended ceiling if it is determined that the ceiling cannot support the additional weight of SheetBlok even with reinforcement. Seal the juncture where the rolled out SheetBlok meets the structure by using the aforementioned tape.

Stage
Pull back the carpet and pad on the stage. Pull up the layer of plywood over the framing members (joists). Insulate between the joists with 6" of insulation to cut down the reflected sound under the stage. Line the bottoms of the joists with SheetBlok to isolate the stage from the structural concrete floor. Install a layer of SheetBlok on the floor of the stage itself, or at least a layer of ¾" MDF and then a layer of ¾" particle board, cross-seamed. Then lay the padding and carpet back down. If the pad is not rebound, replace it with this type or ComfortWear-200, which will offer 5-7dB of additional sound isolation. The stage should be kept as physically separate from the structure as possible. For maximum control, build new walls adjacent to the existing walls as outlined earlier or at least add additional layers of gypsum board to the existing walls with a layer of SheetBlok then a layer of 5/8" gypsum board. The club owner was unwilling to do either of these, so we recommended he apply 4" Studio foam, realizing that it would alleviate at least some of the low frequency sound that is offending the apartment upstairs.

3.7.2 Garage Isolation and Treatment

A one-car 13'x19' garage; carpeted floor; gypsum board walls; no windows; 1 36" solid-core door; acoustical tile ceiling at 8' height. The room is used to teach guitar and rehearse with guitar, bass, drums and drum machine.

The Problem
Excessive slap echo and reverb along with excessive low-end buildup due to drum kit being located in one corner. Owner not overly worried about sound transmission to/from the outside, but would like some additional transmission control.

:What to do
• Roll out unfaced insulation over the top of the suspended ceiling tiles, thus increasing transmission loss through the ceiling while adding low frequency control to the room.
• Treat all four vertical corners with LENRD Bass Traps.
• Treat the walls with 2" Studiofoam, preferably cut into 2'x2' panels and applied in a staggered checkerboard pattern with space between panels, easily adapted so no two parallel walls are mirror-images of each other. This method yields improved absorption and diffusion without costing any more money. Coverage minimum for a room of this size and with this

intended usage is 45%; 60-75% is more appropriate.

The customer originally thought he wanted to purchase Venus Bass Traps and 12" CornerFills for all four (4) wall/ceiling junctures, but we recommended LENRDs instead because of his room's size. We advised 2" Studiofoam for the walls instead of 4" because the slap echo and excessive reverb dictate more coverage, not thicker foam. If the budget allowed, 4" Studiofoam would be a welcome substitution.

Architectural Acoustics

From Wikipedia, the free encyclopedia

5.5: Symphony Hall, Birmingham, an example of the application of architectural acoustics.

Architectural acoustics (also known as room acoustics and **building acoustics**) is the science and engineering of achieving a good sound within a building and is a branch of acoustical engineering. The first application of modern scientific methods to architectural acoustics was carried out by Wallace Sabine in the Fogg Museum lecture room who then applied his new found knowledge to the design of Symphony Hall, Boston.

Architectural acoustics can be about achieving good speech intelligibility in a theatre, restaurant or railway station, enhancing the quality of music in a concert hall or recording studio, or suppressing noise to make offices and homes more productive and pleasant places to work and live in. Architectural acoustic design is usually done by acoustic consultants.

After determining the best dimensions of the room, using the modal density criteria, the next step is to find the correct reverberation time. The reverberation time depends on the use of the room. Times about 1.5 to 2 seconds are needed for opera theaters and concert halls. For broadcasting and recording studios and conference rooms, values under one second are frequently used. The recommended reverberation time is always a function of the volume of the room. Several authors give their recommendations. A good approximation for Broadcasting Studios and Conference Rooms is: TR[1 kHz] = [0,4 log (V+62)] – 0,38 TR in seconds and V=volume of the room in m^3. The ideal RT60 must have the same value at all frequencies from 30 to 12,000 Hz. Or, at least, it is acceptable to have a linear rising from 100% at 500 Hz to 150% down to 62 Hz

To get the desired RT60, several acoustics materials can be used as described in several books. A valuable simplification of the task was proposed by Oscar Bonello in 1979. It consists of using standard acoustic panels of 1 m^2 hung from the walls of the room (only if the panels are parallel). These panels use a combination of three Helmholtz resonators and a wooden resonant panel. This system gives a large acoustic absorption at low frequencies (under 500 Hz) and reduces at high frequencies to compensate for the typical absorption by people, lateral surfaces, ceilings, etc.

Reverberation, in terms of psychoacoustics, is the interpretation of the persistence of sound after a sound is produced. A reverberation, or **reverb**, is created when a sound or signal is reflected causing a large number of reflections to build up and then decay as the sound is absorbed by the surfaces of objects in the space – which could include furniture and people, and air. This is most noticeable when the sound source stops but the reflections continue, decreasing in amplitude, until they reach zero amplitude. Reverberation is frequency dependent. The length of the decay, or reverberation time, receives special consideration in the architectural design of spaces which need to have specific reverberation times to achieve optimum performance for their intended activity. In comparison to a distinct echo that is a minimum of 50 to 100 ms after the initial sound, reverberation is reflections that arrive in less than approximately 50ms. As time passes, the amplitude of the reflections is reduced until it is reduced to zero. Reverberation is not limited to indoor spaces as it exists in forests and other outdoor environments where reflection exists.

An **anechoic chamber** (an-echoic meaning non-reflective, non-echoing or echo-free) is a room designed to completely absorb reflections of either sound or electromagnetic waves. They are also insulated from exterior sources of noise. The combination of both aspects means they simulate a quiet open-space of infinite dimension, which is useful when exterior influences would otherwise give false results.

Anechoic chambers, a term coined by American acoustics expert Leo Beranek, were originally used in the context of acoustics (sound waves) to minimize the reflections of a room. More recently, rooms designed to reduce reflection and external noise in radio frequencies have been used to test antennas, radars, or electromagnetic interference.

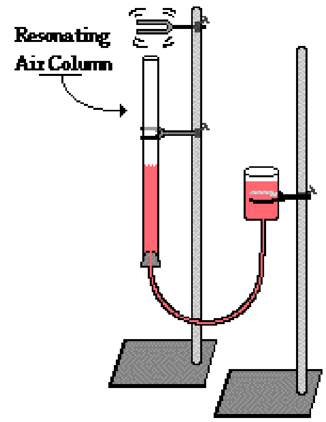
Anechoic chambers range from small compartments the size of household microwave ovens to ones as large as aircraft hangars. The size of the chamber depends on the size of the objects to be tested and the frequency range of the signals used, although scale models can sometimes be used by testing at shorter wavelengths (higher frequencies).

Resonance

Musical instruments are set into vibrational motion at their natural frequency when a person hits, strikes, strums, plucks or somehow disturbs the object. Each natural frequency of the object is associated with one of the many standing wave patterns by which that object could vibrate. The natural frequencies of a musical instrument are sometimes referred to as the **harmonics** of the instrument. An instrument can be forced into vibrating at one of its harmonics (with one of its standing wave patterns) if another *interconnected* object pushes it with one of those frequencies. This is known as **resonance** - when one object vibrating at the same natural frequency of a second object forces that second object into vibrational motion.

The word resonance comes from Latin and means to "resound" - to sound out together with a loud sound. Resonance is a common cause of sound production in musical instruments. One of our best models of resonance in a musical instrument is a resonance tube (a hollow cylindrical tube) partially filled with water and forced into vibration by a tuning fork.

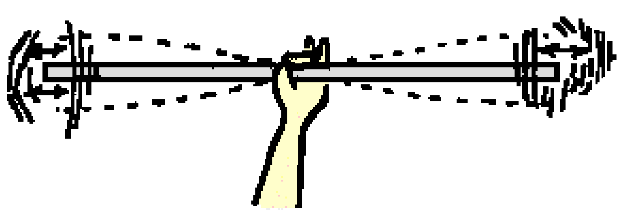
Figure 5.6: Resonance Tube



Tuning fork forcing air coumn into resonance

The tuning fork is the object that forced the air inside of the resonance tube into resonance. As the tines of the tuning fork vibrate at their own natural frequency, they created sound waves that impinge upon the opening of the resonance tube. These impinging sound waves produced by the tuning fork force air inside of the resonance tube to vibrate at the same frequency. Yet, in the absence of resonance, the sound of these vibrations is not loud enough to discern. Resonance only occurs when the first object is vibrating at the natural frequency of the second object. So if the frequency at which the tuning fork vibrates is not identical to one of the natural frequencies of the air column inside the resonance tube, resonance will not occur and the two objects will not sound out together with a loud sound. But the location of the water level can be altered by raising and lowering a reservoir of water, thus decreasing or increasing the length of the air column. As we have learned earlier, an increase in the length of a vibrational system (here, the air in the tube) increases the wavelength and decreases the natural frequency of that system. Conversely, a decrease in the length of a vibrational system decreases the wavelength and increases the natural frequency. So by raising and lowering the water level, the natural frequency of the air in the tube could be matched to the frequency at which the tuning fork vibrates. When the match is achieved, the tuning fork forces the air column inside of the resonance tube to vibrate at its own natural frequency and resonance is achieved. The result of resonance is always a big vibration - that is, a loud sound.

Another common physics demonstration that serves as an excellent model of resonance is the famous "singing rod" demonstration. A long hollow aluminum rod is held at its center. Being a trained musician, teacher reaches in a rosin bag to prepare for the event. Then with great enthusiasm, he/she slowly slides her hand across the length of the aluminum rod, causing it to sound out with a loud sound. This is an example of resonance. As the hand slides across the surface of the aluminum rod, slip-stick friction between the hand and the rod produces vibrations of the aluminum. The vibrations of the aluminum force the air column inside of the rod to vibrate at its natural frequency. The match between the vibrations of the air column and one of the natural frequencies of the singing rod causes resonance. The result of resonance is always a big vibration - that is, a loud sound.



A vibrating metal rod forces the air column inside into vibrations at the same frequency - resonance occurs.

Figure 5.7; Resonance Demondtration

The familiar *sound of the sea* that is heard when a seashell is placed up to your ear is also explained by resonance. Even in an apparently quiet room, there are sound waves with a range of frequencies. These sounds are mostly inaudible due to their low intensity. This so-called background noise fills the seashell, causing vibrations within the seashell. But the seashell has a set of natural frequencies at which it will vibrate. If one of the frequencies in the room forces air within the seashell to vibrate at its natural frequency, a resonance situation is created. And always, the result of resonance is a big vibration - that is, a loud sound. In fact, the sound is loud enough to hear. So the next time you hear the *sound of the sea* in a seashell, remember that all that you are hearing is the amplification of one of the many background frequencies in the room.

Resonance and Musical Instruments

Musical instruments produce their selected sounds in the same manner. Brass instruments typically consist of a mouthpiece attached to a long tube filled with air. The tube is often curled in order to reduce the size of the instrument. The metal tube merely serves as a container for a column of air. It is the vibrations of this column that produces the sounds that we hear. The length of the vibrating air column inside the tube can be adjusted either by sliding the tube to increase and decrease its length or by opening and closing holes located along the tube in order to control where the air enters and exits the tube. Brass instruments involve the blowing of air into a mouthpiece. The vibrations of the lips against the mouthpiece produce a range of frequencies. One of the frequencies in the range of frequencies matches one of the natural frequencies of the air column inside of the brass instrument. This forces the air inside of the column into resonance vibrations. The result of resonance is always a big vibration - that is, a loud sound.

Woodwind instruments operate in a similar manner. Only, the source of vibrations is not the lips of the musician against a mouthpiece, but rather the vibration of a reed or wooden strip. The operation of a woodwind instrument is often modeled in a Physics class using a plastic straw. The ends of the straw are cut with a scissors, forming a tapered *reed*. When air is blown through the reed, the reed vibrates producing turbulence with a range of vibrational frequencies. When the frequency of vibration of the reed matches the frequency of vibration of the air column in the straw, resonance occurs. And once more, the result of resonance is a big vibration - the reed and air column sound out together to produce a loud sound. As if this weren't silly enough, the length of the straw is typically shortened by cutting small pieces off its opposite end. As the straw (and the air column that it contained) is shortened, the wavelength decreases and the frequency was increases. Higher and higher pitches are observed as the straw is shortened. Woodwind instruments produce their sounds in a manner similar to the straw demonstration. A vibrating reed forces an air column to vibrate at one of its natural frequencies. Only for wind instruments, the length of the air column is controlled by opening and closing holes within the metal tube (since the tubes are a little difficult to cut and a too expensive to replace every time they are cut).

4.0 CONCLUSION

Echo, a variant of sound has a number of applications in daily life and in industries. While it is used in distance/depth measurement, position determination as we have for birds like bath, at the manufacturing level ultrasound equipments are useful in hospitals for scanning and so on. Architectures use echo phenomenon extensively to in designing room acoustics and in studios sound proofs rooms are made to restrict interference, reflection, etc.

5.0 SUMMARY

In this unit a lot of sound related applications both in daily life and in industries are discussed. Sound has applications mainly in the echo form for depth determination and sound enhancement in architectural halls. Ultrasound is used at industrial level to produce scan machines used extensively in hospitals to examine babies in fetus, bone fracture, tissue or lung cancer, and so on.

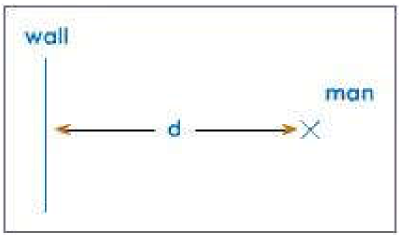
6.0 TUTOR-MARKED ASSIGNMENT

1. List any six properties of ultrasound

2. Explain the form of industrial application of sound in [a] sound enhancement in concert halls [b] baby scan in pregnant women

3. What is resonance? Explain a situation that resonance can be an advantage and another where it can be a disadvantage.

ANSWER TO ACTIVITY 1



3 vibrations - 1s

5 vibrations - $\frac{1}{3} \times 5 = \frac{5}{3}$s

distance = velocity x time

2d = 340 x $\frac{5}{3}$

$d = \frac{340}{2} \times \frac{5}{3}$

= 283.3 m

ANSWER TO ACTIVITY 2

Conditions necessary for echo to take place are:

1. The size of the obstacle/reflector must be large compared to the wavelength of the incident sound (for reflection of sound to take place).
2. The distance between the source of sound and the reflector should be at least 17 m (so that the echo is heard distinctly after the original sound is over).

The intensity or loudness of the sound should be sufficient for the reflected sound reaching the ear to be audible. The original sound should be of short duration

7.0 REFERENCES

Bueche, F . J. & Hecht, E. (2006). *College physics*. Schaum's Outline Series. New York: McGraw-Hill.

Gibbs, K (2011). *Advanced physics, 2nd ed*. Cambridge: Cambridge University Press.

Sears, F. W., Zemansky, M. W. & Young, H. D. (1975).*College Physics 4th* edition. Reading, U. K: Addison-Wesley Pub. Co. Inc.

Vuille, C., Serway, R. A. & Faughn, J. S. (2009). *College physics*, 8th ed. Belmont, USA: Brooks/Cole.

http://www.genesis.net.au/~ajs/projects/medical_physics/ultrasound/

http://www.wikiradiography.com/page/Basic+Ultrasound+Physics

http://en.wikipedia.org/wiki/Echo_sounding

http://www.physicsclassroom.com/class/sound/Lesson-5/Resonance