

**MODULE 1**

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| Unit 2 | Differential Calculus            |
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**1.0 INTRODUCTION**

Thermodynamics is a branch of physics that deals with heat and flow of energy. The basic idea is that objects are made up of atoms and molecules, which are in ceaseless motion. The faster the motion, the hotter the object. However, thermodynamics deals only with the large-scale response of a system, i.e. response that can be observed and measured, to heat flow. This unit examines the basic concepts of thermodynamics as a way of introducing the course.

## 2.0 A: OBJECTIVES

By the end of this unit, you should be able to:

- explain what thermodynamics is;
- mention thermodynamics coordinates and explain change of state;
- describe 3 thermodynamics processes;
- write equation of state and its usefulness; and
- differentiate between a system and its surrounding.

## 2.0 B: HOW TO STUDY THIS UNIT

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

## 3.0 MAIN CONTENT

### 3.1 Definition of Thermodynamics

Thermodynamics is the study of the effects of work, heat, and energy on a system. It deals only with the large-scale response of a system, which can be observed and measured in an experiment, of heat and work. Small-scale gas interactions are described by the kinetic theory of gases.

### 3.2 Idea of a System and its Surrounding

**System** is a restricted region of space or a finite portion of matter one has chosen to study. Or the part of the universe, with well-defined boundaries, one has chosen to study.

**Surrounding** is the rest of the universe outside the region of interest (i.e. the rest of space outside the system).

**Boundary or Wall** is the surface that divides the system from the surroundings. This wall or boundary may or may not allow interaction between the system and the surroundings.

### 3.3 Thermodynamic Properties/Coordinates

These are macroscopic coordinates or properties used to describe or characterise a system. Because they are macroscopic properties or coordinates, they can be observed and measured. Some examples are Temperature (**T**), Pressure (**P**), Volume (**V**), density ( $\rho$ ), mass (**m**), specific heat capacity at constant volume ( $C_V$ ), specific heat capacity at constant pressure ( $C_P$ ), thermal conductivity ( $k$ ), thermal diffusivity ( $\alpha$ ), and chemical potential ( $\mu$ ).

### 3.4 Thermodynamic System

This is a system that could be described in terms of thermodynamic coordinates or properties. Thermodynamic Systems can be categorised into the followings depending on the type of boundary:

**Open System:** This is a system that its boundary allows transfer of mass and energy into or out of the system. In other words, the boundary allows exchange of mass and energy between the system and the surrounding.

**Closed System:** This is a system that its boundary allows exchange of energy alone (inform of heat) between the system and its surrounding (i.e. the boundary allows exchange of energy alone). This type of boundary that allows exchange of heat is called diathermal boundary.

**Isolated System:** This is a system that its boundary allows neither mass nor energy between it and the surrounding. In other words, the boundary does not allow exchange of mass nor energy.

### 3.5 Thermodynamic Processes

A system undergoes a thermodynamic process when there is some sort of energetic change within the system, generally associated with changes in pressure, volume, internal energy, temperature, or any sort of heat transfer.

There are several specific types of thermodynamic processes that happen frequently enough (and in practical situations) that they are commonly treated in the study of thermodynamics. Each has a unique trait that identifies it, and which is useful in analyzing the energy and work change related to the process.

**Adiabatic process:** This is a thermodynamic process in which there is no heat transfer into or out of the system. For this process, change in quantity of heat is zero (i.e.  $\Delta Q = 0$  during this process)

**Isochoric process:** This is a thermodynamic process that occurs at constant volume (i.e.  $\Delta V = 0$  during this process). This implies that during this process no work is done on or by the system.

**Isobaric process:** This is a thermodynamic process that occurs at constant pressure (i.e.  $\Delta p = 0$  during this process).

**Isothermal process:** This is a thermodynamic process that takes place at constant temperature (i.e.  $\Delta T = 0$  during this process)

It is possible to have multiple processes within a single process. A good example would be a case where volume and pressure change during a process, resulting in no change in temperature and no heat transfer. This kind of a process would be both adiabatic and isothermal.

**Cyclic Processes:** These are series of processes in which after certain interchanges of heat and work, the system is restored to its initial state.

For a cyclic process  $\Delta U = 0$ , and if this is put into the first law (unit 5)

$$Q = W$$

This implies that the net work done during this process must be exactly equal to the net amount of energy transferred as heat; the store of internal energy of the system remains unchanged.

**Reversible Process:** A reversible process can be defined as one which direction can be reversed by an infinitesimal change in some properties of the system.

**Irreversible Process:** An irreversible process can be defined as one which direction cannot be reversed by an infinitesimal change in some properties of the system

**Quasi-static Process:** This is a process that is carried out in such a way that at every instant, the system departs only infinitesimal from an equilibrium state (i.e. almost static). Thus a quasi-static process closely approximates a succession of equilibrium states.

**Non-quasi-static Process:** This is a process that is carried out in such a way that at every instant, there is finite departure of the system from an equilibrium state.

## SELF ASSESSMENT EXERCISE 1

Explain the following:

- i. open system
- ii. isolated system
- iii. isochoric process
- iv. isobaric process
- v. quasi-static process

### 3.6 Thermodynamic Equilibrium

Generally, a system is said to be in equilibrium when its properties do not change appreciably with time over the interval of interest (i.e. observation time).

A system is said to be in thermodynamic equilibrium with its surrounding or with another system if and only if the system is in thermal equilibrium, in chemical equilibrium and in mechanical equilibrium with the surrounding or with another system. If any one of the above conditions is not fulfilled, the system is not in thermodynamic equilibrium.

#### 3.6.1 Mechanical Equilibrium

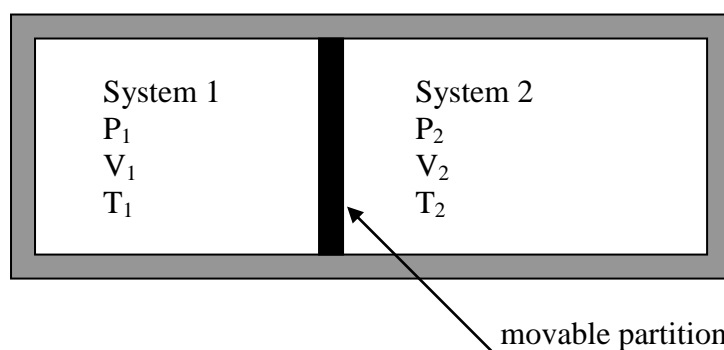
A system attains mechanical equilibrium with its surrounding or with another system when there is no unbalance or net force in the interior of the system and also none between the system and its surroundings or another system. Suppose two systems are separated by a movable boundary that does not allow exchange of mass nor heat as shown in figure 1.1. If  $P_1$  is greater than  $P_2$ , the partition will continue to move toward system 2 until  $P_1$  is equal to  $P_2$ . When this occurs, the two systems are said to be in mechanical equilibrium.

#### 3.6.2 Chemical Equilibrium

A system attains chemical equilibrium when there are no chemical reactions going on within the system or there is no transfer of matter from one part of the system to other due to diffusion. Two systems are said to be in chemical equilibrium with each other when their chemical potentials are same.

#### 3.6.3 Thermal Equilibrium

This occurs when two systems in thermal contact or a system that is in thermal contact with the surrounding attains the same temperature. For example if system 1 with temperature  $T_1$  and system 2 with temperature  $T_2$  are in thermal contact, there will be exchange of heat between the two systems if there is a temperature gradient (i.e. when  $T_1 \neq T_2$ ). This process of heat exchange will continue until thermal equilibrium is attained (i.e.  $T_1 = T_2$ ).



**Fig. 1.1: Two Systems Separated by a Movable Partition**  
**SELF ASSESSMENT EXERCISE 2**

Explain what is meant by thermodynamic equilibrium.

### 3.7 State of a System

This is a specific situation in which macroscopic properties (thermodynamic properties) of a system have certain values (e.g.  $P=10$  Pa,  $V=100$  cm<sup>3</sup>, and  $T=300$  K would be a state of a gas). It is important to note that the state of a pure substance or a system can be defined or specified by any two of its properties.

**Change of state** occurs when there is change in one, two or all the properties of the system. Using figure 1.1 above as example, suppose  $P_1$  is greater than  $P_2$  the partition will continue to move towards system 2 until  $P_1$  is equal to  $P_2$ . When this happens, the system 1 and 2 have a new set of coordinates in which Temperature remain constant for the two systems but pressure and volume changed. Then we say that the state of system 1 and system 2 has changed.

**Note:** Change of state is not exactly the same thing as change of phase (Phase change). Change of phase or phase transition is a special case of change of state and it will be treated fully in module 4, unit 1.

### 3.8 Equation of State

This is the known relationship between the thermodynamic variables or properties. It is an equation which provides a mathematical relationship between two or more state functions associated with matter such as its temperature, pressure, volume, or internal energy.

From the above descriptions, Boyle's law, Charles' law, Dalton's law of partial pressures are examples of equation of state. Some other examples of equation of state are:

### 3.8.1 The Ideal Gas

The equation of state for ideal gas is

$$PV = nRT$$

where **P** is the pressure, **V** is the volume, **R** is the molar gas constant ( $R=8.314 \text{ JK}^{-1}\text{mol}^{-1}$ ), **T** is temperature in Kelvin, and **n** is the number of mole of gas.

### 3.8.2 Van der Waals Equation of State

The equation of state for real gas also known as Van der Waals Equation is

$$\left(P + \frac{a}{V_m}\right)(V_m - b) = RT$$

where quantities **a** and **b** are constants for a particular gas but differ for different gases.

There are many more equations of state besides these two given above.

## 3.9 Extensive and Intensive Properties

Thermodynamic properties of a system can be categorised into two namely:

**Extensive properties:** These are properties of system that depend on the mass of the system (e.g. n, V and total energy U)

**Intensive properties:** These are properties of system that are independent of the mass of the system (e.g. T, P and  $\rho$ ).

#### **Block 1**

As an illustration of these two categories of thermodynamic properties, assuming you cut into two equal parts a hot bar of metal of uniform temperature T. Each half will still have almost the same temperature T. This clearly shows that temperature is independent of mass of the 'system' (i.e. metal bar). But what about the volume of each part, are they going to be the same as the original volume? This indicates that volume of a system is dependent of mass (volume is extensive property).

Specific Value of an extensive property (for example Volume, V) is defined as the ratio of the volume of the property to the mass of the system, or as volume per unit mass.

Specific volume  $V_s$  is

$$V_s = \frac{V}{m}$$

Note that the specific volume is evidently the reciprocal of the density  $\rho$ , defined as the mass per unit volume:

$$\rho = \frac{m}{V} = \frac{1}{V_s}$$

Molar Value of an extensive property (for example Volume,  $V$ ) is defined as the ratio of the volume of the property to the number of moles of the system, or as volume per unit mole.

Molar volume  $V_m$  is

$$V_m = \frac{V}{n}$$

### SELF ASSESSMENT EXERCISE 3

Differentiate between extensive and in-extensive properties of a system.

## 4.0 CONCLUSION

You have learnt in this unit, as way of introducing the course, the basic concepts of thermodynamics. This of course will help you in understanding the succeeding topics.

## 5.0 SUMMARY

In this unit you have learnt that:

- thermodynamics is a branch of physics that deals with heat and flow of energy
- thermodynamics coordinates are used to characterise the state of a system and that a change in any or all of these coordinates brings about a change of state
- based on the type of constraint imposed on a system, the system can undergo thermodynamics processes like adiabatic, isochoric and isothermal processes
- equations of state are the known relation between the thermodynamic coordinate of a system
- a system is a restricted region of space one has chosen to study while the rest of the universe is its surroundings. And that the system and its surrounding constitute the universe.

## 6.0 TUTOR-MARKED ASSIGNMENT

1. Define thermodynamics
2. Describe 2 thermodynamic processes
3. Differentiate between a system and its surrounding
4. List the thermodynamics coordinate and explain the change of state
5. Write out the equation of state and its usefulness



6. The equation for ideal gas is  $PV = nRT$ , suppose an ideal gas is in equilibrium at initial state with temperature  $T = 187\text{ }^\circ\text{C}$ , pressure  $P = 0.75 \times 10^7\text{ Nm}^{-2}$ , and volume  $V = 0.75\text{ m}^3$ . If there is a change of state in which the gas undergoes an isothermal process to a final state of equilibrium during which its volume doubled, Calculate the temperature and pressure of the gas at this final state.

## 7.0 REFERENCES/FURTHER READING

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## UNIT 2 DIFFERENTIAL CALCULUS

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### 1.0 INTRODUCTION

One of the useful mathematical topics in thermodynamics is partial derivative. This unit gives a brief summary of some of the most useful formulas involving partial derivatives that we are likely to use in subsequent units and modules of this course.

### 2.0 A: OBJECTIVES

By the end of this unit, you should be able to:

- explain what partial derivatives are;
- explain what exact differentials are;
- derive partial derivative of a given function;
- solve some problems on partial derivatives; and
- state some rules of partial derivatives.

### 2.0 B: HOW TO STUDY THIS UNIT

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.

4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

### 3.0 MAIN CONTENT

#### 3.1 Partial Derivatives

A partial derivative of a function of several variables is its derivative with respect to one of those variable with the others held constant.

$$x = x(z, y) \quad 2.1$$

From equation 2.1,  $x$  a dependent variable is a function of two independent variables  $z$  and  $y$ . Partial derivative of  $x$  with respect to  $y$  with  $z$  held constant is  $(\frac{\partial x}{\partial y})_z$ .

For example, if

$$x = zy^2 \quad 2.2$$

then, the partial derivative of  $x$  with respect to  $y$  with  $z$  held constant is

$$(\frac{\partial x}{\partial y})_z = 2zy \quad 2.3$$

Similarly, the partial derivative of  $x$  with respect to  $z$  with  $y$  held constant is

$$(\frac{\partial x}{\partial z})_y = y^2 \quad 2.4$$

#### 3.2 Exact Differential

Suppose that there exists a relation among the three coordinates  $x$ ,  $y$ , and  $z$  in such a way that  $x$  is a function of  $y$  and  $z$  (i.e.  $x(z, y)$ ); thus

$$f(x, y, z) = 0 \quad 2.5$$

The exact differential of  $x$  ( $dx$ ) is

$$dx = (\frac{\partial x}{\partial y})_z dy + (\frac{\partial x}{\partial z})_y dz \quad 2.6$$

Generally for any three variables  $x$ ,  $y$ , and  $z$  we have relation of the form

$$dx = M(y, z)dy + N(y, z)dz \quad 2.7$$

If the differential  $dx$  is exact, then

$$\left(\frac{\partial M}{\partial z}\right)_y = \left(\frac{\partial N}{\partial y}\right)_z \quad 2.8$$

### 3.3 Implicit Differential

Consider an equation of the form

$$xy = x^2 y^2 \quad 2.9$$

One can differentiate the two sides of the equation 2.9 using equation 2.6 (i.e. differentiating both the left and right hands side with respect to  $x$  while  $y$  is held constant and with respect to  $y$  while  $x$  is held constant).

$$\left(\frac{\partial(xy)}{\partial x}\right)_y dx + \left(\frac{\partial(xy)}{\partial y}\right)_x dy = \left(\frac{\partial(x^2 y^2)}{\partial x}\right)_y dx + \left(\frac{\partial(x^2 y^2)}{\partial y}\right)_x dy \quad 2.10$$

Equation 2.10 gives

$$ydx + xdy = 2xy^2 dx + 2x^2 y dy \quad 2.11$$

Collecting like term and then factorize to have

$$\frac{dy}{dx} = \frac{2xy^2 - y}{x - 2x^2 y} \quad 2.12$$

Another way to obtain expression for  $\frac{dy}{dx}$  is to consider equation 2.9 as  $f = xy - x^2 y^2$  (i.e. moving the expression in the right side of equation 2.9 to the left side and then equate the result to  $f$ ). Then

$$\frac{dy}{dx} = -\frac{\partial f}{\partial x} / \frac{\partial f}{\partial y} \quad 2.13$$

### 3.4 Product of Three Partial Derivatives

Suppose that there exists a relation among the three coordinates  $x$ ,  $y$ , and  $z$ ; thus

$$f(x, y, z) = 0 \quad 2.14$$

Then  $x$  can be imagined as a function of  $y$  and  $z$

$$dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz \quad 2.15$$

Also,  $y$  can be imagined as a function of  $x$  and  $z$ , and

$$dy = \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz \quad 2.16$$

Insert equation 2.16 in 2.15

$$dx = \left(\frac{\partial x}{\partial y}\right)_z \left[ \left(\frac{\partial y}{\partial x}\right)_z dx + \left(\frac{\partial y}{\partial z}\right)_x dz \right] + \left(\frac{\partial x}{\partial z}\right)_y dz$$

Rearrange to get

$$dx = \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z dx + \left[ \left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x + \left(\frac{\partial x}{\partial z}\right)_y \right] dz \quad 2.17$$

If  $dz=0$  and  $dx \neq 0$ , it follows that

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial x}\right)_z = 1$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z} \quad 2.18$$

Take note of the  $z$  in the expression on both sides of the equation 2.18, indicating that the derivatives in equation 2.18 are at constant  $z$ .

### Block 1

Suppose that there exists a relation among the three coordinates  $x$ ,  $y$ , and  $z$ ; thus  $f(x, y, z) = 0$ . Going by equation 2.18, then the following hold

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}, \quad \left(\frac{\partial z}{\partial x}\right)_y = \frac{1}{\left(\frac{\partial x}{\partial z}\right)_y}, \quad \text{and} \quad \left(\frac{\partial y}{\partial z}\right)_x = \frac{1}{\left(\frac{\partial z}{\partial y}\right)_x}$$

Also from equation 2.17, if  $dx = 0$  and  $dz \neq 0$ , it follows that

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x + \left(\frac{\partial x}{\partial z}\right)_y = 0$$

Move  $\left(\frac{\partial x}{\partial z}\right)_y$  to the other side of equation to get

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x = -\left(\frac{\partial x}{\partial z}\right)_y \quad 2.19$$

Then divide both sides of the equation 2.19 by  $(\frac{\partial z}{\partial x})_y$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1 \quad 2.20$$

This is called minus-one product rule.

### 3.5 Chain Rule of Partial Derivatives

Another useful relation is called chain rule of partial derivatives. Suppose  $\mathbf{T}$  is a function of  $\mathbf{V}$  and  $\mathbf{P}$ , and that each of  $\mathbf{V}$  and  $\mathbf{P}$  is a function of  $\mathbf{Z}$ , then

$$\left(\frac{\partial \mathbf{T}}{\partial \mathbf{V}}\right)_P = \left(\frac{\partial \mathbf{T}}{\partial \mathbf{Z}}\right)_P \left(\frac{\partial \mathbf{Z}}{\partial \mathbf{V}}\right)_P \quad 2.21$$

Equation 2.21 is the chain rule of partial derivative. Going by 1.2.21, the following can as well be written:

$$\left(\frac{\partial S}{\partial P}\right)_T = \left(\frac{\partial S}{\partial V}\right)_T \left(\frac{\partial V}{\partial P}\right)_T \quad 2.22a$$

$$\left(\frac{\partial U}{\partial V}\right)_P = \left(\frac{\partial U}{\partial T}\right)_P \left(\frac{\partial T}{\partial V}\right)_P \quad 2.22b$$

Equations 2.21 and 2.22 are called chain rule of partial derivatives.

### 3.6 Second Derivatives or Second Order Derivatives

Let  $f(x, y)$  be a function with continuous order derivatives, then we can calculate first derivatives to be  $(\frac{\partial f}{\partial x})_z$  and  $(\frac{\partial f}{\partial z})_x$ . One can further calculate the second

derivatives  $\frac{\partial^2 f}{\partial x^2}$ ,  $\frac{\partial^2 f}{\partial z^2}$ ,  $\frac{\partial^2 f}{\partial x \partial y}$ , and  $\frac{\partial^2 f}{\partial y \partial x}$ . Take note of these two second order derivatives

i.e.  $\frac{\partial^2 f}{\partial x \partial y}$  and  $\frac{\partial^2 f}{\partial y \partial x}$ , they are called mixed second derivatives. It can be shown that the

mixed second derivatives are equal, i.e. it does not matter the order will perform the differentiation.

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x} \quad 2.23$$

**SELF ASSESSMENT EXERCISE**

- i. For  $xy = x^2 y^2$ , by using equation 1.2.13 show  $\frac{dy}{dx} = \frac{2xy^2 - y}{x - 2x^2 y}$ .
- ii. Given that  $u = x^2 \ln y$  show that  $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x} = \frac{2x}{y}$ .

**3.7 Functions of More than Two Variables**

Suppose that  $f(x, y, z)$ , the derivative of  $f$  with respect to one of the variables with the other two constant (e.g. derivative of  $f$  w.r.t.  $x$  with  $y$  and  $z$  constant) can be written as

$$\left(\frac{\partial f}{\partial x}\right)_{yz}, \left(\frac{\partial f}{\partial y}\right)_{xz}, \text{ and } \left(\frac{\partial f}{\partial z}\right)_{xy}$$

**4.0 CONCLUSION**

You have learnt in this unit some rules in partial derivatives that you are likely to apply in this course.

**5.0 SUMMARY**

In this unit you have learnt that:

- the partial derivatives for three variables  $xyz$ , if  $x = zy^2$ , the partial derivative of  $x$  with respect to  $y$  with  $z$  held constant is  $\left(\frac{\partial x}{\partial y}\right)_z = 2zy$
- for  $x(z, y)$ ; thus  $f(x, y, z) = 0$ , the exact differential of  $x$  (i.e.  $dx$ ) is  $dx = \left(\frac{\partial x}{\partial y}\right)_z dy + \left(\frac{\partial x}{\partial z}\right)_y dz$
- for any three variables  $xyz$ ,  $\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$ .  
this is called minus-one product rule
- for  $T$  a function of  $V$  and  $P$ , and that each of  $V$  and  $P$  is a function of  $Z$ , then  $\left(\frac{\partial T}{\partial V}\right)_P = \left(\frac{\partial T}{\partial Z}\right)_P \left(\frac{\partial Z}{\partial V}\right)_P$
- for any  $f(x, y)$ ,  $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ .

$$\frac{dy}{dx} = \frac{2xy^2 - y}{x - 2x^2 y}$$

## 6.0 TUTOR-MARKED ASSIGNMENT

1. explain partial derivative and exact derivative
2. state some rules of partial derivative
3. If  $z = 4x^3y^2$ , show that  $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$
4. If  $u = x^2 + y^2$  and  $v = 4xy$ , determine  $\frac{\partial x}{\partial u}$ ,  $\frac{\partial x}{\partial v}$ ,  $\frac{\partial y}{\partial u}$ ,  $\frac{\partial y}{\partial v}$

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## UNIT 3 MEASUREMENT OF TEMPERATURE

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### 1.0 INTRODUCTION

Temperature is one of the seven SI base quantities and it is measure in Kelvin (K). Temperature measurement has its basis in the Zeroth law of thermodynamics. This unit deals with temperature measurements and different thermometers.

### 2.0 A: OBJECTIVES

By the end of this unit, you should be able to:

- explain the concept of heat and temperature;
- state the zeroth law of thermodynamics;
- define thermometric property and mention different thermometers; and
- do some calculations on temperature scales.

## 2.0 B: HOW TO STUDY THIS UNIT

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

## 3.0 MAIN CONTENT

### 3.1 Definition

Temperature ( $T$ ) is defined as the degree of hotness or coldness of a body. Temperature is one of the seven SI base quantities and it is measured in Kelvin (K). Temperature of a body has no upper limit but it has a lower limit (absolute zero or zero Kelvin).

Heat ( $Q$ ) is a form of energy that is transferred from one part of a substance to another, or from one body to another by virtue of a difference in temperature (i.e. temperature gradient). The unit of heat is in Joules (J).

#### Sign of heat ( $Q$ )

$Q$  is positive when there is a flow of heat into the system

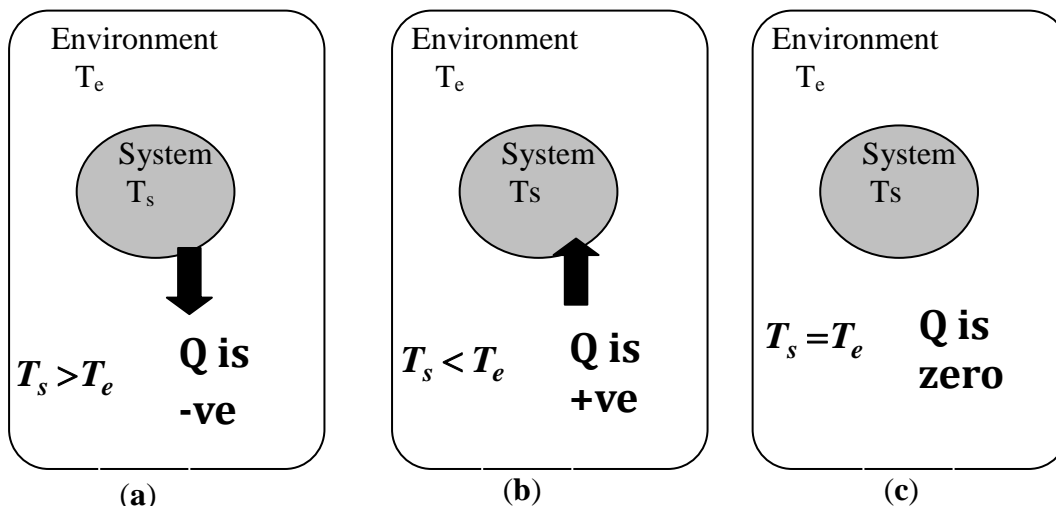
$Q$  is negative when there is a flow of heat out of the system

### 3.2 Temperature and Heat

Temperature is the degree of hotness or coldness of a body. Heat, on the other hand, is a form of energy that flows from a body of higher temperature to a body of lower temperature. From the definition of heat, it is clear that temperature gradient gives the direction of heat flow (i.e. from a body of higher temperature to a body of lower temperature). The process of heat flow is called heat exchange. For two bodies in thermal contact, this process of heat exchange will continue until the two bodies attain thermal equilibrium (i.e. equal temperature).

For example, in figure 3.1 (a) the temperature of the system is greater than that of the surroundings, so heat flows out of the system ( $-Q$ ). In (b) the temperature of the system  $T_s$  is less than that of the surroundings, so heat flows into the system ( $+Q$ ). In

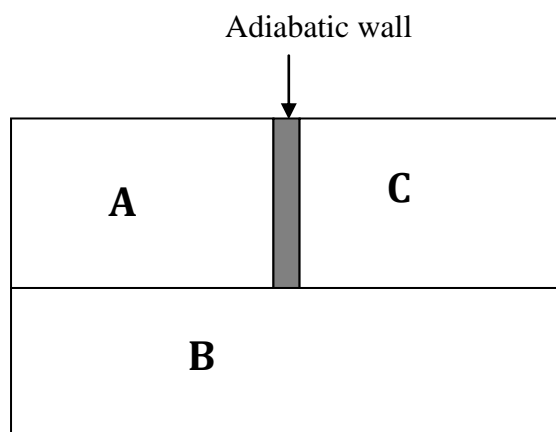
(d) the temperature of the system and that of the surrounding  $T_e$  are equal (i.e. they are in thermal equilibrium), hence no flow of heat ( $Q = 0$ ).



**Fig. 3.1: Heat Flow**

### 3.3 Zeroth Law of Thermodynamics

From the concept of heat and temperature discussed above in section 3.2, it is now clear that if two bodies are in thermal contact there will be heat exchange (provided they are not at the same temperature). This process of heat transfer will continue until the two bodies attain the same temperature. Then it is said that thermal equilibrium is reached. Suppose one of the two bodies is in thermal contact with a third body, after some time thermal equilibrium will be reached between these two bodies. Using figure 3.2 as an illustration, suppose body **B** is in thermal contact with bodies **A** and **C** but bodies **A** and **C** are not in thermal contact. If **B** is in thermal equilibrium with each of **A** and **C**, then **A** and **C** are in thermal equilibrium.



**Fig. 3.2: A Body in Thermal Contact with Two Separated Bodies**

**Zeroth Law of thermodynamics** states that if bodies **A** and **C** are each in thermal equilibrium with a third body **B**, then they are in thermal equilibrium with each other.

### 3.4 Temperature Scales

Available temperature scales are; the Celsius scale (also known as the Centigrade scale), the Fahrenheit scale, the Kelvin scale, the Rankine scale, and the international thermodynamic temperature scale

#### 3.4.1 Triple Point of Water

In setting up a temperature scale, there is need to pick some reproducible thermal phenomenon and, quite arbitrarily, assign a certain Kelvin temperature to its environment; that is, we select a standard fixed point and give it a standard fixed-point temperature. Triple point of water is the temperature at which solid ice, liquid water, and water vapour coexist in thermal equilibrium at the same temperature and pressure. By international convention, the triple point of water is 273.16 K and this value is a standard fixed-point temperature ( $T_3$ ) for the calibration of thermometers.

$$T_3 = 273.16 K \quad 3.1$$

Other fixed-point temperatures besides  $T_3$  are boiling point of water, and absolute zero temperature. The boiling point of water is 100 °C while the absolute zero temperature (0 K) is the all gases have zero volume.

#### 3.4.2 Conversion between Temperature Scales

There is possibility of changing from one temperature scale to another. The conversion formulas are listed below.

##### Kelvin Scale to Celsius Temperature Scale

The relation between Kelvin scale and Celsius scale is

$$T_c = (T - 273.15) ^\circ C , \quad 3.2$$

Where  $T_c$  is the temperature in degree centigrade, and  $T$  is the temperature in Kelvin.

##### Kelvin Temperature scale

The relation between Celsius scale and Kelvin scale is

$$T = (T_c + 273.15) K , \quad 3.3$$

where  $T_c$  is the temperature in degree centigrade, and  $T$  is the temperature in Kelvin.

##### Fahrenheit Temperature scale

The relation between Celsius scale and Fahrenheit scale is

$$T_F = \frac{9}{5} T_c - 32 , \quad 3.4$$

where  $T_c$  is the temperature in degree centigrade, and  $T_F$  is the temperature in Fahrenheit.

### 3.5 Thermometers

Thermometers are instruments used to measure the temperature of a body or a system.

#### 3.5.1 Thermometric Properties

It has been observed experimentally that the properties of many bodies or objects change with temperature. Examples of such properties are volume of liquid, length of metal rod, and electrical resistance of a wire. These properties of material can be used as the basis of an instrument to measure temperature and they are called **Thermometric Properties**.

#### 3.5.2 Calibration

Generally, calibration is the process of comparing the output value an instrument is given with that of a standard instrument (i.e. the one that its output is known to be true or correct value). From this process, adjustment can then be made to the instrument to be calibrated to give a correct output. When this is done, the instrument is said to be calibrated.

The method being employed in calibrating thermometers is to use the device to measure the easily reproducible temperatures like triple point of water and boiling point of water. If the thermometric property being used is recorded at these two temperatures, a linear graph can be plotted using these two set of variables. From this graph, a linear relationship can be obtained between temperature and the thermometric property.

In another way, one can measure the thermometric property, say  $X$ , when the thermometer is placed in contact with the system or body which temperature is to be measured. Then we have

$$T(X) = aX, \quad 3.5$$

where  $T_X$  is the temperature of the body to be measured,  $a$  is a constant, and  $X$  is the value of the thermometric property at  $T_X$ . Equation 3.5 is also applies when the thermometer is placed in contact with easily reproducible temperature. The triple point of water is mostly used. Then we have

$$T_3 = 273.16 \text{ K} = aX_3,$$

so we obtain

$$a = \frac{273.16 \text{ K}}{X_3}. \quad 3.6$$

Insert equation 3.6 into equation 3.5

$$T(X) = (273.16 \text{ K}) \frac{X}{X_3} \quad 3.7$$

Equation 3.7 is generally applicable to all thermometers.

### SELF ASSESSMENT EXERCISE 1

- i. Explain what is meant by triple point of water
- ii. State zeroth law of thermodynamics

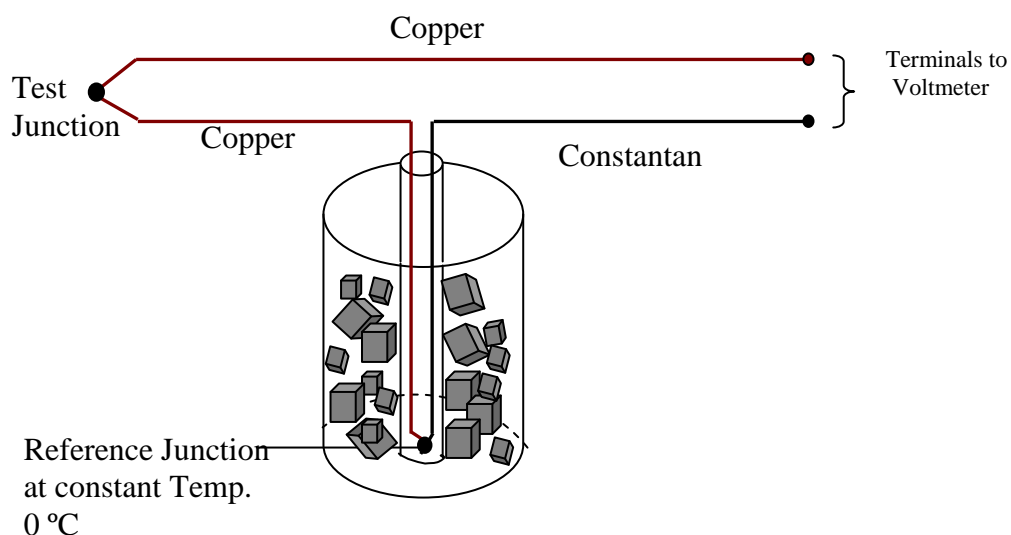
## 3.6 Types of Thermometers

Thermometer is named after the thermometric property used for the construction. For example, thermometric property of the mercury-in-glass thermometer is the length of mercury column, while in the constant-volume gas thermometer the thermometric property is the pressure of the gas. Examples of thermometer are thermocouple thermometer, resistance thermometer, liquid in gas thermometer (e.g. mercury in glass thermometer), and gas thermometer (e.g. constant volume gas thermometer) among others. Three types of thermometer are discussed below.

### 3.6.1 Thermocouple Thermometer

It has been observed that when two dissimilar metals are joined together to make two junctions, an electromotive force (emf) will flow in the circuit. This emf can be measured using a voltmeter and its value depends on the temperature difference between the junctions. The arrangement is called thermocouple and the observation is known as **Seebeck effect**. Thermocouple thermometer is based on the **Seebeck effect**.

**Thermometric property:** emf generated when two junctions made from two different metals are maintained at different temperature.



**Fig. 3.3: Thermocouple Thermometer**

The diagram for Figure 3.3 shows the arrangement for the copper-constantan thermocouple thermometer. The test junction is placed on the body or inside the system whose temperature is to be measured, while the reference junction is maintained at constant temperature at  $0\text{ }^{\circ}\text{C}$ . The potentiometer is connected to the terminals to the voltmeter. The relationship between the *emf* and temperature is

$$emf = a + bT + cT^2 + dT^3 \quad 3.8$$

Where **a**, **b**, **c**, and **d** are constant and they are different for each thermocouple.

Using equation 3.7, we can write

$$T(emf) = (273.16\text{ K}) \frac{emf}{emf_3} \quad 3.9$$

The range of measurement of thermocouple thermometer depends on the choice of metals used. For example, a platinum-10 % rhodium/platinum thermocouple has temperature range of  $0$  to  $1600\text{ }^{\circ}\text{C}$ .

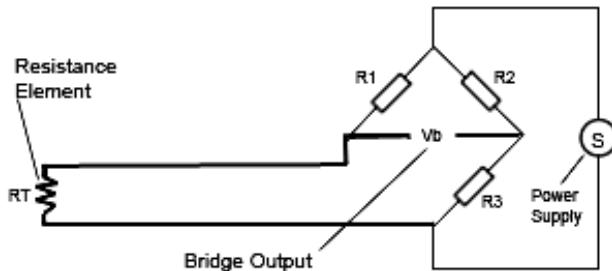
Thermocouple thermometer is used extensively in scientific laboratories.

### 3.6.2 Resistance Thermometer

Electrical conductivity of a metal depends on the movement of electrons through its crystal lattice. The electrical resistance of a conductor, due to thermal excitation, varies with temperature. This forms the basic principle of operation of resistance thermometer. Resistance thermometer therefore, uses the variation in electrical conductivity of a conductor to indicate temperature.

Figure 3.4 shows a generalized form of a resistance thermometer. In the diagram, **RT** is the resistance element which could be any conductor (e.g. platinum). This is usually

wound round a frame constructed so as to avoid excessive strains when the wire contracts upon cooling. **S** is the power supply and the purpose is to maintain a known constant current in the thermometer while measuring the potential difference with the aid of a bridge output (usually a sensitive potentiometer).



**Fig. 3.4 :** Resistance Thermometer

The relationship between the temperature and the electrical resistance is usually non-linear and described by a higher order polynomial:

$$R(T) = R_0(1 + AT + BT^2 + CT^3 + \dots) \quad 3.10$$

Where  $T$  is the Celsius temperature,  $R_0$  is the nominal resistance at a specified temperature, and **A**, **B**, **C** are constants. The number of higher order terms considered is a function of the required accuracy of measurement. The constants (i.e. **A**, **B** and **C** etc.) depend on the conductor material used and basically define the temperature-resistance relationship. The value  $R_0$  is referred to as nominal value or nominal resistance and is the resistance at  $0^\circ\text{C}$ . Material most commonly used for resistance thermometers are Platinum, Copper and Nickel. However, Platinum is the most dominant material internationally.

The calibration of this instrument requires the measurement of  $R(T)$  at various known temperatures and from these the constants in equation 3.10 can be obtained. However, equation 3.7 can be used to obtain

$$T(R) = (273.16 \text{ K}) \frac{R}{R_3} \quad 3.11$$

Where  $T(R)$  is the temperature of the body or system to be measured,  $R$  is the resistance at that temperature, and  $R_3$  is the resistance at triple point of water?

The range of measurement of resistance thermometer depends on the choice of conductor used. Platinum resistance has a very accurate measurement within  $-253$  to  $1200^\circ\text{C}$ .



### 3.6.3 Constant-Volume Gas Thermometer

Constant-Volume gas thermometer is normally referred to as the standard thermometer and is mostly used to calibrate other thermometers. The working principle is based on the pressure of a gas in a fixed volume. Figure 3.5 shows an example of a constant-volume gas thermometer. It consists of a gas-filled bulb connected by a tube to a mercury manometer. By raising and lowering reservoir R, the mercury level on the left can always be brought to the zero of the scale to keep the gas volume constant (Note that variation in the gas volume can affect temperature measurement).

The basic equation is

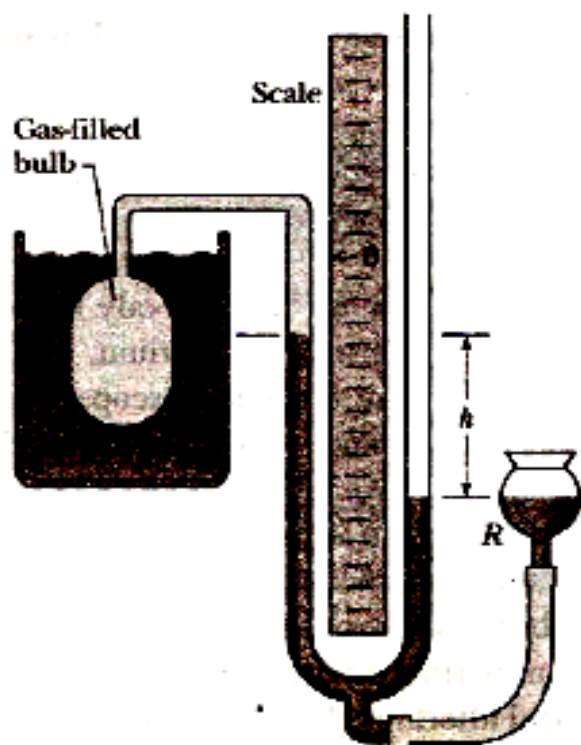
$$P = P_o - \rho gh, \quad 3.12$$

where  $P_o$  is the atmospheric pressure,  $\rho$  is the density of the mercury in the manometer,  $g$  is the acceleration due to gravity, and  $h$  is the measured difference between the mercury level in the two arms of the tube.

The difference in height  $h$  between the two arms of the manometer can be measured when the gas filled bulb is surrounded by the system which temperature is to be measured, and when it is surrounded with water at triple point. Using equation 3.7, the relationship between temperature and the pressure of the gas is

$$T(P) = (273.16 K) \left( \frac{P}{P_3} \right) \quad 3.13$$

Where  $T(P)$  is the temperature of the system which temperature is to be measured,  $P$  is the pressure of the gas at that temperature, and  $P_3$  is the pressure at triple point of water.



**Fig. 3.5: Schematic of Constant-Volume Gas Thermometer (after Halliday *et al.*, 2001)**

### SELF ASSESSMENT EXERCISE 2

- i. Name four types of thermometer and their thermometric properties
- ii. A certain resistance thermometer at triple point of water has resistance  $152.0\ \Omega$ . What is the temperature ( $T$ ) of a system in degree centigrade if the resistance of the thermometer is  $230.51\ \Omega$  when inserted into the system?

### 4.0 CONCLUSION

The zeroth law of thermodynamics establishes the existence of thermal equilibrium and allows temperature to be measured using a thermometer. Thermometers are the instruments used to measure the temperature of a body or a system. The basis for the working of thermometer is the variations in physical properties of materials with temperature. Those properties being used for the construction of thermometers are called thermometric properties and thermometers are named after these properties.

## SUMMARY

In this unit you have learnt that:

- that heat is a form of energy that flows from one point of the system to another or from one system to another; while temperature is the degree of hotness or coldness of a body or system
- thermal equilibrium is reached when systems in contact, directly or indirectly, attain the same temperature-statement of zeroth law
- thermometric properties are the physical properties of materials that change linearly with temperature; and these properties are used for the construction of thermometers
- difference temperature scales exist and one can convert from one temperature scale to another.

### 6.0 TUTOR-MARKED ASSIGNMENT

1. Explain the concept of heat and temperature
2. State zeroth law of thermodynamics
3. Define thermometric properties and list different types of thermometer
4. Derive the equation below:

$$T(X) = (273.16\text{k}) \frac{X}{X_3}$$

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## UNIT 4 HEAT TRANSFER MECHANISM

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  - 3.2 Methods of Heat Transfer
  - 3.3 Conduction
    - 3.3.1 Conductors and Insulators
    - 3.3.2 Conduction of Heat through a Material
    - 3.3.3 Thermal Resistance to Conduction (R-Value)
    - 3.3.4 Conduction of Heat through a Composite Material
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    - 3.5.1 Absorption and Emission of Radiant Energy
    - 3.5.2 Stefan-Boltzmann Law of Radiation
  - 3.6 Newton's Law of Cooling
- 4.0 Conclusion
- 5.0 Summary
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### 1.0 INTRODUCTION

Heat is a form of energy which transferred from one part of the body to another or from one body to another. As discussed in unit 3 the temperature gradient determines the direction of heat flow. The transfer of heat energy is important in many aspects of our lives. This unit is therefore examined the three fundamental processes of heat transfer and their useful applications in our daily lives.

### 2.0 A: OBJECTIVES

By the end of this unit, you should be able to:

- define heat;
- mention and discuss the three (3) methods / processes of heat transfer;
- write the useful equations for the heat transfer;
- state Newton's law of cooling; and
- mention useful applications of heat transfer.

## 2.0 B: HOW TO STUDY THIS UNIT

1. You are expected to read carefully through this unit twice before attempting to answer the activity questions. Do not look at the solution or guides provided at the end of the unit until you are satisfied that you have done your best to get all the answers.
2. Share your difficulties in understanding the unit with your mates, facilitators and by consulting other relevant materials or internet.
3. Ensure that you only check correct answers to the activities as a way of confirming what you have done.
4. Note that if you follow these instructions strictly, you will feel fulfilled at the end that you have achieved your aim and could stimulate you to do more.

## 3.0 MAIN CONTENT

### 3.1 Heat Energy

Heat ( $Q$ ) is a form of energy that is transferred from one part of a system to another or to another system by virtue of a difference in temperature. As earlier discussed in unit 3, temperature gradient determines the direction of heat flow.

### 3.2 Methods of Heat Transfer

The transfer of heat from one part of a system to another or to another system by virtue of a temperature difference can only be by one or more of the three processes namely; conduction, convection, and radiation. Each of these mechanisms or processes is discussed in the preceding sections.

### 3.3 Conduction

This is the process of heat transfer whereby heat energy is transferred directly through a material without any bulk movement of the material.

#### 3.3.1 Conductors and Insulators

Materials can be divided into two groups based on their ability to conduct thermal energy namely; thermal conductor and thermal insulator.

#### Conductors

Materials that conduct heat well are called thermal conductors. Examples of thermal conductors are metals (most metals are conductors) like aluminum, copper, silver, and gold.

**Why do conductors conduct?** The behaviour of conductors in terms of thermal conductivity can be explained with two mechanisms namely; collision mechanism and free electrons in metals.

**Molecular collision:** Atoms and molecules in a hot part of the material vibrate or move with greater velocity (i.e. higher kinetic energy) than those at the colder part. By means of collisions, the more energetic molecules pass on a portion of their energy to their less energetic neighbours. As the more energetic molecules collide with their less energetic neighbours they transfer some of their energy to the neighbours. The collision mechanism does not depend on bulk movement of the material.

**Free electrons in metals:** Good conductors of thermal energy, like metals, have pool of electrons that are more or less free to wander through the volume of the metal. These free electrons are capable of transporting energy round the whole volume of conductors. Free electrons are also responsible for the excellent electrical conductivity in metals.

### Insulators

Materials that conduct heat poorly are called thermal insulators. Examples of thermal insulators are wood, glass, and most plastics. These materials poorly conduct heat energy because the two above discussed mechanisms for conduction are not possible with these materials (i.e. the molecules of these materials are not free to move and or the material don't contain free electrons).

### 3.3.2 Conduction of Heat through a Material

Consider a bar of material of area  $A$  and thickness  $L$ , heat  $Q$  transfer through the material by conduction is

$$Q = \frac{kA\Delta Tt}{L} \quad 4.1$$

where  $\Delta T$  is the temperature difference between the ends of the bar and  $k$  is the thermal conductivity of the material. Unit of  $k$  is  $J/(s.m.C^\circ)$

### 3.3.3 Thermal Resistance to Conduction (R-Value)

There is a term similar to the electrical resistance  $\mathbf{R}$  used for thermal resistance to conduction called R-value. To differentiate this from  $\mathbf{R}$  that we are familiar with,  $\mathbf{R}_t$  is used here for thermal resistance to conduction.

Thermal resistance  $\mathbf{R}_t$  of a slab of thickness  $\mathbf{L}$  is defined as

$$\mathbf{R}_t = \frac{\mathbf{L}}{\mathbf{k}} \quad 4.2$$

High value of  $\mathbf{R}_t$  indicates a bad thermal conduction or a good thermal insulation.

Note:  $R_t$  is a property attributed to a slab of a specified thickness, and not to a material.

### 3.3.4 Conduction of Heat through a Composite Material

Consider two materials of thickness  $L_1$  and  $L_2$  with different thermal conductivities  $k_1$  and  $k_2$  respectively as shown in figure 4.1. The outer surfaces of the slab are in thermal contact with hot reservoir at temperature  $T_H$  and cold reservoir at temperature  $T_C$ . Assuming that the heat transfer through the slabs is a steady rate process i.e. the temperature everywhere in the slab and the rate of energy transfer do not change with time. Then, the conduction rate through the two slabs must be equal. Using equation 4.1, the conduction rate  $P_{cond}$  is

$$P_{cond} = \frac{Q}{t} = \frac{kA\Delta T}{L} \quad 4.3$$

Let  $T_X$  be the temperature of the interface between the two materials, so

$$P_{cond} = \frac{k_2 A (T_H - T_X)}{L_2} = \frac{k_1 A (T_X - T_C)}{L_1} \quad 4.4$$

Solving for  $T_X$  in equation 4.4 gives

$$T_X = \frac{k_1 L_2 T_C + k_2 L_1 T_H}{k_1 L_2 + k_2 L_1} \quad 4.5$$

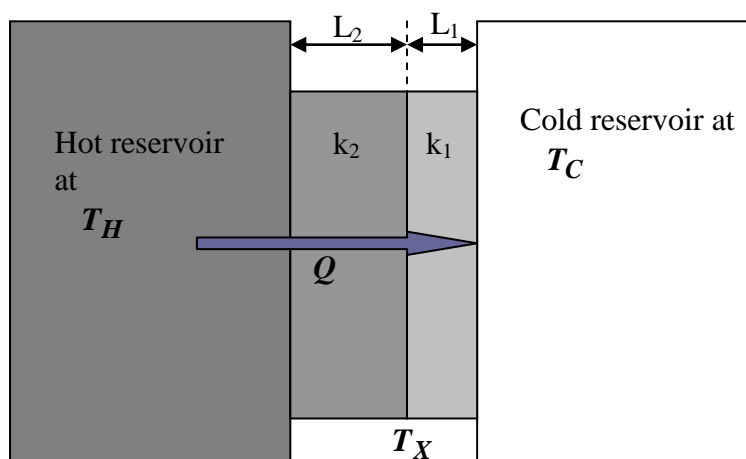
Insert equation 4.5 in equation 4.4 to obtain

$$P_{cond} = \frac{A(T_H - T_X)}{L_1/k_1 + L_2/k_2} \quad 4.6$$

If we apply this to any number  $n$  of materials, equation 4.6 becomes

$$P_{cond} = \frac{A(T_H - T_X)}{\sum_2^n (L_i/k_i)} \quad 4.7$$





**Fig. 4.1 : Heat Transfer through a Composite Slab**

### 3.4 Convection

Convection is the process in which heat energy is transferred from place to place by the bulk movement of a fluid. A good example of this process is convection current in liquid.

This bulk movement of a fluid can be explained in terms of buoyant force. When a portion of a liquid is warmed, such as water, the volume of the liquid or fluid expands, and density decreases ( $\rho = \frac{m}{V}$ ). From Archimedes' principle, the surrounding cooler and denser fluid exerts a buoyant force on the warmer fluid and thus pushes the warmer fluid upward. As the warmer fluid is pushed upward, the surrounding cooler fluid moved downward to replace the warmer fluid. The cooler fluid, in turn, is warmer and pushes upward. This cycle is continuously repeated and this is called convection current. This phenomenon is called natural convection.

The phenomenon described above is called natural convection. Forced convection occurs if the fluid is made to move in a similar way as natural convection by the action of a pump or a fan.

Consider a fluid in contact with a flat or curved wall which temperature is higher than that of the main body of the fluid, the rate of heat transfer due to both conduction through the film and convection in fluid is

$$\frac{Q}{t} = hA\Delta T \quad 4.8$$

where  $h$  is the convection coefficient and it includes the combined effect of conduction through the film and the convection in the fluid,  $A$  is the area of the wall,

and  $\Delta T$  is the temperature difference between the surface of the wall and the main body of the fluid.

Finding the value of  $h$  that is appropriate for a particular arrangement is problematic because  $h$  depends on the following factors:

- whether the wall is flat or curved
- whether the wall is horizontal or vertical
- whether the fluid in contact with the wall is a gas or a liquid
- the density, viscosity, specific heat, and thermal conductivity of the fluid
- whether the velocity of the fluid is small enough to give rise to laminar flow or large enough to cause turbulent flow
- whether evaporation, condensation, or formation of scale takes place.

### 3.5 Radiation

This is a process in which energy is transferred by means of electromagnetic waves. A good example of this is the solar radiation from the sun traveling in all directions in space. Part of this radiation is reaching the earth on daily basis and in actual sense; the bulk of energy on earth is from the sun. All bodies, hot or cold, continuously radiate energy in form of electromagnetic waves. But the amount of this radiation is proportional to the temperature of the body and the nature of its surface.

#### 3.5.1 Absorption and Emission of Radiant Energy

The surface of an object is very important in the determination of the amount of radiant energy a body or object can absorb or emit. An experiment set-up to justify this is shown in figure 4.2. Figure 4.2 consists of two identical blocks, one coated in black and the other coated with silver. If a thermometer is inserted on each of the blocks and they are exposed to direct sunlight as shown in the diagram. It will be observed that the temperature of the block coated in black will rise faster than that of the one coated with silver. The reason for this is that the block coated in black absorbed larger percentage of the solar radiation falling on it and thus rapid rise in temperature due to large heat energy input.

**Perfect Blackbody:** This is a body that absorbs all the electromagnetic waves falling on it.

Generally, all objects can emit and also absorb electromagnetic waves. So when an object is in thermal equilibrium with its surroundings, it implies that the amount of radiant energy the object absorbs balances with the amount the object emits. However, if the absorption is greater than the emission, the object will experience a net gain of radiant energy and thus the temperature will rise. If the emission is greater than the absorption, the object will experience a net loss of radiant energy and the temperature will fall. A good absorber is also a good emitter while a poor absorber is also a poor

emitter. Hence, a perfect black body is a perfect absorber and also a perfect emitter of radiant energy.

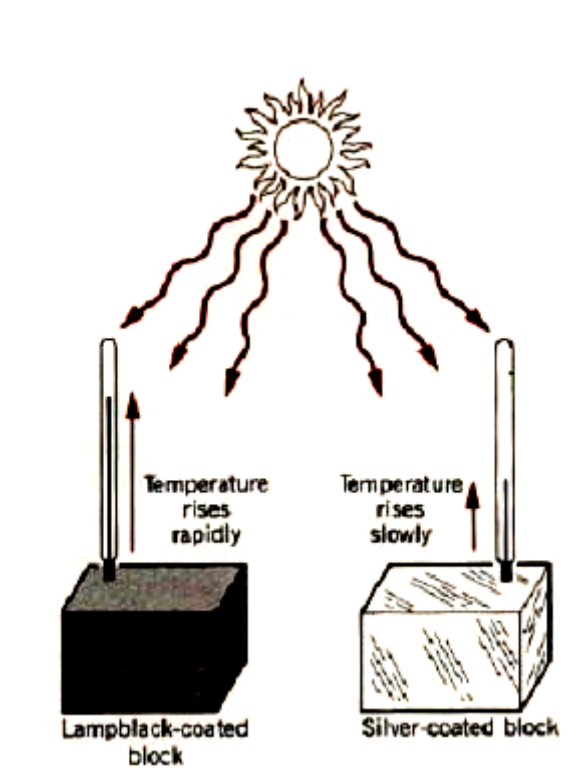


Fig. 4.2: (After Cutnell and Johnson, 1989)

### 3.5.2 Stefan-Boltzmann Law of Radiation

All matter constantly radiate energy in form of electromagnetic waves.

$$\frac{Q}{t} = \sigma \epsilon A T^4 \quad 4.9$$

where  $\sigma$  is the Stefan-Boltzmann constant,  $\epsilon$  is emissivity and it has value between 0 and 1. For a perfect reflector,  $\epsilon=0$  and for a black body  $\epsilon=1$ .

Equation 4.9 is known as the Stefan-Boltzmann law of radiation, and the law stated that the radiant energy, emitted in a time  $t$  by an object that has a Kelvin temperature of  $T$ , a surface area  $A$ , and an emissivity  $e$ , is given by  $Q = \epsilon \sigma T^4 A t$ .

**Emissivity  $\epsilon$  of an Object:** This is the ratio of radiant energy emitted by an object to the one it would have emitted if it were to be a perfect black body.

Assuming that the radiation a body would emit if it were to be a perfect body is represented by  $Rad_{pbb}$ , then

$$\varepsilon = \frac{\text{radiation emitted}}{\text{Rad pbb}} \quad 4.10$$

### 3.6 Newton's Law of Cooling

Energy is lost to the surroundings by conduction, convection, and radiation. The rate at which an object loses energy to the surrounding is determined by the temperature difference between the object ( $T_o$ ) and the surrounding ( $T_s$ ).

By conduction energy loss rate =  $kA(T_s - T_o)/l$

By convection, energy loss rate depends on whether the air is forced to flow (e.g. by a fan) or moves by natural convection. Energy loss rate =  $hA(T_s - T_o)$

By radiation, energy loss rate =  $\sigma A(T_o^4 - T_s^4)$

The total effect of these three processes is to give a rate of energy loss per second that is proportional to the temperature difference between the object and its surroundings. This is known as Newton's law of cooling.

#### SELF ASSESSMENT EXERCISE 1

- i. State the Stefan-Boltzmann Law of Radiation
- ii. In a light bulb, the tungsten filament has a temperature of  $3.0 \times 10^3$  °C and radiates 60 W of power. Assuming the emissivity of the filament is 0.36, estimate the surface area of the filament.  $\sigma = 5.67 \times 10^{-8} \text{ J(s.m}^2.\text{K}^4)$

### 4.0 CONCLUSION

The three (3) mechanisms of heat transfer are conduction, convection and radiation. Heat can be transferred in or out of a system by one or more of these three mechanisms.

### 5.0 SUMMARY

In this unit you have learnt that:

- heat is a form of energy that can be transferred from one body to another
- heat can be transferred by three (3) main mechanisms
- the rate of heat loss by conduction is  $kA(T_s - T_o)/l$ , by convection is  $hA\Delta T$ , and by radiation is  $\sigma A(T_o^4 - T_s^4)$
- the Newton's law of cooling gives the rate at which energy is lost to the surrounding by the three mechanisms.

**6.0 TUTOR-MARKED ASSIGNMENT**

1. Define Heat transfer
2. State Newton's Law of cooling
3. State useful application of heat
4. Write out useful equation for the heat transfer
5. Name the three main methods of heat transfer and write the expression for the rate of heat transfer for each method.
6. A perfect blackbody has a temperature of 605 °C. An identically shaped object whose emissivity is 0.400 emits the same radiant power as the blackbody. What is the Celsius temperature of this second body?
7. The amount of radiant power produced by the sun is approximately  $3.9 \times 10^{26}$  W. assuming that the sun is a perfect blackbody sphere with a radius of  $6.96 \times 10^8$  m, find its surface temperature.

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