MODULE 1

UNIT 1 CONCEPT OF TEMPERATURE

CONTENTS

1.0 INTRODUCTION

You will recall that the concepts of length, mass and time are regarded as fundamental quantities during the study of elementary mechanics. You also learn about derived quantities such as force, momentum and energy. In this unit, you will be introduced to another fundamental quantity called temperature.

Temperature is a fundamental quantity in the study of heat (thermal energy) or thermodynamics. Some students usually get confused with the concept of temperature and heat. Heat is a form of energy. Temperature is a sensation of hotness and coldness. Hence in this unit we shall make an attempt to differentiate between the two. We would then explain the concept of temperature by using the zeroth law of thermodynamic. Having done this, we would then establish how the temperature of a body is measured through the use of thermometers with emphasis on their thermometric properties. In the next unit, you will learn about the types of thermometers.

2.0 OBJECTIVES

After the end of this unit, you should be able to:

- differentiate between temperature and heat
- define the concept of temperature
- explain the terms used for the zeroth law of thermodynamics
- state the law of thermodynamics
- define temperature scales
- solve problems on conversion of temperatures on Celsius Fahrenheit and Absolute scales.

3.0 MAIN CONTENT

3.1 Concept of Temperature and Heat

The concepts of temperature and heat are two kinds of concepts in physics that are so closely related that may confuse you as a learner in the study of physics. You need to be clear in your mind what exactly they mean.

You will remember that in your study of mechanics you learnt various forms of energy. Heat was among them. Heat measurement is usually referred to as **Calorimetry**. Also in the study of Integrated Science, you will also recall that temperature is the degree of coldness or hotness of a body. Temperature measurement in physics is referred to as **thermometry**. ***Then the question arises: Why Heat a form of energy?*** It is a form of energy because we use it to do work for us.

In the history of Industrial Revolution, heat engines were used to perform various kinds of work as in the textiles factories and in the locomotive engines used for transport. Besides, at home we use it for cooking and ironing our clothes.

Heat is an intangible agency that causes increase of hotness of a body. A body is said to be warmer if it receives heat and colder when it losses heat. You will therefore observe that increase in heat content of a body can be caused by any of the following ways:

- direct application of mechanical work (as in friction between two bodies);
- churning of a liquid;
- contact with a hotter body.

Heat therefore flows from a body whose degree of hotness (temperature) is greater to the body of lower degree of hotness (temperature). Consequently, it is the heat energy that is transferred and not the temperature. As a rule, when two bodies of different temperatures are placed in contact with each other, by definition, the heat lost by one body is absorbed by the other body provided the two bodies neither lose nor gain heat from the surrounding. In this case we would describe such heat energy interaction as being conserved in the system.

In this unit, we would discuss in details what is meant by temperature and how it is measured using the appropriate scales. Using the zeroth law of thermodynamics will do this. The concept pf heat and its measurement will be dealt with later.

SELF ASSESSMENT EXERCISE 1

Distinguish between the concepts of heat and temperature.

3.2 Concept of Temperature

The concept of temperature is one of the fundamental concepts in physics. It is rather difficult to define this physical quantity as compared with, say, the length of a body. A meter is a standard length of a bar of platinum – iridium kept in Paris. We can directly copy this bar and use it to find out how many times an object is as long as this bar at anytime and place. This is not the case with temperature. Temperature is measured through an indirect method, as you will observe later in our discussion. ***But the question you may ask now: What is temperature? How can we measure the temperature of a body?***

Qualitatively, temperature of a body is the degree of hotness or coldness of a body. However, this answer does not lead us to the quantitative definition of temperature or its operational definition. This is because the sensations of hotness and coldness are highly subjective. The way you feel is not the same as any other person. Other adjectives used include cold, cool, tepid, warm, etc. Using our feeling of heat to estimate degree of hotness/coldness (temperature) is very personal and very unreliable since its measurement is personal and not a standard one. Hence when we say the temperature of a body is cold, hot or lukewarm such descriptions are rather too vague to comprehend quantitatively. Consequently, an independent scale of temperature measurement is therefore highly essential. We would now discuss how we have evolved the scale of temperature measurement.

3.2.1 Thermal Equilibrium

Two bodies may be at different temperature – one hot and the other cold. The hot one is said to possess more heat energy than the colder body. In another sense, the temperature of the hot body is higher than the colder body.

However, if the two bodies are now in contact with each other, heat energy flows from the hot body to the cold body until the temperatures of the two bodies are the same. The two bodies are then described as being in thermal equilibrium with each other.

Therefore, a thermal equilibrium exists between two bodies when they are in thermal contact with each other and there is no net flow of heat between them.

It is the temperature of a body that determines the direction of flow of heat from that body to another. It will flow until two temperatures are the same i.e. there is a thermal equilibrium. Once, there is a thermal equilibrium between two bodies, then it means that the two temperatures are the same – no net flow. *But now the basic question arising: How do we then define the temperature of a body quantitatively?* Let us discuss this concept.

3.2.2 The Zeroth Law of Temperature

The zeroth law of thermodynamics helps us to quantify the concept of temperature objectively. Quantitative definition of temperature involves terms of operations that must be independent of our sense perceptions of hotness or coldness. That is, temperature has to be measure objectively and not subjectively.

It has been observed that there are some systems in which a measurable property of the system varies with hotness or coldness of the system.

For example:

(i) the length L of a mercury column in a thin tube will change variation in temperature (fig. 3.1).

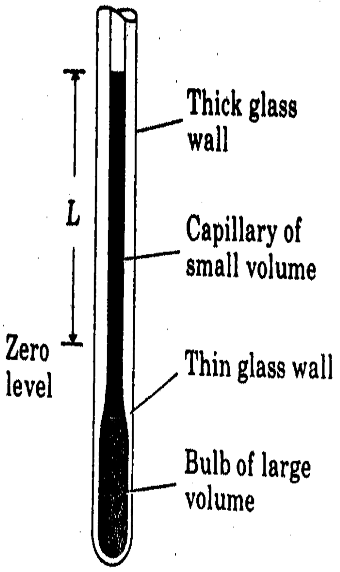

Fig. 3.1: A System Whose State is Specified by the Value L.

(ii) the pressure P of a constant volume container, measured by a pressure gauge or a manometer (fig. 3.2).

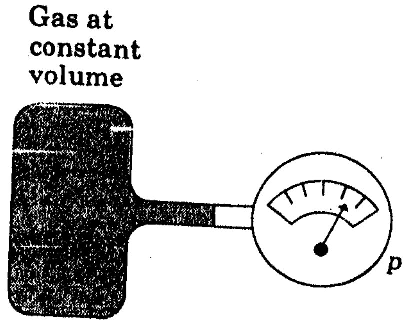

Fig. 3.2: A System Whose State is Given By the Value P (Pressure)

(iii) the electrical resistance (R) of a wire which varies with hotness or coldness as with the platinum resistance (fig. 3.3).

Fig 3.3: Platinum Resistance

(iv) the electromotive force (E) of a thermo-junction varies also with hotness or coldness of the system (fig. 3.4).

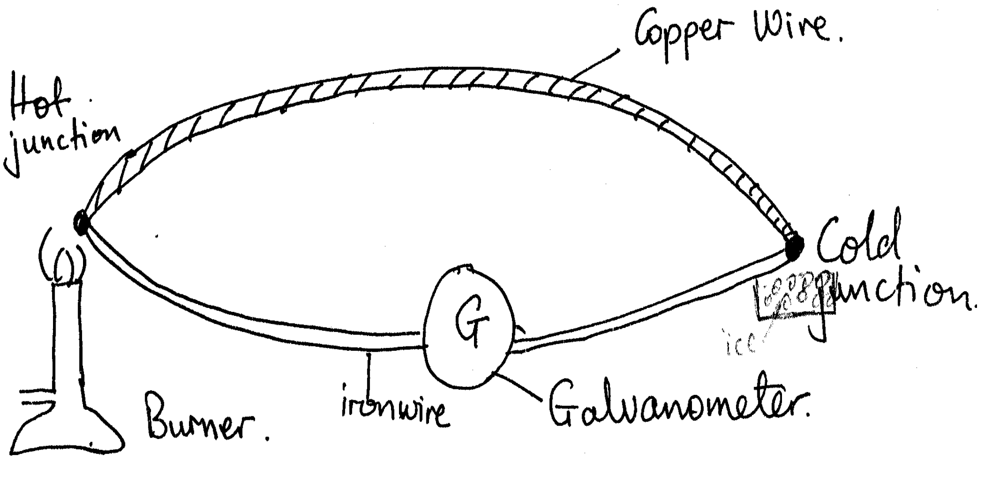

Fig. 3.4: A Thermocouple

In each case, discussed above the quantity describing the varying state of the system such as L, P, R and E are called a state of coordinate for the system.

In thermodynamics, bodies are brought into contact in order to establish the common temperature using one of these coordinates. The two bodies may be in direct contact or they may be separated by two types of wall. The types of wall are namely adiabatic and diathermic walls. Adiabatic walls are those through which no heat can be transmitted whereas the walls through which heat can be transmitted are known as diathermic walls. In thermodynamics, these two words are used to describe the process of thermal equilibrium that is, of being at thermal equilibrium. They will help you to understand the zeroth law.

Let us consider two systems A and B such that system A is a mercury-in-glass tube with a state coordinates L and system B, a constant-volume gas container with state coordinate P (fig. 3.5).

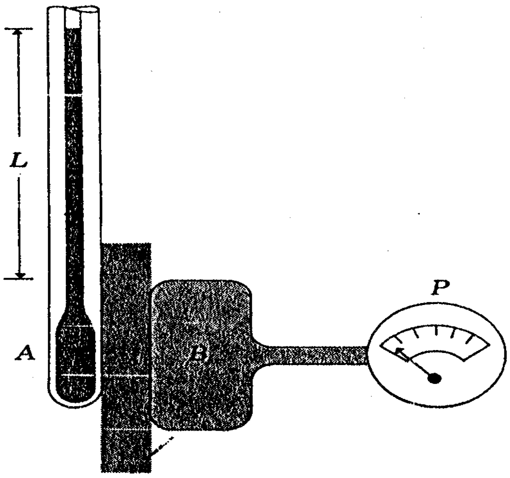

Fig. 3.5: Adiabatic Wall

If system A is at a higher temperature than system B, when brought in contact with each other, then their state coordinates change. However, if the two systems are separated by insulating such as wood, plastic or fibre glass, the change in the state coordinate will be low or none. Such an insulating material is described as an adiabatic wall. Therefore, in general, an adiabatic wall is such in which, in an ideal situation, there is no change in the state of coordinates (fig. 3.5).

The opposite of adiabatic wall will then be a wall or a partition that will allow the systems A and B to influence each other (fig. 3.6).

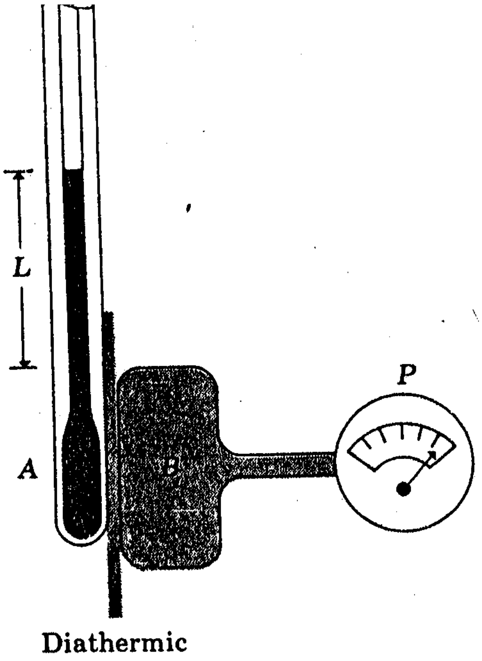

Fig. 6.3: A Diathermic Wall

In this case it will allow free exchange of heat energy. Such a wall is called a diathermic wall. Examples of such diathermic walls are copper and aluminum. Thus when the two systems A and B are separated by a diathermic wall, initially there may be no change but eventually a state is reached when no further change in the state coordinates of A and B takes place. This joint state of both systems that exist when all changes in the state coordinates have ceased is called thermal equilibrium. Note the reduction in length of L and the increase in P of the pressure at thermal equilibrium. These changes are used to measure temperature. We would use the above illustration to explain the zeroth law of thermodynamics and subsequently temperature measurement.

The Zeroth Law

Consider two systems A and B separated from each other by an adiabatic wall but each system is in contact with a third system **C** separated through a diathermic wall. Consider the whole systems surrounded by an adiabatic wall (fig. 3.7). This is to ensure that no heat energy is lost to or gained from the surrounding. Experiments have shown that systems A and B will attain a thermal equilibrium with **C**. If however the adiabatic wall is replaced by a diathermic wall as shown in (fig. 3.8).

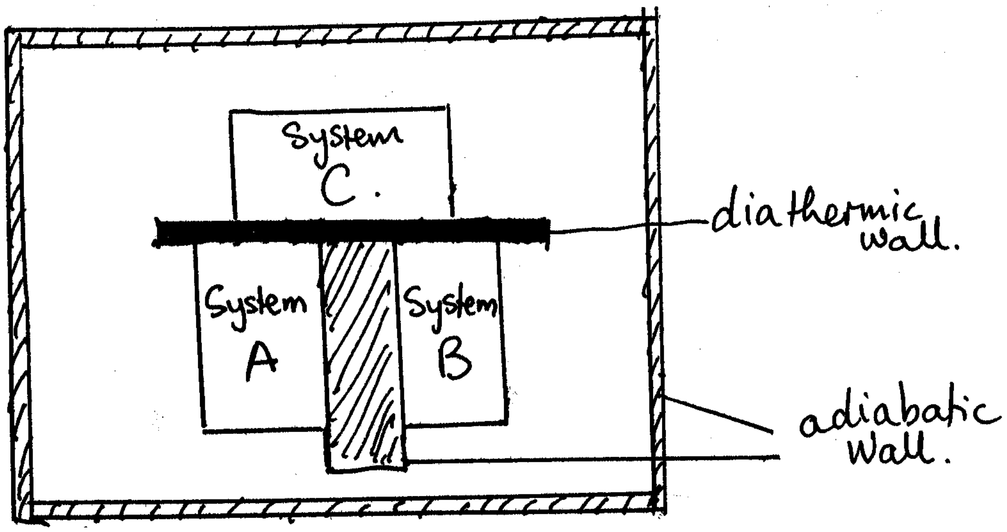

Fig. 3.7: A and B are each in Thermal Equilibrium with C

Instead of allowing both systems A and B to come to equilibrium with C at the same time, we can first have equilibrium between A and C and then equilibrium between B and C making sure that the state of C is the same in both cases, (fig. 3.7) then A and B are brought in contact through a diathermic wall, they will be found to be in thermal equilibrium (fig. 3.8). It means that no further changes occur in systems A and B

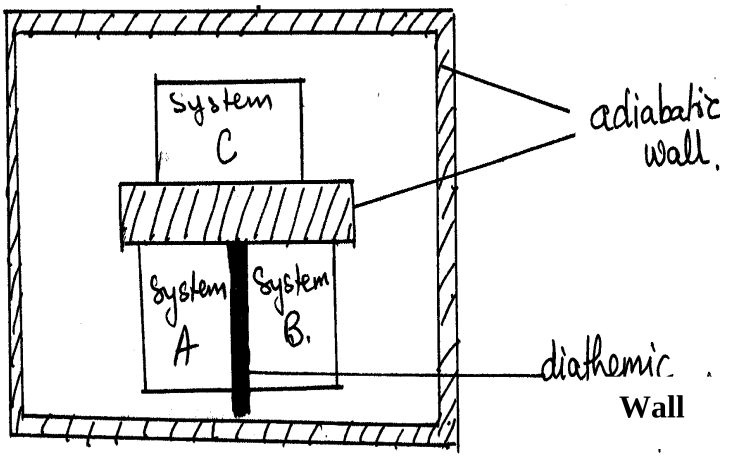

Fig. 3.8: A and B are in Thermal Equilibrium with Each Other.

That is systems A and B are already in equilibrium with each other.

The above principle is called the zeroth law of thermodynamics. The zeroth law of thermodynamics states that:

"Two thermodynamics systems A and B are separately in thermal equilibrium with a third system C, then the systems A and B are in Thermal equilibrium with each other".

It is called the zeroth law because the most important principles of thermodynamics have hitherto been identified as the first, second and third laws of thermodynamics. You will be introduced to these laws later in the course of your physics programme.

It is the property called temperature that determines whether or not given two systems are in thermal equilibrium. The temperature of a system is that property that determines whether or not it will be in thermal equilibrium with other system when two or more systems are in thermal equilibrium, they are said to have the same temperature. If the two systems are not in thermal equilibrium, their temperatures must be different.

The zeroth law is thus used in establishing the temperature of a body quantitatively and objectively. But before it is measured, a scale must be established with the aid of a physical property which varies with temperature – the thermometer.

We shall now consider the establishment of scales of temperatures in the next section.

3.3 Scale of Temperature

There are some principles underlying the establishment of temperature scales. These principles are based on the fact that when the temperature of a body changes, all the magnitudes of almost all its physical properties also change. The condition, of course, is that this variation of properties must be linear, that is, uniform. Therefore any property of any substance that varies uniformly with temperature can be used. The variation of one of these properties is chosen to represent the accompanying change of temperature.

To measure temperature, therefore, we need to select a physical property or parameter of a chosen substance which varies uniformly with temperature. A parameter or property is a variable which is assigned a constant value during a discussion or event. As we have discussed in section 3.3.2, that some of the examples of these parameters are:
(i) the volume of a liquid;
(ii) the volume of a gas at constant pressure;
(iii) the pressure of a gas at constant volume;
(iv) the electrical resistance of a conductor;
(v) the emf change of a thermocouple when there is a temperature difference between the junctions of a thermoelectric thermometer.

For the establishment of temperature scale, the following are required:

(a) specification of fixed points;
(b) specification of the method of interpolation.

Now we will discuss briefly about these concepts required for the establishment of temperature scale.

3.3.1 Specification of Fixed Points

Fixed points are temperatures chosen which are fixed and reproducible. They are useful as reference temperatures. Changes in the parameters from the fixed points are assigned numbers called degrees on a calibrated scale. Two such fixed points are:

(i) The Lower fixed point (ice point): That is the temperature of equilibrium between ice, water and air saturated at standard pressure. This temperature is) 0^oC.

(ii) The Upper fixed point (steam point): That is the temperature of steam rising from pure water boiling under standard atmospheric pressure. That is, the temperature of one standard atmosphere. This temperature is 100^oC.

(iii) The fundamental interval: This is the difference between the upper fixed point and the Lower fixed point divided into equal parts.

Other fixed points such as the sulphur point also exist for reference.

3.3.2 Factors for Changes in Fixed Points

1. Changes in the atmospheric pressure and latitude cause variation in freezing and boiling points. Changes caused by pressure in freezing point can be ignored. That due to impurities cannot be ignored.

2. Freezing point depression and Boiling point elevation are caused by impurities of slats. Hence water used in determining these points is required to be pure.

3. Daily floatation of barometric reading call for the correction of the boiling point. In the neighbourhood of standard atmospheric pressure, the boiling point rises by 0.37^oC when the height of mercury barometer increases by 1.0 cm. Therefore true boiling point is given as on the Celsius scales as:

   $$6^oC = 100^oC + 0.37(B - 76)^oC \ldots\ldots\ldots\ldots\ldots (1.1)$$

   Where B is any atmospheric pressure in cm of mercury.

SELF ASSESSMENT EXERCISE 2

Mention two factors that can change the fixed points on a temperature scale.

3.3.3 The Temperature Scales

The systems of temperature scales are:

(i) The Celsius scales whose ice point is 0^oC and the steam point is at 100^oC. Each part represents 1^oC.

(ii) The Fahrenheit scale whose ice point is 32°F while the steam point if 212°F. The fundamental interval is 180 divisions. Each division represents 10°F.

(iii) The absolute scale of temperature, the thermodynamics scale. This will be discussed later.

3.4 Specification of Interpolation

The way we establish the temperature of a body on either the Celsius scale or the Fahrenheit scale is what we refer to as the specification of interpolation. This therefore establishes the scale of temperature, which decides upon the temperature below, between and above the fixed points, are to be established.

We then choose a thermometric substance and its particular property, which will serve as a temperature indicator.

3.4.1 Definition of Temperature on Celsius Scale

If X represents the property of the thermometric substance, which serves as temperature indicator, by adopting the Celsius scale. Let X_o be the values of X of the thermometric substance when surrounds by the melting ice for a long time. Let X_{100} be the value of X when the substance has reached an equilibrium with steam at standard pressure (1 atmosphere).

Hence, the fundamental interval is defined as the change of X between the ice and steam points = $X_{100} – X_o$.

Consequently, the size of the Celsius degree, which results from our choice of property X, is defined at that range of temperature which causes a change in property which is Z.

Hence $Z = \frac{X_{100} - X_o}{100}$ ………………………………………..…..……. (1.2)

If X_t is the value of X of the substance in the neighbourhood of another body whose temperature is to be determined then, the number of degrees by which the Celsius temperature t_c of the thermometric substance exceeds the temperature of melting ice 0°C is equal to the number of items the quantity Z is contained in $(X_t – X_o)$.

$\therefore \quad (t_c – 0^oC) \times Z = (X_t - X_0)^oC$ ……………………...….…..……. (1.3)

But from Eq. 1.2, you know that

$$Z = \frac{X_{100} - X_o}{100}$$

Substituting Eq. (1.2) in Eq. (1.3), we get

$$\therefore \quad (t_c - 0) \times \frac{X_{100} - X_o}{100} = (X_t - X_o)^oC$$

$$\therefore \quad t_c = \frac{(X_t - X_0)\ X\ 100^oC}{(X_{100} - X_o)} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.4)$$

SELF ASSESSMENT EXERCISE 3

The lengths of the mercury column of a mercury thermometer are 1.06cm and 20.86cm respectively at the standard fixed points. What is the temperature of body, which produces 7.0cm of this mercury column?

3.4.2 Definition of Temperature on Fahrenheit Scale

In the case of the Fahrenheit scale, one can also state that

$$(t_F - 32) \quad Z = (X_t - X_{32})\ ^oF \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.5)$$

$$\text{Where, } Z = \frac{(X_{212} - X_{32})}{(212 - 32)}\ ^oF$$

$$Z = \frac{X_{212} - X_{32}}{180}\ ^oF \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.6)$$

Substituting Eq. (1.5) in Eq. (1.6), we get the expression

$$\therefore \quad t_F - 32 = \left(\frac{X_t - X_{32}}{X_{212} - X_{32}}\right) X180^o\,F$$

$$\therefore \quad t_F = \left(\left(\frac{X_t - X_{32}}{X_{212} - X_{32}}\right) X180 + 32\right)^o F \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.7)$$

Furthermore, the value of the property of X at any definite temperature e is independent of the method of numbering temperature.

Hence $X_{212} = X_{100}$ and $X_{32} = X_o$

Now inserting these parameters in Eq. (1.7), we get

$$\therefore \quad t_F = \left(\left(\frac{X_t - X_0}{X_{100} - X_0}\right) X180 + 32\right)^o F \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.8)$$

Also from Eq. (1.4), we get

$$\frac{t_c}{100} = \frac{X_t - X_0}{X_{100} - X_0} \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1.9)$$

Therefore, a relation between t_F and t_c can be obtained as

$$t_F = \left(\frac{tc}{100}X180+32\right)^o F$$

$$\therefore\ t_F = \left(\frac{9}{5}t_c+32\right)^o F$$

The Eq. (1.10) enables us to convert a temperature measurement from one scale to the other.

SELF ASSESSMENT EXERCISE 4

Covert 50^{o}F to Celsius scale.

Now, let us discuss the examples of X property for different thermometers with Celsius scale.

(a) Platinum thermometer

X is in terms of resistance (R), thus

$$t = \frac{R_t - R_o}{R_{100} - R_0}X100^o C \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.11)$$

(b) Mercury thermometer

X is in terms of length of mercury L.

$$t = \frac{L_t - L_o}{L_{100} - L_0}X100^o C \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.12)$$

(c) Constant Volume thermometer

X is in terms of the pressure P of the gas at constant volume

$$t = \frac{P_t - P_o}{P_{100} - P_0}X100^o C \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.13)$$

(d) Constant Pressure thermometer

X is in terms of the volume of the gas at constant pressure

$$t = \frac{V_t - V_o}{V_{100} - V_0}X100^o C \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1.14)$$

SELF ASSESSMENT EXERCISE 5

A platinum resistance thermometer has a resistance of 10.40 ohms at 0^oC and 14.35 ohms at 100^oC. Assuming that the resistance changes uniformly with temperature, what is

(a) The temperature when the resistance is 11.19 ohms?
(b) The resistance of the thermometer when the temperature is 45^oC?

3.5 Thermodynamic Scale (Absolute Scale) of Temperature

The thermodynamic scale is the standard temperature scale used in scientific measurements. The symbol on this scale is T and it is measure in Kelvin after Lord Kelvin.

On the thermodynamic scale, the reference point is the triple point of water where saturated water vapour, pure water and melting ice are in equilibrium to each other. The temperature of the triple point of water has been found to be 273.16K. The ice point is 273.15K. The slight difference with the triple point is due to the pressure in the two cases.

There is variation of Pressure (P) with temperature T is shown in fig. 3.9. When the graph is extrapolated, it meets the temperature axis at -273.15^oC. This value of temperature is called absolute zero (ok) by Kelvin. It is to be noted that the value of pressure at this temperature reduce to zero.

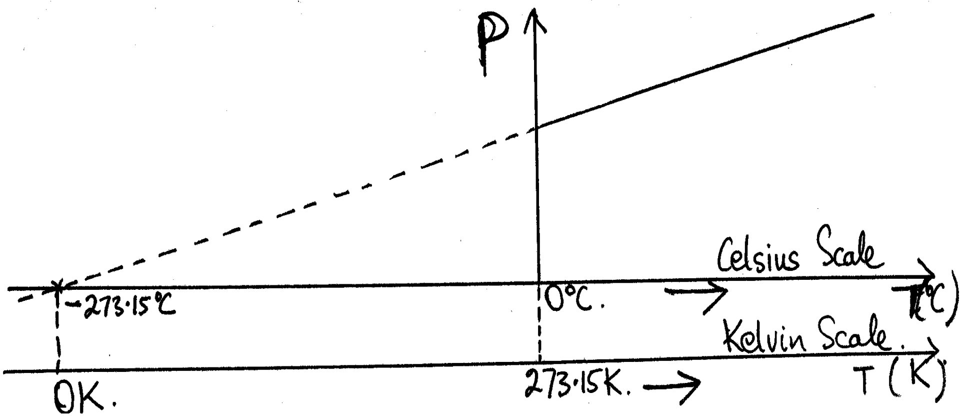

Fig. 3.9

On the Celsius temperature scale,

$$-273.15^oC = 0$$
$$0^oC = 273.15K$$

Hence the change in 1^oC on the Celsius scale is equal to the change of 1K on the Kelvin (Thermodynamics) scale.

4.0 CONCLUSION

What you have learnt in this unit concerns the concept of temperature and how it is measured. The property of a substance which varies with temperature is used to measure the temperature of a body through an appropriate scale. You have learnt about the Celsius, Fahrenheit and the Absolute scale. The property of a substance is used in the construction of a thermometer.

5.0 SUMMARY

In this unit, you have learnt:

- the difference between temperature and heat;
- the concept of temperature using the sensation of human feelings which is always sensitive;
- how the zeroth law is used to conceptualize the concept of temperature objectively by using some measurable properties such as length of a liquid, pressure on a gas at constant volume, resistance and the electromotive force;
- the scales of temperature measurements;
- that on the thermodynamics scale the triple point of water is chosen as the fixed point and is defined as 273.16K.

ANSWER TO SELF ASSESSMENT EXERCISE 1

- Heat is a form of energy while temperature is the degree of hotness or coldness in a body.
- Heat flows from higher temperature to a lower temperature. Thus the difference in temperature dictates the direction of transfer of the heat energy.

ANSWER TO SELF ASSESSMENT EXERCISE 2

See the text.

ANSWER TO SELF ASSESSMENT EXERCISE 3

Using the Eq. (1.4) as

$$t_c = \frac{(X_t - X_o)\ X\ 100^oC}{(X_{100} - X_o)}$$

Where, $X_t = 7.00cm$, $X_{100} = 20.86cm$, $X_o = 1.06cm$

Substituting this parameter in the Eq. (1.4) we get

$$\therefore \quad t_c = \left(\frac{7.00cm - 1.06cm}{20.86cm - 1.06cm}\right) x100cm\,^oC$$

$$= \frac{5.94cm}{19.80cm} x100\,^oC$$

$$t_c = 30^oC$$

ANSWER TO SELF- ASSESSMENT EXERCISE 4

Using Eq. (1.10) as

$$t_F = \left(\frac{9}{5}t_c + 32\right)^o F$$

Substitute $t_F = 50^oF$

$$\therefore\ 50^oF = \left(\frac{9}{5}t_c + 32\right)^o F$$

On rearranging the terms, we get

$$\therefore\ 50^oF = \frac{9}{5}t_c + 32$$

$$18 = \frac{9}{5}t_c$$

$$\therefore t_c = \frac{18X5^o}{9}C = 10^oC$$

ANSWER TO SELF ASSESSMENT EXERCISE 5

Use Eq. (1.11) as

(a) $t = \dfrac{R_t - R_o}{R_{100} - R_0} X100^oC$

Substitute R_t = 11.19 ohms, R_{100} = 14.35 ohms

R_o = 10.40 ohms

$$\therefore\ t = \frac{11.19\Omega - 10.40\Omega}{14.35\Omega - 10.40\Omega} X100^oC$$

$$t = \frac{0.79\text{ohms}}{3.95\text{ohms}} X100^oC$$

$$= \quad 20^oC$$

(b) Again use Eq. (1.11)

$$t = \frac{R_t - R_o}{R_{100} - R_0} X100^oC$$

Substituting t = 45°C, R_o and R_{100} take their usual values

On arranging the terms, we will get,

$$\therefore\ 45^oC = \frac{R_t - 10.40\Omega}{14.35\Omega - 10.40\Omega} X100^o C$$

$$\therefore\ 45^oC = \frac{R_t - 10.40\Omega}{3.9\Omega} X100^o C$$

$$\therefore\ \frac{45}{100} X3.95\Omega = R_t - 10.40\Omega$$

$$\therefore\ 1.7775\,\Omega = R_t - 10.40\,\Omega$$

$$\therefore\ R_t = (1.7775 + 10.4)\,\Omega$$

$$R_t = 12.18\,\Omega$$

6.0 TUTOR-MARKED ASSIGNMENT

1. At what temperature do the Fahrenheit and the Celsius scales coincide?

2. The normal boiling point of liquid oxygen is -182.97°C, what is this temperature on the Kelvin and Fahrenheit scales?

3. What is the body temperature of a normal human being on the Celsius scale? What will this value be on the Fahrenheit scale?

4. An ungraduated mercury thermometer attached to a millimeter scale reads 22.8mm in ice and 242mm in steam at standard pressure. What will the millimeter read when the temperature is 20°C?

7.0 REFERENCES/FURTHER READINGS

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UNIT 2 TYPES OF THERMOMETERS

CONTENTS

1.0 INTRODUCTION

In Unit 1, you learnt about the difference between temperature and heat. You also learnt about the concept of temperature by defining it qualitatively and quantitatively. The zeroth law was used to define temperature objectively and quantitatively. This was done by the use of measurable property of a substance that varies linearly or uniformly with changes in temperature.

In this unit, we shall go further by considering the thermometric properties of various substances used in the construction of thermometers. Finally, we shall describe some types of thermometers.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state the conditions necessary for the choice of properties and the substance used for temperature measurement
- list the different types of thermometers and their thermometric properties
- show the appropriate use of thermometers in life.

3.0 MAIN CONTENT

3.1 Thermometric Properties

In the last unit, we discussed about the temperature scale. You learnt that to establish such a scale, there is a requirement of some physical properties of a substance and the fixed point.

But, ***suppose you are asked to construct a thermometer which they may be used to measure the temperature of a body, what are the things you must consider in the choice of the property of the substance to be used?***

The following are some necessary conditions governing the choice of properties and materials for temperature measurement by a thermometer.

- The substance must have a considerable variation over a small increase in temperature. That means, it must be very sensitive to a small change of temperature.
- The change in the property of the substance must occur readily.
- The range of temperature over which the thermometer can be used should be as large as possible.
- The thermometric substance should have a small thermal capacity. This means that it must readily acquire the temperature of its surrounding.
- The time taken to acquire this temperature should be small. This will minimize time lag when it is used and when variation of temperature should follow.

SELF ASSESSMENT EXERCISE 1

Enumerate five thermometric properties that must be taken into consideration before choosing a substance to be used to measure the temperature of a body.

Notice, as we proceed further how these properties are used in the construction of thermometers that will now be discussed.

3.2 Types of Thermometer

Thermometers may be classified according to the thermometric properties used in constructing such thermometers. For example, we have:

- **Liquid-in-Glass Thermometers**: Such thermometers use the expansion of the liquid in the glass tube. As the liquid volume increases with temperature rise, the length of the liquid along the tube varies with temperature.

- **Constant Volume-Gas Thermometer**: Which makes use of a given mass of gas whose pressure varies with temperature at constant volume.

- **Resistance Thermometer**: Makes use of the variation of resistance of a given conductor such as platinum with temperature.

- **Thermo-electric Thermometer**: Which makes use of the variation of electromotive force (emf) developed between the pair of junctions of two dissimilar metals with temperature.

We shall now discuss each one of them in details.

3.2.1 Liquid-in-Glass Thermometers

The liquid-in-glass thermometer makes use of two liquids such as alcohol or mercury. The mercury-in glass thermometers are commonly used in the laboratory or in the hospitals as clinical thermometers. Whereas, the alcohol-in-glass thermometers are used in temperate countries because alcohol has a much lower melting point than mercury. (See table 3.1).

Table 3.1 Properties of Mercury and Alcohol as Thermometric Liquids

<table>
<tr><th></th><th>Mercury</th><th></th><th>Alcohol</th></tr>
<tr><td>1</td><td>Mercury solidifies at -39^oC</td><td>1</td><td>Alcohol solidifies at</td></tr>
<tr><td colspan="4">Hence alcohol may be used for much lower temperature than mercury’s.</td></tr>
<tr><td>2</td><td>Mercury boils at 357^oC</td><td>2</td><td>Alcohol boils at 78^oC</td></tr>
<tr><td colspan="4">Thus, alcohol is not suitable for temperatures above 50^oC or 60^oC whereas mercury could be used to measure temperatures well above
500^oC even as high as 800^oC provided the space above the mercury is filled with Nitrogen or some inert gases which increases its boiling point.</td></tr>
<tr><td>3</td><td>Mercury does not wet glass. It is able to move with jerky action.</td><td>3</td><td>Alcohol wets glass. The thread of alcohol has therefore the tendency to stick to the stem.</td></tr>
<tr><td>4</td><td>Mercury expands less than alcohol for a given rise in temperature.</td><td>4</td><td>Alcohol expands more than mercury for a given rise in temperature.</td></tr>
<tr><td colspan="4">Alcohol thermometers are more sensitive than mercury.</td></tr>
<tr><td>5</td><td>Mercury is opaque. Silvery in colour it is therefore easily seen.</td><td>5</td><td>Alcohol is transparent. It has to be coloured before it can be seen.</td></tr>
<tr><td>6</td><td>A good conductor of heat</td><td>6</td><td>A poor conductor of heat.</td></tr>
<tr><td>7</td><td>It has low specific heat capacity.</td><td>7</td><td>Has high specific heat capacity.</td></tr>
</table>

From the above table it will be observed that in certain areas alcohol has some advantages over mercury while in some areas mercury also has some advantages over alcohol.

SELF ASSESSMENT EXERCISE 2

State the advantages and disadvantages of the use of alcohol over mercury as thermometric liquids.

The increase in length (L) of the mercury-in-glass thermometer is used as a property as the temperature increases. Using the Celsius scale therefore, the temperature t is defined as:

$$t = \left(\frac{L_t - L_o}{L_{100} - L_0}\right) X 100^o C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.1)$$

where, L_t = length of the mercury column at t^oC
L_{100} = length of the mercury at 100^oC
L_o = length of the mercury at 0^oC

3.2.1.1 The Clinical Thermometers

The clinical thermometer is usually used in the hospital clinics, hence, the name. It is specifically adapted for measuring the temperature of the human body. The stem of the clinical thermometer is graduated from 35^oC to 45^oC.

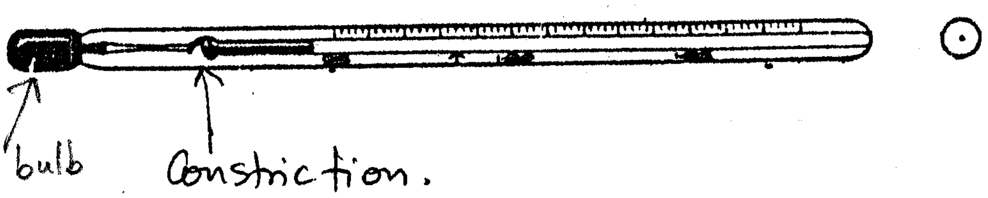

Fig. 3.1 The Clinical Thermometer

It has a constriction in the stem near the bulb, which prevents the mercury from entering the bulb. Thus the measurement can be read at leisure. Jerking the instrument in order to drive the mercury back then returns the broken thread of the mercury.

The laboratory thermometers are longer than the clinical thermometer s and they have no constrictions in them. Besides, they are usually longer with longer range of temperatures from -10^oC to about 150^oC.

3.2.1.2 The Maximum and Minimum Six's Thermometers

Apart from the laboratory and the clinical thermometers, there is yet another mercury-in-glass thermometer called the Maximum and Minimum (six's) thermometer.

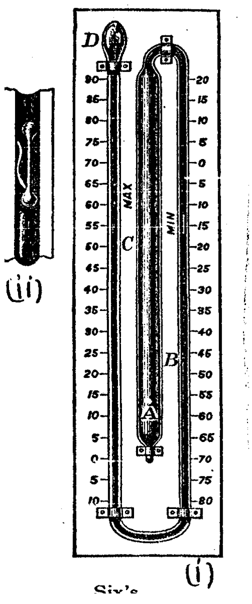

Fig. 3.2: Maximum and Minimum (Six's) Thermometer

The Six's thermometer is used to measure the maximum and minimum temperatures on a particular day (fig. 3.2).

The thermometer is made up of long bulb A that contains alcohol or nay other suitable liquid. BC is a thread of mercury. Above C, there is more alcohol which partly fills the bulb D. Above the surface of the mercury thread at B and C are light steel indices (fig 3.2) with a spring attached to each steel to prevent it from slipping.

When the alcohol in A expands, the mercury thread in BC is pushed round sending the index at C upwards and leaving g the index B in position. When the alcohol in A contracts, the mercury thread BC is drawn back leaving the C index to record the maximum temperature reached. The index B similarly records the minimum temperature reached. A magnet is used to reset each index by drawing them down to the surface of the mercury thread.

The Minimum and Maximum thermometer is therefore used to record the minimum and maximum temperatures of the day. The knowledge of such temperatures over a period of time may be useful in predicting the weather.

3.2.2 Constant Volume Gas Thermometers

The two properties that are kept constant in the construction of this thermometer are the volume (V) and the amount of the gas in moles (n). Thus the varying properties are the pressure (P) on the gas and the temperature (t).

Consider a fixed mass of gas maintained at constant volume in a vessel capable of measuring the pressure P. If P_0 and P_{100} are the pressures at ice and steam points respectively and Pt is the pressure at some unknown temperature t^oC. Assuming that equal changes in pressure denote equal changes in temperature, then

$$\frac{P_t - P_o}{P_{100} - P_0} X 100^o C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.2)$$

Jolly's Constant volume air thermometer will be described to show how the pressures are measured at constant volume but at varying temperatures (fig.3.3).

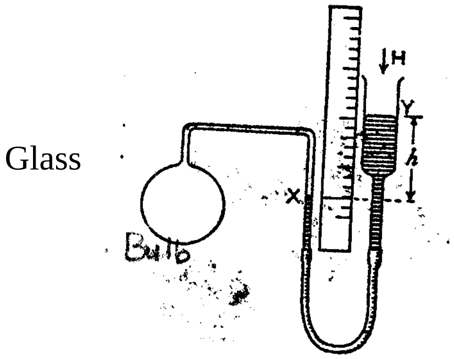

Fig. 3.3 Constant Volume Air Thermometer

It consists of a glass bulb of about 100cm^3 in volume containing dry air. This bulb is connected to a fine glass capillary tube and a rubber pressure tube, which in turn is connected to a moveable reserviour of mercury.

There is mark X on the capillary tube. This is the constant volume mark. The total pressure on the air is (H + h) where H is the atmosphere pressure and h is the height of the mercury above the mark X.

If $P_0 = H + h_0$

$P_{100} = H + h_{100}$ and

$P_t = H + h_t$

Then $$t = \left(\frac{P_t - P_o}{P_{100} - P_0}\right) x 100^o C$$

$$t = \left(\frac{h_t - h_o}{h_{100} - h_0}\right) x 100^o C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.3)$$

The Jolly's constant-volume gas thermometer is not very convenient to use, when compared with the mercury-in-glass, resistance and thermo-electric thermometers.

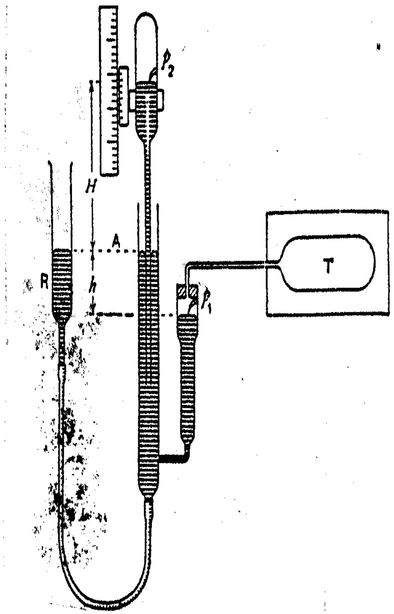

Fig. 3.4: Standard Constant–Volume Hydrogen Thermometer

Fig. 3.4 is a standard constant-volume hydrogen thermometer as developed by Chappius. It consists of Platinum Iridium cylinder T which is connected to a manometer. The constant volume is achieved through the devise P_1. A barometer is also incorporated with the manometer using a device P_2. The vertical distance between P_1 and P_2 gives the total pressure.

The Chappius version is an improvement of Jolly’s simple gas thermometer. It is to overcome the inaccuracies in Jolly’s instrument due to fluctuations in the barometric heights in the course of the experiment. It is also to reduce errors due to capillary depression on the mercury which come to play when using tubes of narrow bore in Jolly’s instrument.

3.2.3 Platinum Resistance Thermometers

As the name of the thermometer suggests, it is a thermometer that depends on the resistance of a wire whose values change with temperature.

Callendar found out that the resistance of pure metals increases as the temperature increases. He showed that the resistance of pure platinum varies with the gas thermometer temperature t according to the equation.

$$R_t = R_0 (1 + A_t + B_t^2) \ldots\ldots\ldots\ldots\ldots (2.4)$$

Where, R_t and R_0 are the resistances at t°C and 0°C respectively. A and B are some constants for a given specimen.

On the platinum resistance scale of temperature, equal changes in resistance denote equal changes in temperature.

Fig. 3.5 Platinum Resistance

Fig. 3.5 shows a typical platinum resistance which is used in connection with the Wheatstone (Meter) Bridge devised by Callendar and Griffiths (Fig. 2.6). The bridge is used to obtain a balance point which enables the experiment to determine the various values of R_t , R_{100} and R_0

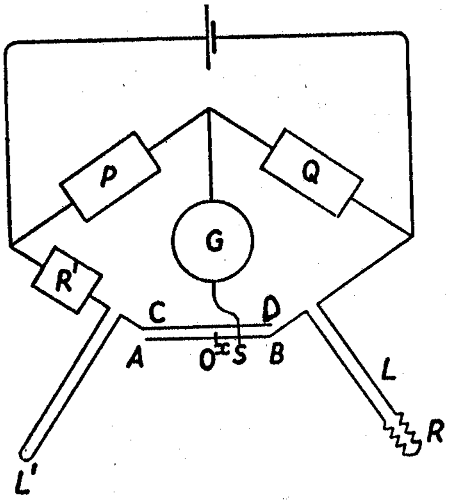

Fig. 2.6 Callendar and Griffiths Wheatstone Bridge

Let R_0 and R_{100} be the resistances of ice and steam respectively and R_t the resistance of the platinum at t_p°C.

$$T_P = \frac{R_t - R_o}{R_{100} - R_0} X 100^\circ C \text{ ………….……….. } (2.5)$$

The platinum resistance thermometer has a wide range noted for its extreme accuracy over the length of this range. However, its chief disadvantage is the long time needed for it to assume the temperature of its surroundings and the time required for making an observation so tat it cannot follow rapidly changing temperatures. This is where the thermo-electric thermometer is a good substitute.

3.2.4 Thermo-Electric Thermometers

The thermo-electric thermometer is otherwise called thermo-couple. This type of thermometer is constructed by using the Seebeck effect. First, we will discuss about the Seebeck effect.

Seebeck effect simply states that if two dissimilar metals, such as copper and iron are joined to make a complete circuit, then on heating one end of the junctions, a current flows round the circuit.

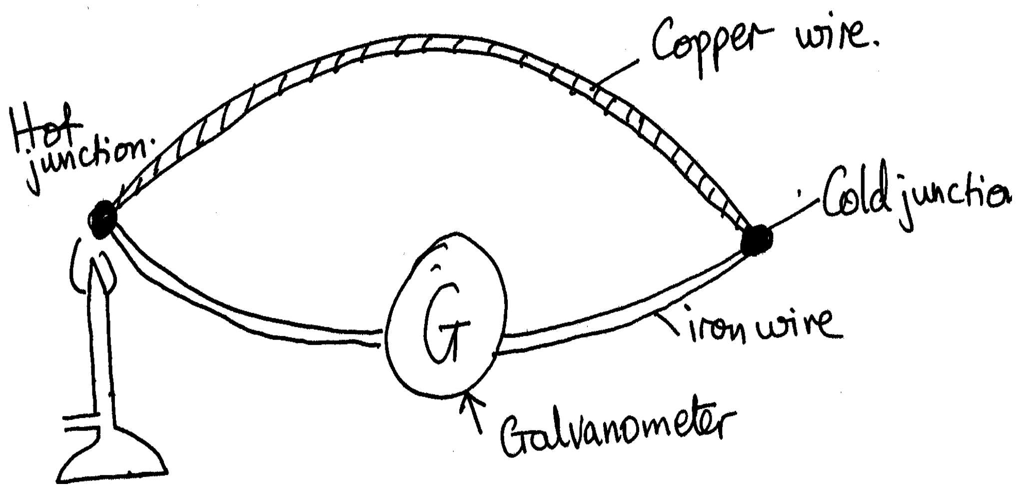

Fig. 3.7: Thermo-Electric Couple

The presence of the galvanometer is to detect and measure the magnitude of the current that flows in the circuit.

This type of arrangement is called a thermo-couple. The emf established round the circuits depends on the nature of the metals used to form the couple and also on the temperature e difference between the hot and cold junctions.

The cold junction is usually maintained at ice point. It has been shown experimentally that when the other junction is at some temperature t, the thermo-electric emf set up depends on the temperature accordingly as

$$E = A + Bt + Ct^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (2.6)$$

Where A, B and C are constants depending on the metals used. The graph between emf versus t is shown in fig. 3.8.

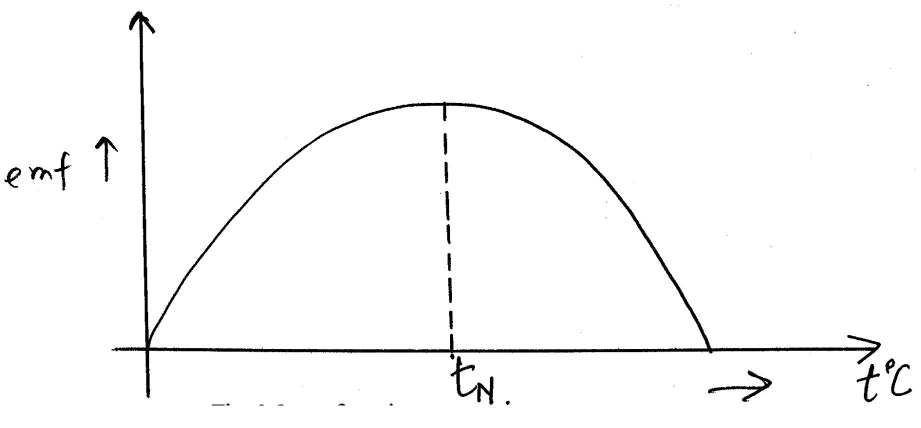

Fig. 3.8: emf against t

The temperature where the emf is maximum for the two metals is called the neutral temperature t_N.

It is however more convenient to observe the thermo-electric behaviour of the two metals graphically by plotting their thermo-electric power (P) against temperature.

Thermo-electric power is the change in the thermo-electric emf per degree Celsius in temperature between the hot and cold junctions.

From the Eq. (2.6) if t is measure from the ice point, then A is equal to zero. Then,

$$\therefore \quad E = Bt + Ct^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.7)$$

The graph of E versus t is either fig. 3.9 (i) or fig. 3.9 (ii) as shown below.

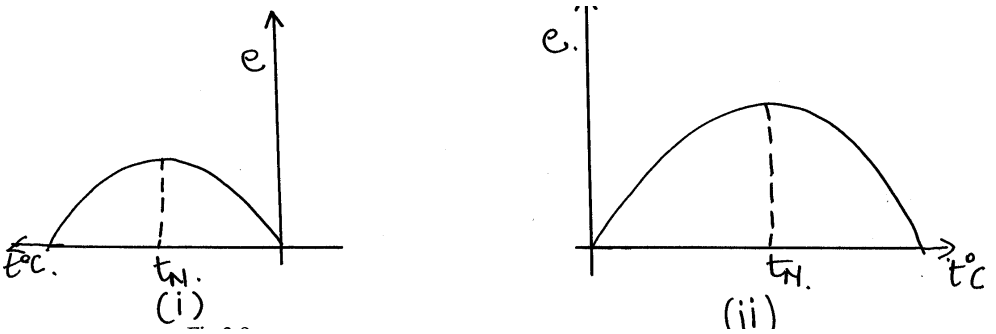

Fig. 3.9

But Power (P) is given by

$$\frac{de}{dt} = B + 2Ct \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2.8)$$

Thus the plot of $\frac{de}{dt}$ of versus t is either given by fig. 3.10 (i) or fig. 3.10 (ii)

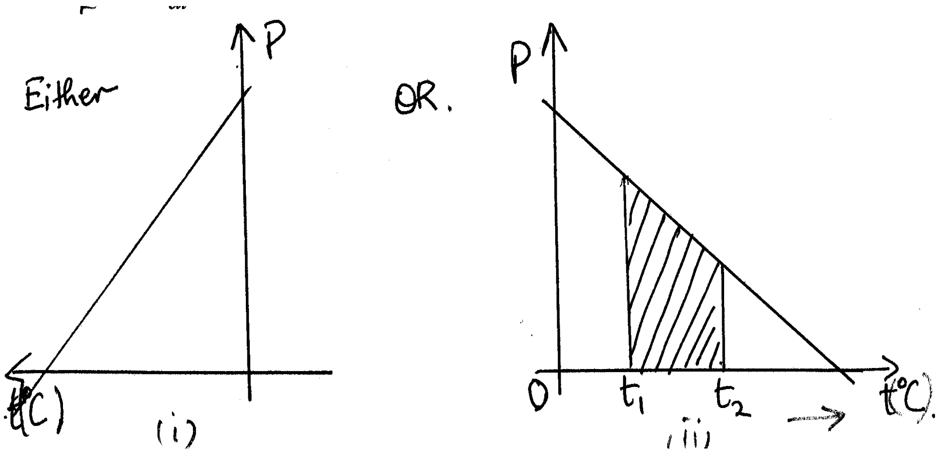

Fig. 3.10

Since we are increasing the temperature from zero to higher temperature fig 3.10 (ii) will be a better choice.

The intercept on the t – axis indicates where $\frac{de}{dt} = 0$ and that is the neutral temperature. It is possible to establish from this graph the emf when the junctions are kept at temperatures t_1 and t_2. It is equal to the area under the curve between t_1 and t_2.

$$E = \int_{t_1}^{t_2} P dt \qquad \text{……………………………………………….. (2.9)}$$

In theory, it is more satisfactory to measure the electromotive force (emf) developed instead of the current. This is because the emf depends on the total resistance of the circuit which alters as the temperatures changes. In the industrial set up, a high resistance millivoltmeter is used. The thermo-electric couple is capable of measuring up to 1500°C.

4.0 CONCLUSION

You have learnt in this unit the different types of thermometers used in measuring temperatures. A thermometer uses the property of a substance which varies uniformly with changes in temperature. It is this thermometric property that is used in the construction of thermometers. Each thermometer has its own range of measurement.

5.0 SUMMARY

In this unit, you have learnt the following:

- Thermometers ate constructed using a specified physical property known as thermometric property.
- Such thermometric properties are volume of a liquid, pressure of a gas at constant volume, resistance of a metal and the electromotive force of a thermocouple which vary with temperature.
- There are different types of thermometers. Each type is grouped according to the kind of thermometric property used in its construction. There are:

  (i) Liquid-in-glass thermometers
  (ii) Resistance thermometers
  (iii) Thermo-electric thermometer

- The merits and demerits of each type have been discussed.

ANSWER TO SELF ASSESSMENT EXERCISE 1

Refer to section 3.1 of the text.

ANSWER TO SELF ASSESSMENT EXERCISE 2

Refer to section 3.2.1 for solution.

6.0 TUTOR-MARKED ASSIGNMENT

1. State the five conditions necessary for the choice of properties and the substance used for temperature measurement.
2. Mention the types of thermometers that exist and their appropriate thermometric properties used for their construction.

3. A body has temperature of about 1200°C. Which thermometer would you use and why?

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UNIT 3 HEAT MEASUREMENT

CONTENTS

1.0 INTRODUCTION

Heat and temperature are not the same but they are closely related. Heat, as you must have learnt is a form of energy while temperature is the degree of coldness or hotness of a body, may the body be solid, liquid or gas. This degree of coldness or hotness is measured objectively and quantitatively by suing the thermometer. ***The question arises is: "how would you measure heat energy?"*** The study of heat measurement is known as Calorimetry in physics.

The nature of substance plays an important role in the study of thermal phenomena. For example, a large iron tank and an aluminum kettle have a different heat capacity. It depends on their respective masses and on the metal need.

In this unit you will be introduced to the measurement of heat. The nature of substance plays an important role in the study of thermal phenomena. For example, a large iron tank and an aluminum kettle have a different heat capacity. It depends on their respective masses and on the metal used. Some of the major concepts crucial to the measurement of heat are: heat capacity, specific heat capacity, specific latent heat of fusion and vapourization which you will also study here. You will also learn about how temperature measurement is related to the measurement of heat energy.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain the concept of heat

- define the heat (thermal) capacity of a substance
- define specific heat capacity of a substance
- define (a) the specific latent heat of fusion (b) specific latent heat of vapourization
- solve problems involving the concepts mentioned above.

3.0 MAIN CONTENT

3.1 Concept of Heat

Prescot Joule first showed that heat is a form of energy in an experiment in which mechanical work (energy) was transformed into heat. Other scientists also showed that heat from fuel, such as gasoline in an engine, may be transformed to mechanical energy when the engine is used to drive carts, trains and aeroplanes. In an electric power station, heat from fuel is changed to electrical energy.

The unit of energy is Joule (J) which is also the unit of measuring mechanical energy and electrical energy. We also have larger units such as kilojoule (KJ) and megajoule (MJ).

Power, you will remember, is the rate of doing work. It is also defined as the rate at which heat energy is given out by a source. For example, the heat energy delivered per second by a gas burner is its power. This power is measured in watts (W). One watt is therefore defined as one joule per second (J/s or Js^{-1}). Other larger units are kilowatt (KW) and megawatt (MW).

Sources of heat energy are the sun, fuels such as coal, gas, oil and electricity.

Can you think of other sources of heat energy?

3.2 Heat Capacity

You would have observed that a source of heat will transfer its heat energy to another body. The source is usually at a high temperature while the other body being heated is at a lower temperature. When the source and the other body are in contact, the rise in temperature takes place in colder body.

Let $t_1{}^oC$ be the initial temperature of the body and $t_2{}^oC$ be the final temperature when Q joules of heat has been supplied.

Then the change in temperature,

$$\Delta\theta = (t_2 - t_1)^oC$$

The amount of heat in joules that is capable of changing its temperature through 1^oC is $\frac{Q.J}{\Delta\theta}$

This amount of heat to change the temperature of the body is described as the heat capacity of the body. It is usually represented by the symbol H.

By definition, the heat (thermal) capacity (H) of a body, is the quantity heat (Q) in joules required to change its temperature by one degree (Celsius or one Kelvin).

$$\therefore \quad H = \frac{QJ}{\Delta\theta\,^{o}C}$$

Where, $\Delta\theta = (t_2 - t_1)$

$$\therefore \quad H = \frac{QJ}{(t_2 - t_1)^{o}C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.1)$$

Thus the unit of heat (thermal capacity is expressed in joules per Celsius or joules per Kelvin (JK^{-1} or $J\,^{o}C^{-1}$).

The values of H for different bodies are not the same. They vary from one body to another.

EXAMPLE 3.1

A metal container of heat capacity $200J\,^{o}C^{-1}$ is heated from $15^{o}C$ to $45\,^{o}C$. What is the total quantity of heat required to do so?

SOLUTION 3.1

Using Eq. (3.1)

$$H = \frac{Q}{\Delta\theta}$$

$$\therefore \quad Q = H\Delta\theta$$

$$= H\,(t_2 - t_1)$$

Substituting the values, we get

$$= 200\ J/\,^{o}C \times (45 - 15)^{o}C$$
$$= 200 \times 30J$$
$$Q = 6000J$$
$$Q = 6KJ$$

3.3 Specific Heat Capacity

If there are different masses of a substance m_1, m_2 and m_3, it will be observed that to raise their temperatures through $1^{o}C$ each, they will require different quantities of heat energy Q_1, Q_2 and Q_3.

Experiments have shown that the quantities of heat (Q) required to change their temperature through 1°C is proportional to the corresponding masses (m).

$$\therefore \; Q \,\alpha\, m \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (3.2)$$

However, if we fix the mass of the substance to 1kg an we transfer various quantities of heat Q to it, there will be various corresponding changes in temperature$\Delta\theta$. Again, it will be found that the quantities of heat Q to change the temperature of 1kg mass of the body will be proportional to the corresponding changes in temperature$\Delta\theta$. Hence we can write

$$Q \,\alpha\, \Delta\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (3.3)$$

Combining these two factors, we get

$$Q \,\alpha\, m\Delta\theta$$

Or $$Q = Cm\Delta\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (3.4)$$

Where C is a constant of proportionality known as the specific heat capacity of the substance

$$\therefore \quad C = \frac{Q}{m\Delta\theta}$$

The specific heat capacity of a substance is therefore defined as the amount of heat Q (in joules) required to raise the temperature of 1kg mass of substance through unit degree (1°C or 1°K).

The unit of C is J/kg °C or $Jkg^{-1 0}C^{-1}$ or $Jkg^{-1}K^{-1}$. It can also be measured in cal. $g^{-1}\,k^{-1}$

The value of C differs from one substance to another.

The values of specific heat capacity for some common substances are given in Table 3.1 below.

Table 3.1: Specific Heat Capacity for Some Substances

Substance	Specific heat capacity in $Jkg^{-1o}C^{-1}$
Iron	460
Copper	400
Lead	120
Aluminum	800
Water	4200

EXAMPLE 3.2

How much heat is needed to bring 10g of water from 50°C to boiling point? (Specific heat capacity of water = 4200 $Jkg^{-1o}C^{-1}$)?

SOLUTION 3.2

Using Eq. 3.4 $Q = mC\Delta\theta$

$= mC\,(t_2 - t_1)$

Substituting the values, in the Eq. 3.4 we get,

$= 10g/1000kg \times 4200J/kg\,^{o}C \times (100 - 50)\,^{o}C$

$Q = 2100J$

$= 2.1KJ$

3.3.1 Simple Method of Mixtures

In this section we shall consider exchange of heat between two bodies in such a way that one body is at a high temperature and the other is at a low temperature. In the simple method of mixtures, we are simply looking at hot and cold substances being mixed without considering the container in which they are being mixed. The principle of conservation of heat energy is being observed very closely. **Here, very briefly, we will discuss the principle of conservation of heat energy**.

This principle states that ***"the heat lost by a hot body is equal to the heat gained by the cold body in any system provided there is no heat exchange between the substances involved and their surrounding"***

But you may now ask: Is this always true?

Unfortunately, this heat exchange cannot be completely true. However, the heat lost to the surrounding or gained from it can be reduced to a negligible amount by surrounding the container (calorimeter) with a bad conductor of heat (insulator). This process is called lagging.

In a laboratory, use the calorimeter as the container with which you observe exchange of heat between two substances, keeping them usually at different temperatures. The cold body is usually in form of water or any liquid and the hot body could be a solid body or liquid at a higher temperature.

Did you understand?

Let me illustrate this principle with this example,

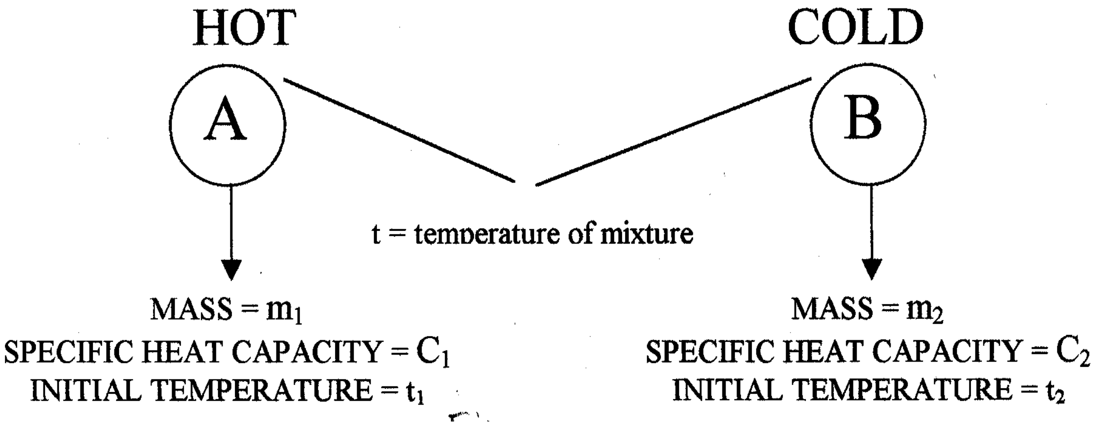


Fig. 3.1

If body A is the hot body at temperature t_1 and B is the cold body at temperature t_2, A and B are then mixed up to give a final temperature of mixture t. **Let us see then what has happened?**

(1) The hot body has lost some heat to the cold body B. The heat lost can be find out.

Let Q_1 be the amount of heat

$Q_1 = m_1c_1\Delta\theta_1$

∴ $Q_1 = m_1c_1\Delta\theta(t_1 - t)$

Or $= Q1 = -m_1c_1\Delta\theta(t - t_1)$ ………………………………………. (3.5)

(2) The cold body B has gained some heat. The heat gained is,

Let Q_2 be the amount of heat gained.

∴ $Q_2 = m_2c_2\Delta\theta_2$

$Q_2 = m_2c_2\Delta\theta(t - t_2)$ ………………………….. (3.6)

(3) How do you relate Q_1 and Q_2? We relate them together by suing the principle of conservation of heat energy, which says that

"In any heat exchange provided heat is not lost to or gained from the surrounding."

Heat lost = Heat gained

∴ $Q_1 = Q_2$

∴ $m_1c_1\Delta\theta_1 = m_2c_2\Delta\theta_2$

∴ $m_1c_1(t_1 - t) = m_2c_2(t - t_2)$ ……………. (3.7)

Out of all the seven quantities m_1, c_1, t_2, t, m_2, c_2 and t_1 all will be known except one. Now try to look for this only unknown quantity.

Let us solve an SAQ to understand this more clearly.

SELF ASSESSMENT EXERCISE 1

A piece of iron of mass 0.20kg is heated to 64°C and then dropped gently into 0.15kg of water at 16°C. If the temperature of the mixture is 22°C, what is the specific heat capacity of iron?

3.3.2 Inclusion of Calorimeter in Method of Mixtures

In section 3.3.1, we talked about simple method of mixtures in which the idea of the container taking part in the heat exchange was excluded. In this section, we are going to consider the calorimeter as one of the major players in the heat exchange (see Fig. 3.2). Once again, we shall still consider the principle of conservation of heat energy in our discussion.

Here we have the hot body A at a higher temperature than the liquid contained in the calorimeter. Thus, the liquid and the calorimeter are considered as the cold body gaining heat from the hot body.

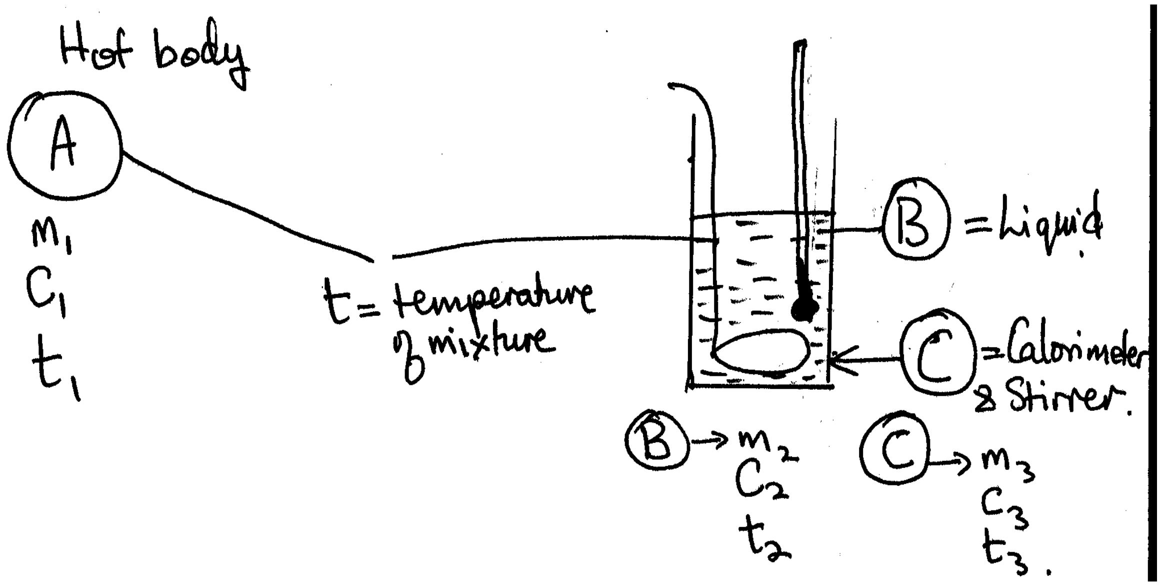


Fig. 3.2

Consider the hot body (A) with mass m_1, Specific heat capacity C_1 its initial temperature t_1 and Final temperature of mixture is = t.

Then, Heat lost by hot body:

$$Q_1 = m_1c_1\Delta\theta_1$$
$$Q_1 = m_1c_1(t_1 - t) \quad \ldots\ldots (3.8)$$

Again consider the cold liquid (B) with mass m_2, specific heat capacity C_2 with initial temperature t_2. The final temperature of the mixture is t.

Heat gained by cold liquid (B) = Q_2

$$Q_2 = m_2C_2\Delta\theta_2$$
$$Q_2 = m_2C_2\,(t - t_2) \quad \ldots\ldots \quad (3.9)$$

Consider the cold calorimeter (C) with mass m_3, specific heat capacity C_3, Specific heat capacity = C_3 Initial temperature t_3 and Final temperature of mixture t.

Heat gained by cold calorimeter C is Q_3.

$$Q_3 = m_3C_3\Delta\theta_3$$
$$Q_3 = m_3C_3\,(t - t_3) \quad \ldots\ldots \quad (3.10)$$

It can be noted here that the liquid (B) and calorimeter (C) are both gaining heat. Therefore their changes in temperature will be the same.

$$\therefore \quad \Delta\theta_2 = \Delta\theta_3 \quad \ldots\ldots \quad (3.11)$$

$$\therefore \quad (t - t_2) = (t - t_3)$$

$$\therefore \quad Q_3 = m_3C_3\,(t - t_3) \quad \ldots\ldots \quad (3.12)$$

In most cases, m_3 and C_3 may not be provided for you in the heat exchange. Rather, a property of the container, i.e. the calorimeter may be provided in the form of the thermal capacity (H) of the calorimeter. Hence, the quantity of heat gained by the calorimeter is expressed as

$$Q_3 = H\Delta\theta_3 = m_3C_3\Delta\theta_3 \quad \ldots\ldots \quad (3.13)$$

$$\therefore \quad Q_3 = H(t - t_2) \quad \ldots\ldots \quad (3.14)$$

By applying the principle of conversation of heat energy, the relation between the quantities Q_1, Q_2 and Q_3 is

Heat lost = Heat gained

$$\mathbf{Q_1} = \mathbf{Q_2} + \mathbf{Q_3} \quad \ldots\ldots \quad (3.15)$$

$$\therefore \quad m_1C_1\,(t_1 - t) = m_2C_2\,(t - t_2) + m_3C_3\,(t - t_2)$$

Or $\quad m_1C_1\,(t - t_1) = m_2C_2\,(t - t_2) + H\,(t - t_2) \quad \ldots\ldots \quad (3.16)$

SELF ASSESSMENT EXERCISE 2

A calorimeter contains 0.30kg of water at 12°C. When poured in, the temperature of the mixture is found to be 52°C. What is the heat capacity of the calorimeter?

3.4 Latent Heat

When matter is heated, you will recall, there are three observable effects. Heat causes matter to:

- Expand
- Change its temperature
- Change its state

Let us look closely at the last two effects, changes in temperature and state. The first effect, expansion, will be considered later. The effect of the temperature change has been used to measure the quantity of heat absorbed or given out by a substance. The expression used in = $mC\Delta\theta$ (where $\Delta\theta$ is the change in temperature).

But what about the change in state?

We shall now look critically at the effect of heat on a solid ice-block as it is heated from say -10°C to the boiling point 100°C.

Fig 3.3 shows the graph of temperature versus time which allows us to understand the concept of Latent heat which is under discussion.

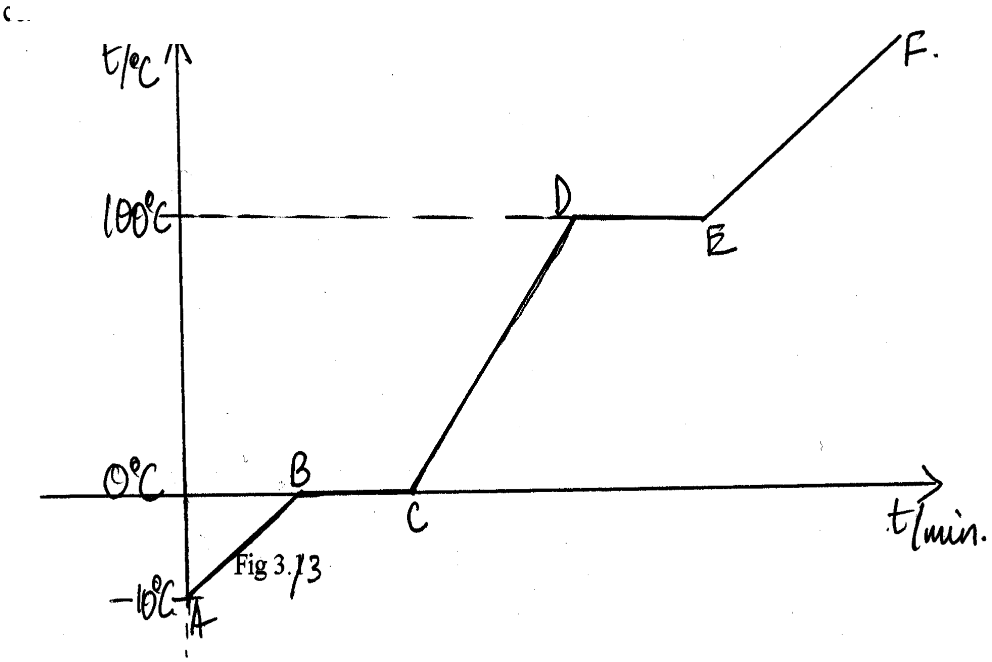


Fig. 3.3

- AB shows increase in temperature as the time increases
- BC shows no increase in temperature with respect to time
  At this point the solid ice is observed to change its state from ice (solid) to water (liquid). This takes place as 0°C – the melting/freezing point of water. The process is called melting. The reverse process of melting is freezing.
- CD shows an increase in temperature with respect to time
- DE again shows no increase in temperature as the water is being heated. This happens at another fixed temperature of 100°C which is the boiling point of water. The liquid water changes its state from water to steam (gas). The process is called vapourization. The reverse process of vapourization is condensation.
- EF shows that there is an increase in temperature of steam with respect to time. In this region water behaves as a gas whose behaviour will be studied later.

It is significant to note that BC and DE show regions at which water changes its state from solid to liquid and from liquid to gas respectively. The reverse could also take place by extracting heat from the system then we have condensation, the reverse of vapourization and then freezing, the reverse of melting.

But, how do we measure the heat content when water or any substance changes its state?

To answer this question will therefore be involved with energy-temperature relationship of the processes of melting and vapourization or condensation and freezing.

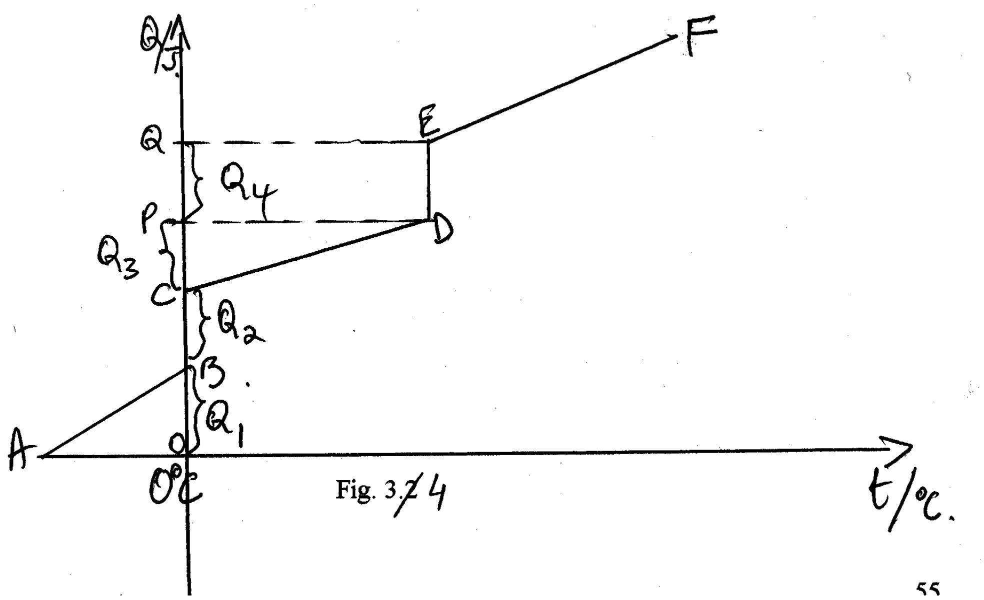


Fig. 3.4

Refer to figure 3.4

- AB indicates the absorption of heat by the solid material to change its temperature from -10^{o}C to 0^{o}C. The amount of heat supplied is OB = Q_1.

  Where $Q_1 = m_1C_1\Delta\theta_1$. Here m_1, represents mass of ice, and m_1 = mass of ice, C_1 is the specific heat capacity of solid ice is the change in temperature from -10^{o}C to 0^{o}C

  $\Delta\theta_1 = (0 - (-10))^oC = 10^oC$
  $\therefore \quad Q_1 = m_1C_1 \times 10J$

- BC corresponds to the absorption of heat energy without any change in temperature. The energy appears latent, that is, hidden since there is no change in temperature.

  The amount of heat supplied to BC is $Q_2 = mL_F$, where, m is the mass of ice and L_F is the specific heat of fusion.

Latent heat of fusion L_F for ice = 3.3×10^5 Jkg^{-1}

Specific latent heat of fusion is defined as the amount of heat required to change 1kg of mass of solid at its melting point to liquid at the same temperature".

- CD corresponds to the increase in temperature of substance in the liquid state. CP = $Q_3 = mC_2\Delta\theta$ is the quantity of heat supplied to change its temperature from its melting point to 100^{o}C, the boiling point. Here m is the mass of the liquid converted from solid ice, C_2 is the specific heat capacity of water, and $\Delta\theta_2$ is the change in temperature from 0^{o}C to 100^{o}C.

- DE shows that there is an increase in heat energy while the temperature does not change. De therefore corresponds to the change in state from liquid to vapour. The heat energy appears latent, that is, hidden, since there was no change in temperature. The quantity of heat supplied is CP.

  CP = $Q_4 = mL_V$

Where, m is the mass of water and Lv is the specific latent heat of vapourization at its boiling point.

Therefore, we can define the specific latent heat of vapourization of a liquid.

"The specific latent heat of vaporization Lv of a liquid is the quantity of heat required to change 1kg of a liquid at its boiling point to vapour at the same temperature".

The value of Lv for water is $2.4 \times 10^6 Jkg^{-1}J$ at 100^oC and external pressure of 760mmHg.

- EF corresponds to the increase in temperature of the vapour. The heat absorbed by the vapour will depend on the condition under which it is treated i.e. whether it is treated at constant pressure or at constant volume. These ideas will be discussed under the concept of molar heat capacity of gases at a later stage of your physics course.

  You therefore need to understand the effects of heat on matter under consideration and to know the process it is undergoing when you are solving problems involving changes in temperatures and or changes in state. For example, the total heat energy required to change the mass m of ice at -10^oC to vapour at 100^oC is Q where

$$Q = Q_1 + Q_2 + Q_3 + Q_4$$

$$Q = m_1C_1\Delta\theta_1 + mL_F + m_2C_2\Delta\theta_2 + mLv_s \text{ ………… (3.17)}$$

SELF ASSESSMENT EXERCISE 3

1. How much heat is needed to melt 1.5kg of ice and then to raise the temperature of the resulting water 50^oC?

2. The water at 0^oC is changed to 50^oC

$$\begin{aligned} Q_2 &= m_1C_1\Delta\theta_1 \\ &= 1.5kg \times 4200Jkg^{-1}\,{}^oC^{-1} \times (50-0)^oC \\ &= 1.5kg \times 4200Jkg^{-1}\,{}^oC^{-1} \times 50^oC \\ &= 3.15 \times 10^5J \end{aligned}$$

3. Total amount of heat required is

$$\begin{aligned} Q &= Q_1 + Q_2 \\ &= 1.5kg \times 3.3 \times 10^5Jkg^{-1} + 1.5kg \times 4200Jkg^{-1}\,{}^oC^{-1} \times 50^oC \\ &= 4.95 \times 10^5J + 3.15 \times 10^5J \\ &= 8.1 \times 10^5J \end{aligned}$$

It can be concluded that there are two kinds of latent heat – the one at the melting point and the other at the boiling point. Thus we define

- **Latent heat of fusion as the heat required to melt a whole mass of solid at melting point to liquid at the same temperature.**
- **Latent heat of vapourization is the heat required to convert a whole mass of liquid at boiling point to vapour at the same temperature. It is expressed in joules per kilogram (Jkg^{-1}).**

3.5 Explanation of Latent Heat

In this section, we would like to explain the meaning of latent heat of fusion and vapourization on the basis of kinetic molecular theory of matter.

3.5.1 Specific Latent Heat of Fusion

A solid consists of atoms or molecules held in affixed structure by forces of attraction between them. These atoms or molecules vibrate about their mean position. When heat is therefore supplied to the solid, the kinetic energy of vibration increases thus increasing the temperature of the solid. The heat supplied is measure by $mC\Delta\theta$. At the melting point, the heat given to the solid is used to overcome the forces of attraction between the atoms or molecules, which keep the solid in its rigid form, and then the solid melts.

At this point we define the specific latent heat of fusion (L_F) as ***the quantity of heat required to change 1kg mass of a solid at its melting point to liquid at the same temperature.***

EXAMPLE 3.3

Explain the statement “the specific latent heat of fusion of ice 3.3 x 10^{-5} Jkg^{-1}”

Solution 3.3

This means that 3.3 x 10^{5} joules of heat energy is required to change 1kg of solid ice at 0^{o}C to 1kg of water at the same 0^{o}C temperature.

3.5.2 Specific Latent Heat of Vapourization

Unlike solids, a liquid has no definite form; it usually takes the shape of its container. It molecules move in random manner inside although the molecules are close enough to attract each other. Some of the molecules, which have the greatest kinetic energy, are able to escape through the surface. They then exist as vapour outside the liquid. This process is called evaporation and it takes place at all temperatures.

However, boiling occurs at a definite temperature, the boiling point which depends on the external pressure. Water for example boils at 100^{o}C and at a pressure 760mmHg. It does so at a lower temperature when the external pressure is lower e.g. boiling point of water at the top of a mountain is less that 100^{o}C. Boiling occurs throughout the whole volume of the liquid whereas evaporation is a surface phenomenon.
At this point, we would define the specify latent heat of vapourization as ***the quantity of heat required to change 1kg mass of liquid at boiling point to vapour at the same temperature***.

SELF ASSESSMENT EXERCISE 4

How much heat is given out when 50g of steam at 100^{o}C cool to water at 28^{o}C? (Specific latent heat of vapourization of water = 2.3×10^{6} J/kg).

3.5.3 Latent Heat and Internal Energy

When a liquid reaches its boiling point, the energy needed to change it to vapour is:

(i) the energy or work needed to separate the liquid molecules from their mutual attraction until they are relatively far apart in the gaseous state.
(ii) the energy or work needed to push back the external pressure so that the molecules can escape from the liquid.

The latent heat of vapourization is used in point (i) which is needed to change the internal energy of the liquid whereas point (ii) is the external work done against the external pressure.

The work done in this case is defined as

$$W = p\Delta V \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.18)$$

Where, p is the external pressure and ΔV is the change in volume
1g of water changes to about 1672cm^3 of steam

$$\therefore \quad \Delta V = (1672 - 1)\text{cm}^3$$
$$= 1671 \times 10^{-6}\text{cm}^3$$

Assuming the external pressure
$P = 1.013 \times 10^{5}$ N m^{-2}

$$\therefore \text{ The work done (W)} = p\Delta V \text{ is therefore}$$
$$= 1.0\ p\Delta V\ 13 \times 10^{5}\ \frac{\text{N}}{\text{m}^2} \times 1671 \times 10^{-6}\,\text{m}^3$$
$$= 169.3\text{J}$$

The latent heat of vapourization per gram of water = 2260J

Thus the internal energy part of the latent heat of vaporization
$= (2260 - 169.3)$J
$= 2090.7$J

This is much greater than the external work done.

The latent heat of fusion is about 340J. So the energy needed to overcome the bonds between molecules in the solid state is much less than the energy to form the gaseous molecules from the liquid state.

SELF ASSESSMENT EXERCISE 5

1g of steam condenses to water 100 °C. Estimate the change in potential energy per molecule, given that the latent heat of vapourization is 2240J and that Avogadro's constant is 6 x 10^{23} per mole. (Assume no heat is gained from or lost to the surrounding).

4.0 CONCLUSION

The three basic effects of heat have been identified under this unit. The effects are expansion, change in temperature and change in state. The change in temperature and change in state have been associated with the measurement of heat. With the change in temperature, the quantity of heat is expressed as $Q = mC\Delta\theta$, whereas for the change in state the heat required is given as $Q = mL_F$ or $Q = mL_V$.

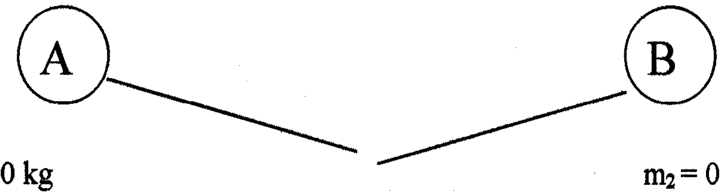
5.0 SUMMARY

In this unit you have learnt that:

- Heat is a form of energy and is measured in joules (J)
- The heat capacity of a substance is the quantity of heat required to change the temperature of the body through 1°C.
- The specific heat capacity of a substance is the quantity of heat in joules that is required to change the temperature of 1kg mass of the substance through 1°C.
- The specific latent heat of fusion of a solid is the quantity of heat in joules that is required to change the state of 1kg mass of the solid at the melting point to liquid at the same temperature.
- The specific latent heat of vapourization of a liquid is the quantity of heat in joules required to change 1kg mass of the liquid at its boiling point to gas at the same temperature.

ANSWER SELF ASSESSMENT EXERCISE 1

HOT – IRON **COLD – WATER**



$m_1 = 0.20$ kg
$c_1 = ?$
$t_1 = 64\ ^0C$

$t = 22\ ^0C$ (temperature of mixture)

$m_2 = 0.15$ kg
$c_2 = 4200$ J/kg ^0C
$t_2 = 16\ ^0C$

Heat lost by iron,

$$
\begin{aligned}
Q_1 &= m_1c_1\Delta\theta_1 \\
&= m_1c_1(t_1 - t) \\
&= 0.20\text{kg} \times c_1 \times (64 - 22)^oC
\end{aligned}
$$

Heat gained by water, $Q_2 = m_2c_2\Delta\theta_2$

$= m_2c_2(t - t_2)$

$= 0.150\text{kg} \times 4200\ \text{Jkg}^{-1o}\text{C}^{-1} \times (22 - 16)^o\text{C}$

Using the principle of conversation of heat energy,

∴ Heat lost by iron = Heat gained by water

∴ $Q_1 = Q_2$

$0.2\text{kg} \times c_1 \times (64 - 22)\ ^o\text{C} = 0.150\text{kg} \times 4200\ \text{Jkg}^{-1o}\text{C}^{-1} \times (22 - 16)^o\text{C}$

$$\therefore \quad c_1 = \frac{0.15 \times 4200 \times 6\text{J}}{0.02\text{kg} \times 42^o\ \text{C}}$$

$c_1 = 450\text{Jkg}^{-1o}\text{C}^{-1}$

ANSWER SELF ASSESSMENT EXERCISE 2

The question is centered on determining H, the heat (thermal) capacity of the container (Calorimeter).

1. Heat lost by hot water = Q_1 and
   $Q_1 = m_1C_1\Delta\theta_1$
   Where $m_1 = 0.4$kg,
   $C_1 =$ specific heat capacity of water = $4200\ \text{Jkg}^{-1o}\text{C}^{-1}$ and
   $\Delta\theta_1 = (t_1 - t) = (84 - 52)\ ^o\text{C}$
   $Q_1 = 0.4\text{kg} \times 4200\ \text{Jkg}^{-1o}\text{C}^{-1} \times (84 - 52)^o\text{C}$
   $= 0.4 \times 4200 \times 32\text{J}$
2. Heat gained by cold water in the calorimeter = Q_2 and
   $Q_2 = m_2C_2\Delta\theta_2$
   Where $m_2 = 0.3$kg,
   $C_1 =$ specific heat capacity of water = $4200\ \text{Jkg}^{-1o}\text{C}^{-1}$ and
   $\Delta\theta_2 = (t - t_2) = (52 - 12)\ ^o\text{C} = 40\ ^o\text{C}$
   $Q_2 = 0.3\text{kg} \times 4200\ \text{Jkg}^{-1o}\text{C}^{-1} \times 40^o\text{C}$
   $= 0.3 \times 4200 \times 40\text{J}$

3. Heat gained by cold water calorimeter = Q_3 and
   $Q_3 = H\Delta\theta_3$ (where H = thermal heat capacity of the calorimeter)
   $= H\Delta\theta_2$
   $= (t - t_2)$
   $= H(52 - 12)^o\text{C}$
   $= H \times 40^o\text{C}$

By applying the conversation of heat energy

Heat lost = Heat gained

$\therefore \quad \mathbf{Q_1 = Q_2 + Q_3}$

$\therefore \quad m_1C_1\,(t_1 - t) = m_2C_2\,(t - t_2) + m_3C_3\,(t - t_2)$

$\therefore \quad 0.4 \times 4200 \times 32J = 0.3 \times 4200 \times 40J + H40^oC$

$\therefore \quad 53760J = 50400J + H \times 40^oC$

$\therefore \quad 53760J - 50400J = H \times 40^oCH$

$$\therefore \quad H = \frac{3360J}{40^oC}$$

$= 84J/\,^oC^{-1}$

The heat capacity of the calorimeter is 84J/^{o}C

ANSWER SELF ASSESSMENT EXERCISE 3

$$\text{SOLID} \xrightarrow{0^oC} \text{WATER} \xrightarrow{50^oC} \text{STEAM}$$

1. The ice melts at
   Quantity of heat used in melting = Q and
   $Q_1 = mL_F$
   Where, m = mass of ice and L_F = specific latent heat of fusion
   $= 1.5kg \times 3.3 \times 10^5 Jkg^{-1}$
   $= 4.95 \times 10^5$

ANSWER SELF ASSESSMENT EXERCISE 4

The steam condensed at 100^{o}C.
Therefore, heat given out is $Q_1 = mLF$

Where m = mass of steam = (50/1000) kg
Lv = specific latent heat of vapourization of water
$= 2.3 \times 10^6 J/kg$

$\therefore \quad Q_1 = (50/1000)kg \times 2.3 \times 10^6 J/kg$

$= (50/1000) \times 2.3 \times 10^6 J$

$Q_1 = 0.155 \times 10^6 J$

To cool from 100^{o}C to 28^{o}C, Heat given out by steam

$Q_2 = mC\Delta\theta$

Where, m = (50/1000)kg mass of water

C = specific heat capacity of water 4200 $Jkg^{-1o}C^{-1}$

= change in temperature = $(10 – 28)^oC$

$\therefore$ Q_2 = (50/1000)kg x 4200 $Jkg^{-1o}C^{-1}$ x $(100 – 28)^oC$

= (50/1000) x 4200 x 72J

= 15120J

$\therefore$ Total heat given out = Q = Q_1+Q_2

Q = 0.115 x 10^6J + 15120J

= 1.15 x 10^5J + 0.1512 x 10^5J

Q = 1.30 x 10^5J.

ANSWER SELF ASSESSMENT EXERCISE 5

1 mole of water (H_2O) = 18g
If 18g of H_2O contains 6 x 10^{23} molecules
$\therefore$ 1g of water contains $\frac{6 \times 10^{23}}{18}$ molecules
Hence the change in Potential energy (ΔPE) per molecule

$$= 2240J \div \frac{6 \times 10^{23} \text{molecules}}{18}$$

$$= 2240J \times \frac{18}{6 \times 10^{23} \text{ molecule}}$$

$$= \frac{2240 \times 18}{6} \times 10^{-23} \frac{J}{\text{molecule}}$$

$$= 6.72 \times 10^{-20} \frac{J}{\text{molecule}}$$

6.0 TUTOR-MARKED ASSIGNMENT

1. An iron casting of mass 30kg at 400°C is being cooled and it gives out heat on an average of 920J/s. Calculate its temperature after 1 hour? (Specific heat capacity of iron = 460$Jkg^{-1o}C^{-1}$).

2. A piece of iron of mass 0.27kg is immersed in boiling water and then dropped into 0.10kg alcohol at 27°C. If the final mixture is 50°C, what is the specific heat capacity of alcohol? (Specific heat capacity of iron = 460$Jkg^{-1o}C^{-1}$).

3. A copper calorimeter of mass 150g contains 100g of water at 16^oC. 250g of a metal at 100^oC are dropped into the water and the temperature of the mixture is 37^oC. What is the specific heat capacity of the metal? (Specific heat capacity of copper = $400Jkg^{-1o}C^{-1}$).

4. What mass of ice is needed to cool 60g of water from 43^oC to 20^oC?

5. Specific Latent Heat of Fusion of Lead is 2.1×10^4J/kg and its melting point is 328^oC. How many joules will be needed to melt 7.0kg of lead at 13^oC? (Specific heat capacity of lead = $120Jkg^{-1o}C^{-1}$)

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UNIT 4 THERMAL EXPANSION

CONTENTS

1.0 INTRODUCTION

In unit 3, we discussed the three effects of matter, namely, expansion, change in temperature and change in state. We were able to show how heat can be measured using the ideas of change in temperature, the specific heat capacity, the mass of the body, specific latent heat of fusion and vapourization to determine the heat absorbed or given out by the body in question.

Broadly, there are these states of matter, solids, liquids and gases. We shall consider the expansion of solid and liquids in this unit. In this unit we are going to examine the expansion/contraction of a material when it is heated or cooled. The expansion of gases will be the subject of the next unit.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain thermal expansion in solids and liquids
- define linear, superficial and cubical expansion on a matter
- apply the expansion of matter to day-to-day activities
- solve problems on the expansion of solids and liquids.

3.0 MAIN CONTENT

3.1 Thermal Expansion in Solids

We know, from our elementary knowledge, that matter is anything that has weight and occupies space. In that case, solids, liquids and gases are forms of matter. When they are therefore heated, experience has shown that they expand. In this unit, we shall examine the expansion of solids and liquids only. It is interesting to know that not only expansion is noticeable in matter when it is heated, but also a change in temperature is also noticeable as heat is absorbed by or removed from the body.

When solids are heated, the effect of heat on them could be found in the change of:

- the length
- the area and
- volume of the solids as temperature changes.

The changes in length, area and volume of the solids depends on:

- the material making up the solid;
- the range of the temperature change;
- the initial dimensions of the solid.

From the above three factors we could deduce that

- the expansion of solids varies from one material to the other;
- the greater the range of temperature change, the greater the expansion;
- expansion depends on the original length, area and volume of the solid.

3.1.1 Linear Expansion

Here, we shall discuss the linear i.e. straight-line expansion of the material. This means we are considering the expansion of a solid in one dimension only.

Consider a metal rod with an original length ℓ_O (fig. 3.1(i)). If such a length of material is heated from an initial temperature t_1°C to t_2°C, the change in temperature $\Delta\theta$ is given as

$$\Delta\theta = (t_2 - t_1)^oC$$

It will be noticed that the length of the metal rod increased from ℓ_O to ℓ_t (fig. 3.11(ii)) ℓ_t is the new length at temperature t_2°C and ℓ_O is the original length at temperature t_1°C.

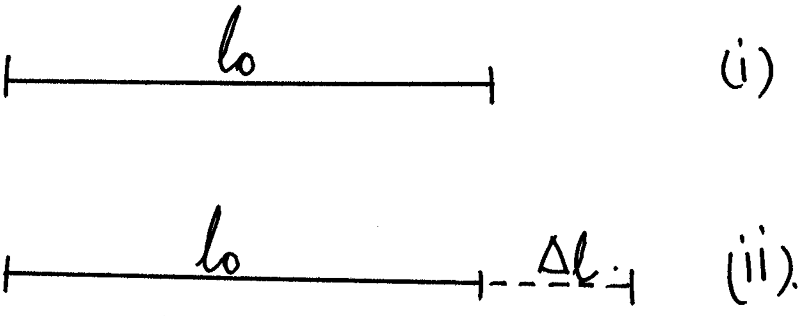

Fig. 3.1

The new length ℓ_t is therefore given as

$$\ell_t = \ell_O + \Delta\ell$$

Where $\Delta \ell$ is the change in length of the rod when heated from $t_1{}^oC$ to $t_2{}^oC$

$$\therefore \quad \Delta \ell = \ell_t - \ell_O \quad \text{.............................} \; (4.1)$$

It is found experimentally that for a given material, the increase in length,$\Delta \ell$, is proportional directly to

(i) the original length ℓ_O and
(ii) the change in temperature $\Delta\theta$

$$\therefore \quad \Delta \ell = \alpha \ell_O \Delta\theta \quad \text{..} \; (4.2)$$

Where, α is the constant of proportionality which is known as the coefficient of linear expansion of the solid or in short, linear expansivity. Consequently α can be given as,

$$\alpha = \frac{\Delta \ell}{\ell_O \Delta\theta}$$

Thus by definition,α, the linear expansivity is "**The increase in length of the material ($\Delta \ell$) per the original ℓ_O length per degree Celsius change in temperature ($\Delta\theta$).**"

The unit of α is $^oC^{-1}$or per degree Celsius.
On comparing Eq. (4.1) and (4.2), we get

$$\therefore \quad \ell_t - \ell_O = \alpha \ell_O \Delta\theta$$

$$\therefore \quad \ell_t - \ell_O = \alpha \ell_O \Delta\theta$$

$$\therefore \quad \ell_t - \ell_O = \alpha \ell_O (t_2 - t_1) \text{....................} \; (4.3)$$

Example 1

An iron rail is 20m long. How much will it expand when heated from 10^oC to 50^oC (linear expansivity of iron = 1.2 x 10-5 $^{-5o}C^{-1}$)?

Solution

Let the original length of iron = ℓ_O = 20m
Let the initial temperature $t_1{}^oC$ = 10^oC
And the final temperature $t_2{}^oC$ = 50^oC
$\therefore$ Change in temperature $\Delta\theta$ = $(t_2 - t_1)^oC$
= $(50 - 10)^oC$
= 40^oC
Given that α for iron = 1.2 x 10-5$^oC^{-1}$
From $\Delta \ell$ = $\alpha \ell_O \Delta\theta$

$\therefore$ The change in length $\Delta\ell = \alpha\ell_O\Delta\theta$

$$\begin{aligned}\therefore \Delta\ell &= 1.2 \text{ x } 20\text{m x } 40^{o}\text{C} \\ &= 0.0096\text{m} \\ \Delta\ell &= 0.96\text{cm.}\end{aligned}$$

The iron rail would have expanded by 0.96cm.

After this example, you must have observed the values of α and $\Delta\ell$.

You would have observed that the value of α is very small not only for iron but for most materials as you will further observe in Table 3.1. below.

Linear Expansivities for Some Materials

Table 3.1

Substance	Value
Pure Metals	
Aluminum	$2.55 \text{ x } 10^{-5o}$C
Copper	$1.67 \text{ x } 10^{-5o}$C
Gold	$1.395 \text{ x } 10^{-5o}$C
Iron	$1.20 \text{ x } 10^{-5o}$C
Nickel	$1.28 \text{ x } 10^{-5o}$C
Platinum	$0.80 \text{ x } 10^{-5o}$C
Silver	$1.88 \text{ x } 10^{-5o}$C
Alloys	
Brass	$1.89 \text{ x } 10^{-5o}$C
Constantan	$1.70 \text{ x } 10^{-5o}$C
Invar	$0.10 \text{ x } 10^{-5o}$C
Phosphor – Bronze	$1.68 \text{ x } 10^{-5o}$C
Solder (2pb: 1Sn)	$2.50 \text{ x } 10^{-5o}$C
Steel	$1.10 \text{ x } 10^{-5o}$C

3.1.2 Determination of Linear Expansion (α)

In the earlier section, you have learnt about linear expansion α. We can measure the value of α by various methods.

We have the following methods used in determining the linear expansivity (α) of a metal:

- Optical lever method

- Screw gauge method
- Comparator method
- Henning’s tube method
- Fizeau’s method

All the above methods are different in the manner in which the increase in length is measured. The specimen to be measured is in form of a bar or tube and this involves:

- the measurement of the length of the bar,
- the rise in temperature during the experiment and
- the increase in length of the bar consequent on this rise in temperature.

The first two measurements do not present any great difficulty, but the actual measurement of the expansion that takes place. It is therefore this measurement of the increase in expansion that has called for use of elaborate vernier microscope, micrometer screw gauge and the optical lever method. We shall describe the screw gauge method here.

The Screw Gauge Method

This is one of the laboratory methods for determining the coefficient of linear expansion of a metal. The apparatus used for the determination of α is as shown in fig. 3.2.

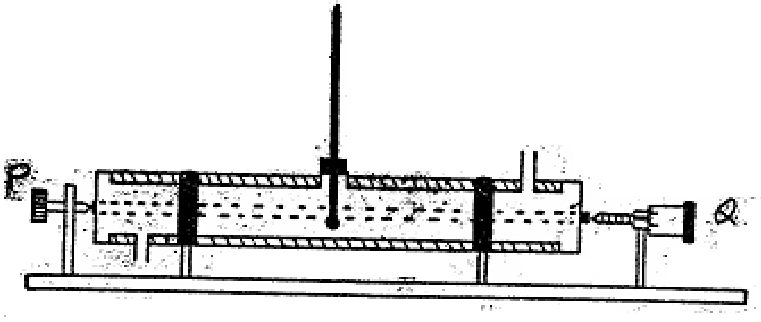

Fig. 3.2: Linear Expansivity Apparatus with Screw Gauge

The metal rod which is placed in the tune AB is about a meter in length. It is supported horizontally by pillars P and Q on affirm base.

The end A of the tube is in contact with the fixed end of the pillar of the apparatus. The screw gauge makes contact with the other end B as shown in fig. 3.2.

The reading of the screw gauge is taken when it is initially in contact with the end B at the room temperature t_1.

The screw gauge is then screwed backward to give room for the expansion of the rod inside the tube.

Steam is allowed into the tube at end A and out through end B for a considerable length of time so that the rod acquires at temperature of 100°C, the temperature of steam (t_2).

The screw is then screwed up to make contact with the rod when fully expanded. The new reading on the crew gauge is then taken. The difference of the two readings on the screw gauge gives the increase in length of the rod due to expansion.

If he original length of the rod is ℓ_O, the increase in length is $\Delta\ell$ and the change in temperature is $\Delta\theta = (t_2 - t_1) = (100 - t_1)^oC$, where t_1 is the room temperature, then the coefficient of expansion α is determined as

$$\alpha = \frac{\Delta\ell}{\ell_O \Delta\theta}$$

$$\alpha = \frac{\Delta\ell_{(m)}}{\ell_{O(m)}(100 - t_1)^\circ C} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.4)$$

3.1.3 Superficial Expansion

Under this section, we shall consider the expansion of material in two dimensions (-length and breadth) to produce an area expansion.

When a solid is heated, the area increases. In this case the expansion or the change in area is in two dimensions as shown in fig. 3.3(ii).

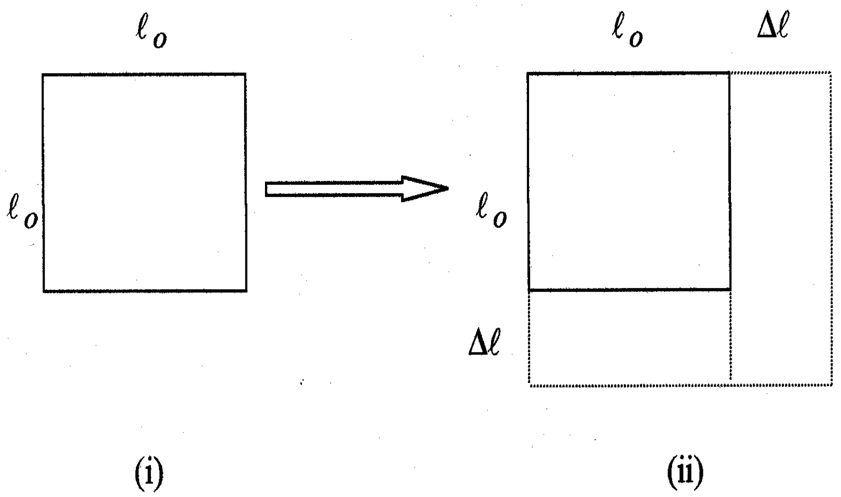

Fig. 3.3

Consider a square solid object of length ℓ_O (fig.3.3(ii). By definition its original area at initial temperature t_1^oC is A_o ($= \ell_O^2$).

If the object is heated from temperature t_1^oC to t_2^oC, then each length would have increased by$\Delta\ell$ (fig. 3.3 (ii)).

Then the new area $A_t = (\ell_O + \Delta\ell)^2$

Thus, the change in area $\Delta\ell$ is given as

$$\Delta A = A_t - A_o$$

and the change in temperature $\Delta\theta$ is given as

$$\Delta\theta = (t_2 - t_1)^oC$$

Again, experiments have shown that for a given material, the change in area (ΔA) is directly proportional to the original area A_o and the change in temperature$\Delta\theta$.

$$\therefore \quad \Delta A = \beta A_o \Delta\theta \text{ ………………………………….. } (4.5)$$

where, β is the constant of proportionality otherwise known as the coefficient of superficial expansion or in short area expansivity of the material.

$$\therefore \quad A_t - A_o = \beta A_o \Delta\theta$$
$$\therefore \quad A_t = A_o + \beta A_o \Delta\theta$$
$$\therefore \quad A_t = A_o (1 + \beta A_o \Delta\theta) \text{ ……………….. } (4.6)$$

So far we have learnt about α andβ. Let us see, that there is any relationship between the linear expansivity (α) of a material and the area expansivity (β) of the same material. We shall proceed to show this relationship.

Given that from Eq. (4.6), that $A_t = (\ell_O + \Delta\ell)^2$

On expanding the expansion $(\ell_{O+}\Delta\ell)^2$, we get

$$\therefore \quad A_t = (\ell_O^2 + 2\ell_O\Delta\ell + \Delta\ell)^2 \text{ …………….} (4.7)$$

You know that $\Delta\ell = \alpha\ell_O\Delta\theta$ ……………………….... (4.8)

But

Therefore, Eq. (4.7) becomes

$$A_t = \ell_O^2 + 2\ell_O\alpha\ell_O\Delta\theta + \alpha^2\ell_O^2(\Delta\theta)^2$$
$$= \ell_O^2 + 2\alpha\ell_O^2\Delta\theta + \alpha^2\ell_O^2(\Delta\theta)^2$$

But $\ell_O^2 = A_o$

On arranging the terms, we get

$$\therefore \quad A_t = A_o + 2\alpha A_o\Delta\theta + \alpha^2 A_o(\Delta\theta)^2 \text{ ………………………..} (4.9)$$

You will recall that the value of α is very small (10^{-5}) therefore α^2 will be so small that term α^2 the expression can be ignored.

$\therefore \quad A_t = A_o + 2\alpha A_o\Delta\theta$ compare with

$$A_t = A_o + \beta A_o\Delta\theta$$

Hence $\beta = 2\alpha$ ……………………………..… (4.10)

That is, the coefficient of superficial expansion β is twice the value of the coefficient of linear expansion.

It means that when problems are set on superficial expansion, you will not be given the value of β but you will be given the value α of for the material. At this point, you must remember the relationship.

$$\beta = 2\alpha$$

SELF ASSESSMENT EXERCISE 1

A square sheet of steel has a side of 15cm at 0^oC. Determine its area at 40^oC.

Given: coefficient of linear expansivity of steel = $1.1 \times 10^{-5}C^{-1}$)

3.1.4 Cubical Expansion

The volume of a solid increases as a result of heat. Let the original volume of the cube as shown in fig. 3.4(i) be V_o at initial temperature $t_1{}^oC$. If the cube is heated to a temperature $t_2{}^oC$, then the new volume at temperature $t_2{}^oC$ is V_t (fig. 3.4(ii).

There is a change in volume given by

$$\Delta V = V_t - V_o$$

The change in volume is directly proportional to the

$$(\ell_O = \Delta\ell)$$

$$(\ell_o + \Delta\ell)$$

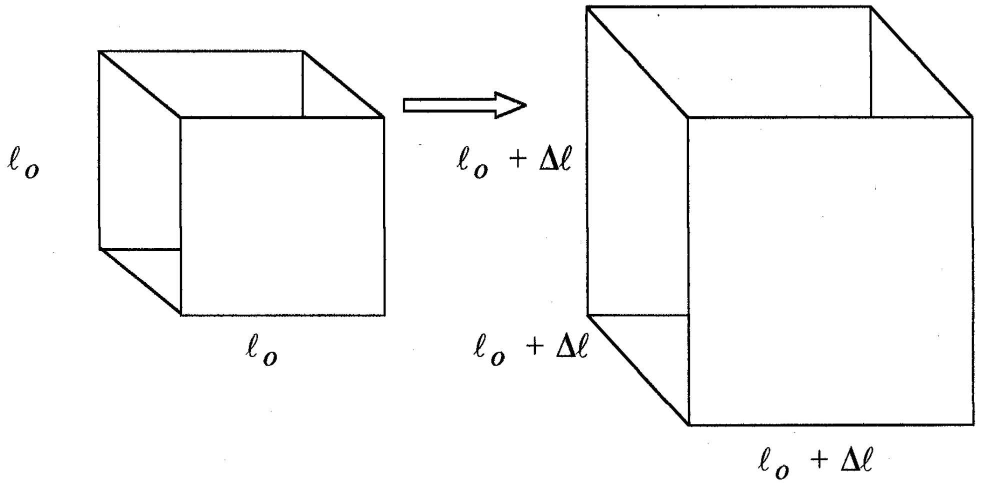

Fig. 3.4

Original volume V_o and the change in temperature $\Delta\theta$

$$\therefore \quad \Delta V = \gamma V_o \Delta\theta$$

where γ is a constant of proportionality the coefficient of cubical expansion, or cubical expansivity. α is defined as,

$$\gamma \frac{\Delta V}{V_o \Delta\theta} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.11)$$

The increase in volume per original volume per degree Celsius rise in temperature.

If $\Delta V = \gamma V_o \Delta\theta$

But $V_t = V_o + \Delta V$

$\therefore$ $V_t = V_o + \gamma V_o \Delta\theta$ $V_t =$

$V_o \quad (1 + \gamma\Delta\theta) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.12)$

Again, one can establish between the cubical expansivity γ and the linear expansivity α.

From fig. 3.4(i) we know that

$$V_o + \ell_O^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.13)$$

and from fig. 3.4(ii) we also note the

$$V_t = (\ell_O + \Delta\ell)^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.14)$$

by expanding $(\ell_O + \Delta\ell)^3$ we obtain

$$V_t = \ell_O^3 + 3\alpha\ell_O^3\Delta\theta + 3\ell_O^3\alpha^2(\Delta\theta)^2 + \alpha^3\ell_O^3\Delta\theta^3$$

By using the previous argument that the value of α is very small, therefore α^2, α^3 will be so small that the expressions in terms of α^2 and α^3 can be ignored.

Consequently

$$V_t = \ell_O^3 + 3\alpha\ell_O^3\Delta\theta \quad \text{since} \quad \ell_O^3 = V_o$$

$$V_t = V_o + 3\alpha V_o\Delta\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.15)$$

But, $V_t = V_o + \gamma V_o \Delta\theta \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4.16)$

On comparing Eq. (4.15) and Eq. (4.16), we can conclude that γ is three times as large as α

That is the volume of expansivity is three times r $=3\alpha$ the linear expansivity ………………………………………………...…….. (4.17)

SELF ASSESSMENT EXERCISE 2

The volume of a small piece of metal is 5.000cm^3 at 20^{o}C and 5.014cm^3 at 100^{o}C. Determine the cubic expansivity of the metal?

3.1.5 Applications of Expansivity

The following are some practical applications of expansivity:

1. Some metals such as platinum and tungsten have their linear expanisivity very close to that of glass. Beside the fact that the linear expansivity of platinum in almost equal to that of glass, the behaviour of these solids are very much alike. This characteristics feature therefore enables us to seal electrodes through glass without the occurrence of breakage through cooling and heating processes.

2. Linear expansivity is also applied in the formation of bimetallic element which are used as:
   - (i) thermostatic control switches;
   - (ii) in the construction of expansion loops for use in steam lines;
   - (iii) bimetallic thermometers.

3. Linear expansivity is also used in the construction of bridges where gaps are left between the girders to accommodate expansion. Such gaps are also between the iron rails in the construction of railway lines.

Example 2

Have you ever noticed that the electric wires on the NEPA poles are always left sagging? Can you explain why this is so?

Solution

This is to allow for changes in length as the temperature changes. If they are taut further cooling in the atmosphere may make them snap.

3.2 Thermal Expansion in Liquids

As expansion takes place in solids so also in liquids. The expansion in liquids is a bulk affair hence we would talk of volume expansivity for liquids.

3.2.1 Cubical Expansion

The coefficient of volume expansion of a liquid is the fractional change in volume per the original volume fro degree change in temperature. From Eq. (4.12) given below,

$$V_t = V_o (1 + \gamma\Delta\theta)$$

It is necessary to note that γ for liquids is not of the same order as 3α where α is the linear expansivity of solids.

In actual fact γ liquids $\approx 10\ \gamma$ solid

i.e. γ liquids is greater that γ solid

The observed increase in the volume f the liquid is the difference between the expansion of the liquid and that of the liquid and that of the container and it is therefore called relative or apparent change in volume.

For water, between 0^oC and 40^oC there is a decrease in volume i.e. increasing in density? But between 4^oC and 100^oC, the volume of water increases uniformly while its density decreases. This is what is being described as the anomalous behaviour of water. Such behaviour of water preserves the lives of marine creatures.

3.2.2 Real and Apparent Expansion of Liquids

Experience has shown that it is impossible to measure the real or absolute thermal expansion of a liquid by direct volume determinations. This is because liquids are contained in vessels which also expand when heated. Hence, the expansion of the content of a vessel is always relative or apparent.

Apparent expansion of the liquid is therefore less that the real expansion of the liquid. Volume dilatometers are used in the determination of thermal expansion of liquids. The mean coefficient of apparent expansion of a liquid (α_{app}) between temperature t_1 and t_2 is given as:

$$\alpha_{app} = \frac{V_2 - V1}{V_1(t_2 - t_1)}$$

Where V_2 is the finial volume at $t_2{}^oC$, V_1 is the initial volume and $(t_2 - t_1)$ the change in temperature.

This is a general definition of the coefficient of apparent expansion. It applies to such experiments as the volume dilatometer, weight thermometer, relative density bottle and sinker methods of determining the coefficient of apparent expansion.

In the last three examples, weights of the volumes of the liquid between t_1 and t_2 are compared which will be equal if the vessel and the sinker did not expand.

$$\alpha_{app} = \frac{\text{mass of liquid expelled}}{\text{mass remaining x temperature change}}$$

Thus $\alpha_{real} = \alpha_{app} + \gamma$

where γ is the coefficient of cubical expansion of the material of vessel, α_{app} is the apparent coefficient of expansion of the liquid and α_{real} is the real coefficient of the liquid.

4.0 CONCLUSION

When matter is heated, it expands. The ways materials expand when they are subjected to heat are described by their coefficients of expansion linear, superficial cubical expansivities.

Expansion in liquids is greater than that of solids. Besides, the study of expansion in liquids is more complicated than solids. This is because as the liquid expands, its container also expands. Thus we talk of relative expansion in liquids rather than of expansion. In the next unit we would consider the expansion in gases. This will be studied under the gas laws.

5.0 SUMMARY

You have learnt the following in this unit:

- When matter is heated, it expands;
- There are three types of coefficient of expansion - linear expansivity, superficial expansivity and cubical expansivity;
- Expansion of a material depends on its nature, temperature range and the initial dimensions of the material;
- Superficial expansivity is twice the linear expansivity of solid material;
- The cubical expansivity is thrice the linear expansivity of a solid material;
- There is no absolute expansion of a liquid because it is contained in a container, which also expands.

ANSWER SELF ASSESSMENT EXERCISE 1

Using Eq. (4.6), we get

$$A_t = A_o + \beta A_o \Delta\theta$$

where, A_t = area of the plate at 40^oC = ?

A_o = original area of plate = $15cm^2$

$\beta = 2\alpha = 2 \times 1.1 \times 10^{-5}\,{}^oC^{-1}$

= area expansivity of steel.

$\Delta\theta$ = change in temperature = $(40 - 0)\,{}^oC$

= $40\,{}^oC$

$$\therefore A_t = A_o + 2\alpha A_o \Delta\theta$$

$$= 15^2cm^{2\,+}\ 2 \times 2 \times 1.1 \times 10^{-5}\,{}^oC^{-1} \times 40^oC \times 15^2cm^2$$

$$= (225cm^2 + 0.198cm^2)$$

$$A_t = 225.198cm^2$$

ANSWER SELF ASSESSMENT EXERCISE 2

Given:

$V_o = 5.000\text{cm}^3$

$V_t = 5.014\text{cm}^3$

and $t_1 = 20^oC$

and $t_2 = 100^oC$

$\therefore \Delta\theta = t_2 - t_1 = (100 - 20)^oC = 80\,^oC$

$\Delta V = V_t - V_o$

Using the Eq. (4.11)

$$\therefore \quad \gamma = \frac{\Delta V}{V_o \Delta\theta}$$

$$= \frac{V_t - V_o}{V_o \Delta\theta}$$

Substituting the values, we get

$$= \frac{(5.014 - 5.000)\text{cm}^3}{5.000\text{cm}^3 \text{ x } 80^o C}$$

$$= \frac{0.014}{5.000 \text{ x } 80^o C}$$

$\gamma = 0.000035^oC^{-1}$

$\gamma = 3.5 \text{ x } 10^{-5o}C^{-1}$

Cubical expansivity of the metal is $3.5 \text{ x } 10^{-5o}C^{-1}$

6.0 TUTOR-MARKED ASSIGNMENT

1. A brass measuring scale is exactly two meters long at 15^oC. Determine its length at 40^oC? (α for brass = $1.8 \text{ x } 10^{-5o}C^{-1}$).

2. If the linear expansivity of a metal is $2.0 \text{ x } 10^{-5o}C^{-1}$, calculate the approximate value of its superficial expansivity.

3. The density of aluminum at 0^oC is 2.76g (cm^{-3}). Determine its density at 200^oC (for aluminum = $2.5 \text{ x } 10^{-5o}C^{-1}$).

7.0 REFERENCES/FURTHER READING

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UNIT 5 GAS LAWS

CONTENTS

1.0 INTRODUCTION

When solids and liquids are heated, they are not significantly affected by changes in pressure as their temperatures change. This is why we did not consider the effect of pressure during the expansion of solids and liquids. However, for a given mass of gas, the expansion of a gas is considerably affected by pressure. It is to be noted that in describing the behaviour of gases, when subjected to heat, four variables are usually considered.

They are:

- pressure (P)
- volume (V)
- temperature (T) and
- the number of moles (n) of the gases.

These four properties or parameters are used to describe the state of a given mass of a gas. In this unit, we shall first discuss the relationship between the temperature, pressure and volume of a gas. Then we will examine the behaviour of gases using these parameters to deduce the various gas laws.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state the different gas laws
- explain the gas laws through the use of graphs
- distinguish between a real gas and an ideal gas
- express the equation of state of an ideal gas
- solve problems on these gas laws.

3.0 MAIN CONTENT

3.1 GAS LAWS

You will recall that four properties are used to describe the behaviour of gases; namely:

Pressure (P), Volume (V), Temperature (T), and Amount of the gas in moles (n).

When any two of these properties are kept constant, the other two are then subjected to change in order to show how the gas behaves. The first two will be considered while temperature and the number of moles of the gas is kept constant.

3.2 Boyle's Law

Boyle (1662) investigated the relationship between the pressure (P) and the volume (V) of a given mass of gas when the temperature (T) and the number of moles (n) are kept constant. Boyle's law states that:

"The pressure on a given mass of gas is inversely proportional to its volume (V) provided its temperature is kept constant".

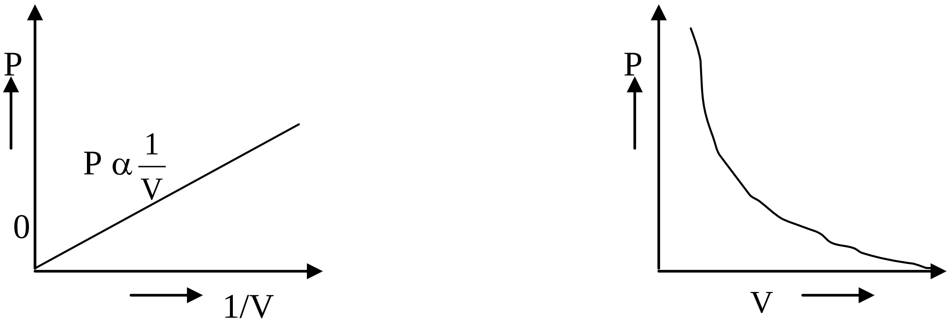
Symbolically, this statement is written as:

$$P \propto \frac{1}{V} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.1)$$

$$\therefore \quad P = \frac{K}{V} \text{ where, K is a constant of proportionality.}$$

$$\therefore \quad PV = K = \text{Constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.2)$$

If you plot a graph P versus $\frac{1}{V}$, then the graph would be as given below in fig. 5.1(i) and fig. 5.1(ii).



(i) **(ii)**

Fig. 5.

3.2.1 Experimental Proof of Boyle's Law

Boyle's law may be demonstrated by suing a 'J' tube as shown in fig. 5.2 such that one end is opened and the other end closed. Thus AB in fig. 5.2 contains the trapped air by the column of mercury. The mercury head (h) constitutes the pressure on the trapped air in addition to the pressure due to the atmosphere (H_o). The cross sectional area of the 'J' tube is assumed to be uniform. That is, the circular area is uniform.

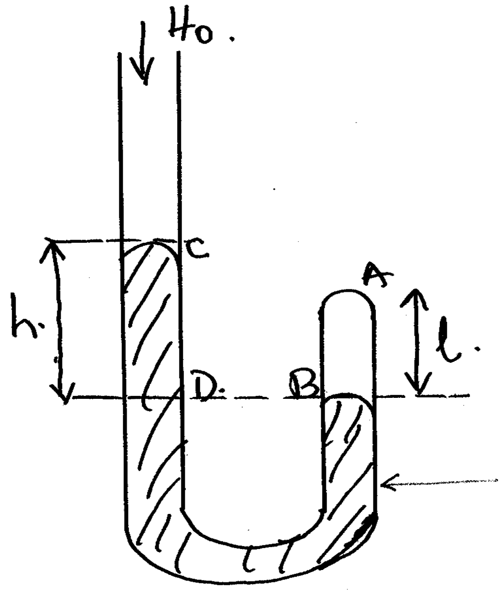


Fig. 5.2

You would have noticed that two variables can be identified in this set up.

(i) the volume of the air trapped V = $A\ell$ ……………….(5.3)
where A is the area of cross-section of the tube and ℓ is the length of the air trapped.

(ii) the pressure P on the tapped air which is made up of the atmospheric pressure H_o and the mercury column h.

From Eq. (5.3),

$V = A\ell$ we say that Volume (V) is proportional to the length of the air.

$$\therefore \quad V \alpha \ \ell \text{ and } \frac{1}{V} \alpha \ \frac{1}{\ell} \quad \ldots\ldots \quad (5.4)$$

Thus the measurement of ℓ is proportional to V

As you know, the pressure = $h\rho g$

Where, h is the height of the mercury column, ρ is the density of mercury, and g is the acceleration due to gravity.

ρ and g are constants.

This also means that the pressure P on the gas is proportional to the height (h) of the mercury.

$$P \propto h$$

That is the total pressure on the gas $P \propto (H_0 + h)$ where H_0 is the barometric height – atmospheric pressure and h is the mercury head in the 'J' tube.

Thus the measurement of $H_0 + h$ will be proportional to the pressure P. Pouring more mercury through the open end varies the length ℓ of the air column. For each measured height h of the mercury, the corresponding length ℓ of the air column is measured.

First, we then plot the graph of $(H_0 + h)$ against ℓ with the $(H_0 + h)$ on the vertical axis and ℓ on the horizontal axis. We would obtain a graph as shown in fig. 5.3(i). This shows that as $(H_0 + h)$ increases, ℓ decreases.

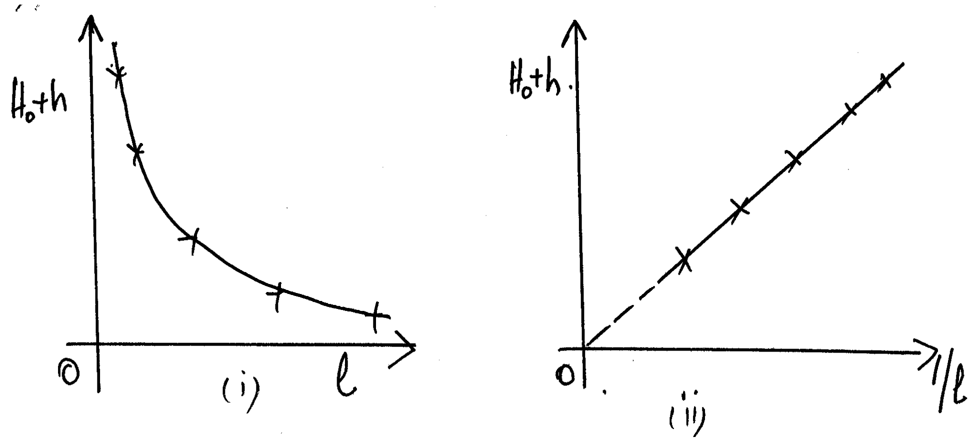


Fig. 5.3: (i) Plot of $(H_0 + h)$ versus ℓ (ii) Plot of $(H_0 + h)$ versus $(\frac{1}{\ell})$

Secondly, if we plot the graph of $(H_o + h)$ against $\frac{1}{\ell}$, with $(H_o + h)$ on the vertical axis and $\frac{1}{\ell}$ on the horizontal axis. A graph as shown in fig. 5.3 (ii) will be obtained. We find that the plot is a linear graph.

The two graphs are in consonance with Boyle's law. Fig. 5.3(i) says that as the pressure increases, the volume decreases while fig. 5.3(ii) is also saying that the pressure is inversely proportional to the volume of the gas.

$$(H_o + h) \propto \frac{1}{\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.5)$$

or

$$P \propto \frac{1}{V}$$

$$P = \frac{K}{V}$$

$$\therefore \quad PV = K$$

Thus for volume V_1 and pressure P_1, $P_1V_1 = k$ and for volume V_2 and pressure P_2, $P_2V_2 = K$

The meaning of these statements is that when an amount of a gas (n) and its temperature t are kept constant we can conveniently state that

$$P_1V_1 = P_2V_2 = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.6)$$

However, if n changes then

$$P_1V_1 \neq P_2V_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5.7)$$

and the conditions of the experiment are no longer consistent with the conditions of the law even when the temperature is kept constant.

Boyle's law is applied in air compressors and exhaust (vacuum) pumps.

SELF ASSESSMENT EXERCISE 1

Under a Pressure of $14Nm^{-2}$, some air has a volume of $1.5m^3$. Determine its volume when its pressure is $10Nm^{-2}$. Assuming the temperature is kept constant.

3.3 Charles's Law

Charles's law deals with the behaviour of a given mass of gas at constant pressure. Under this law, we would consider the variation of volume (V) with temperature (T) when the pressure (P) and the amount of the gas (n) are kept constant.

The original Charles's state that:

"At constant pressure, the volume of a given amount of gas increases by a constant fraction of its volume at 0^oC for each Celsius degree rise in temperature".

The Mathematical expression for this can be written as:

$V \alpha T$ (at constant n and P) ……………….…………………………….. (5.8)

The above statement brings out the idea of volume coefficient, r, where r is defined as the increase in volume of a unit volume of the gas at 0^oC for each degree Celsius rise in temperature when the fixed mass of that gas is heated at constant pressure.

The volume coefficient is called volume expansivity. For more detail, you can see the unit 4, section 3.1.4.

If V_o is the volume of the gas at 0^oC and V_o is the volume of the gas at t^oC, then r is expressed as:

$$r = \frac{\Delta V}{V_o \Delta\theta}$$

$$= \frac{V_t - V_o}{V_o(t-0)}$$

$$\therefore \quad r = \frac{V_t - V_o}{V_o t}$$

$$\therefore \quad V_t = V_o + r V_o t$$

$$V_t = V_o (1 + rt) \text{ ……..……………….… (5.9)}$$

Note that V_o stands for the volume of the gas at 0^oC and not just the original volume at any selected initial temperature. And that t is the actual temperature using the Celsius scale and not for any selected temperature rise.

The valve of r for most gases is $\left(\frac{1}{273}\right)$. Now substituting the value of r in Eq. (5.9), we get

$$V_t = V_o \left(\frac{273+t}{273}\right) \text{……………………..……………….. 5.10)}$$

But as you know from the absolute scale,

$$\left.\begin{aligned} (273 + t) &= T\# \text{ and that} \\ 273 &= T_o \end{aligned}\right\} \text{…………….. (5.11)}$$

Then putting the values in Eq. (5.11) into Eq. (5.10), we get

$$\therefore \quad V_t = \frac{V_o T}{T_o}$$

On rearranging the terms, we obtain

$$\therefore \quad \frac{V_t}{T} = \frac{V_o}{T_o} \quad = \text{Constant}$$

$$\therefore \quad V_t = K\# \qquad \text{i.e. } V_t \propto T$$

Thus the volume of the gas (V) is directly proportional to its absolute temperature (T). The equation $V_t = KT$ is a deduction, or consequence of Charles's law. It is not the law.

3.3.1 Verification of Charles's Law

We shall describe here, the experimental procedure for the determination r, the coefficient if volume expansion of a gas. The experimental set up is as shown in fig. 5.4 below:

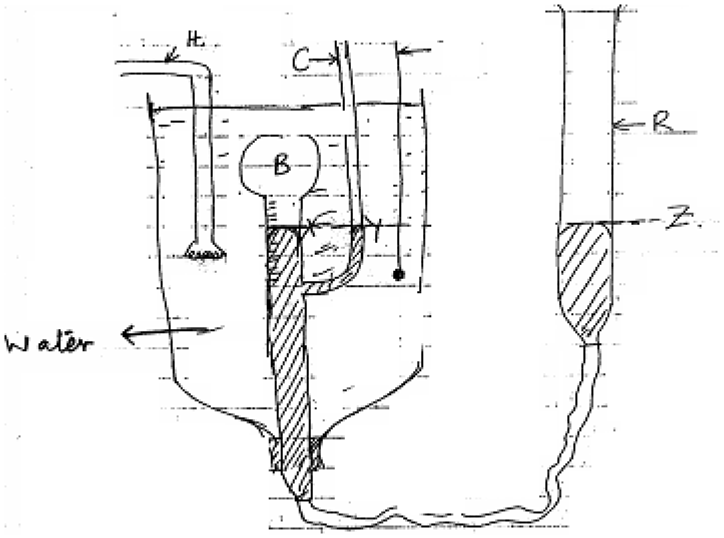


Fig. 5.4: Charles's Law Apparatus

The apparatus is made up of a glass bulb B filled with dry air. The bulb is continuous with a graduated tube. The mercury column encloses the dry air. A branch tube C is also connected to the graduated bulb B that is open to the atmosphere. The mercury levels X in bulb B and raising or lowering the reservoir R adjusts Y in tube C to the same level. In that case the pressure on the gas is the same as that of the atmosphere pressure.

The bulb B and tube C are surrounded by a water bath, which contains an electric heater H, which is also used as a stirrer. A thermometer T is inserted to measure the temperature of the bath. The initial volume V_1 and the initial temperature t_1 of the gas in the bulb are measured. The temperature of the gas is the same as that of the water in the bath. They are both recorded when the level X, Y and Z have been adjusted to be the same.

The heater is then switched on until there is difference of 20oX rise in temperature. It is then switched off and used to stir the water thoroughly. The level X, Y and Z are then adjusted again to obtain a new volume V_1 and its new corresponding temperature t_2°C. The above procedure is repeated for another set of five or six volume measurements between the room temperature t_2°C and 100°C.
When then plot the graph of volumes V on the vertical axis against their corresponding temperature t on the horizontal axis. A linear graph as shown in fig. 5.5 is obtained.

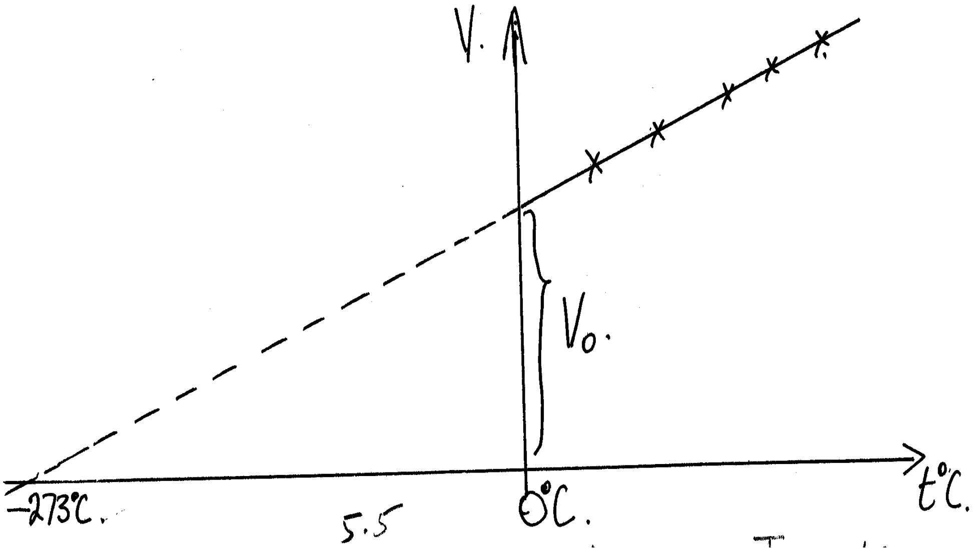


Fig. 5.5: A graph of Volume versus Temperature

The volume at 0°C, V_o may be extrapolated so that r, the volume coefficient could be determined.

$$r = \frac{V_t - V_o}{V_o \times t}$$

Further extrapolation of the graph enables us to determine the absolute zero temperature. This is found to be -273°C. In conclusion, the coefficient of the volume expansion (volume expansivity) will be found to be 0.003663, which is approximately equal to $\frac{1}{273}$.

Using the absolute scale temperature, it will be observed from the graph that

$$\frac{V}{T} = \text{Constant.}$$

$$\therefore \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Thus, the volume of a given mass of gas is directly proportional to its absolute temperature provided the pressure is kept constant – another form of Charles's law.

SELF ASSESSMENT EXERCISE 2

Some hydrogen gas a volume of 200cm^3 at 15^{o}C. If the pressure remains constant, at what temperature will its volume be 150cm^3?

3.4 Pressure Law

In this section we shall once again examine the behaviour of a gas by observing how its pressure **(P)** varies with temperature **(t)**, when its volume **(V)** and the amount of the gas **(n)** are kept constant. The study under this section is described as pressure law or Gay-Lussac's law.

It will be observed that the law of increase with increase in temperature art constant volume is the same as the law of increase in volume with increase in temperature at constant pressure provided there is no change in the amount of the gas.

The pressure law states that:

"For a given mass of a gas at constant volume, its pressure increases by a constant fraction of pressure at 0^{o}C for each Celsius degree rise in temperature".

Let us consider a fixed mass of gas of volume V_1at t_1^{o}C and pressure P_1.

Suppose the gas is then heated to some temperature t_2^{o}C at which the volume is zV_1. Where z is a fraction.

We can reduce this new volume zV_1 to V_1 at higher temperature by increasing the pressure to. Using Boyle's law.

$$P_2V_1 = P_1 zV_1$$

$$\therefore \quad P_2 = zP_1$$

Thus, when the temperature is raised, the volume can be maintained at $V_1$by increasing the pressure. That is, the rise in temperature, which causes an increase in volume from V_1 to zV_1if the pressure is kept at P_1, also causes an increase in pressure from $P_1$to zP_1 if the volume is kept constant at V_1. If Boyle's law is not obeyed perfectly then the theoretical basis fails.

However, experiment have shown that when a fixed mass of gas is heated at constant volume, its pressure increase s by a constant fraction of the pressure at 0°C for each degree Celsius rise in temperature.

The above statement thus defines the pressure coefficient β or pressure expansivity.

The pressure coefficient β is defined as the increase in pressure expressed as a fraction of the pressure at 0°C for one Celsius degree rise in temperature when a fixed mass of that as is heated at constant volume.

If Po is the pressure of he gas at 0°C and P, the pressure at t°C, then is defined as:

$$\beta = \frac{\Delta P}{P_o t}$$

$$\therefore \quad \beta = \frac{P_t - P_o}{P_o t}$$

$$\therefore \quad P_t - P_o = \beta Pot$$

$$\therefore \quad P_t = P_o(1 + \beta t) \quad \text{..............................} \quad (5.13)$$

3.4.1 Constant volume Gas Thermometer

Jolly's constant volume air thermometer s used to determine the pressure coefficient β for a gas (fig. 5.6).

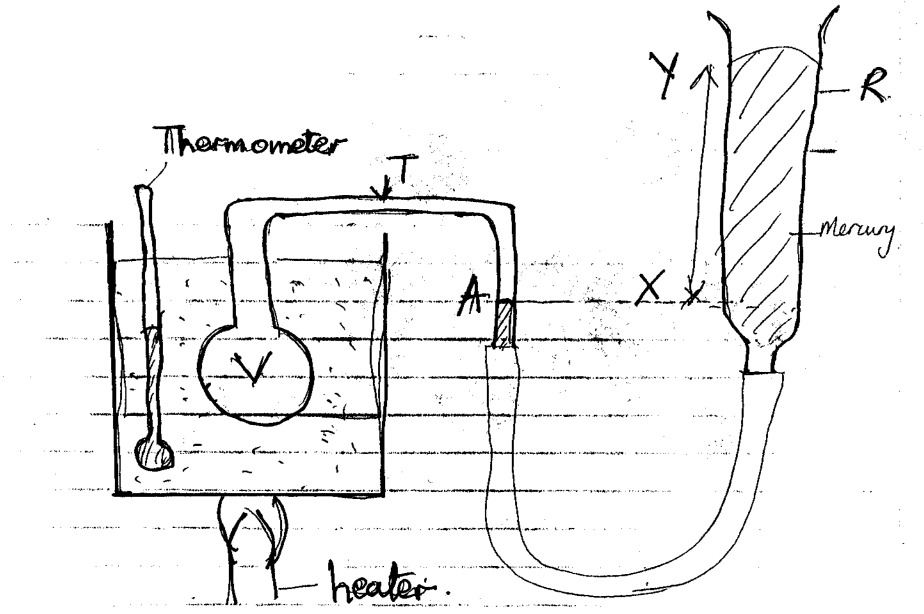


Fig. 5.6: Constant Volume Air Thermometer

It consists of a glass bulb V of volume 10cm filled with dry air. The bulb V is connected to a glass capillary tube T that in turn is connected to a mercury manometer R which measures pressure. A fixed reference mark is made on the capillary tube at A.

The moveable arm of the manometer R is usually adjusted up and down as may be necessary to ensure that the mercury level at A remains the same. This marks the constant volume before any reading is taken.

The initial pressure of the gas at the room temperature is noted from the difference in the mercury levels at X and Y. This is done by arranging the bulb in a water bath, which is well stirred, and with the mercury thermometer in the water bath. The room temperature is taken as the temperature of the water bath on which the thermometer is inserted.

The pressure of the atmosphere is read first from the barometer. Then the initial temperature of the water bath is taken. When the mercury level is first brought to level A the level Y is noted. The difference between Y and X given the mercury head, h.

Thus, the total pressure on the volume of gas is

$$P \quad \alpha \quad (H + h)\text{cm of mercury.}$$

The water bath is gently heated through, about 20°C when the heating is stopped and stirred thoroughly. The moveable arm of the manometer is adjusted, the mercury level A at a steady temperature to enable the reading of anew level of Y. The new temperature is taken and the new corresponding pressure head h is measured. The above procedure is repeated for a set of five or six readings. The barometric height is read again as a check at the end of the experiment. The graph of pressure readings is plotted against the corresponding values of the temperature readings. The graph is shown in fig. 5.7.

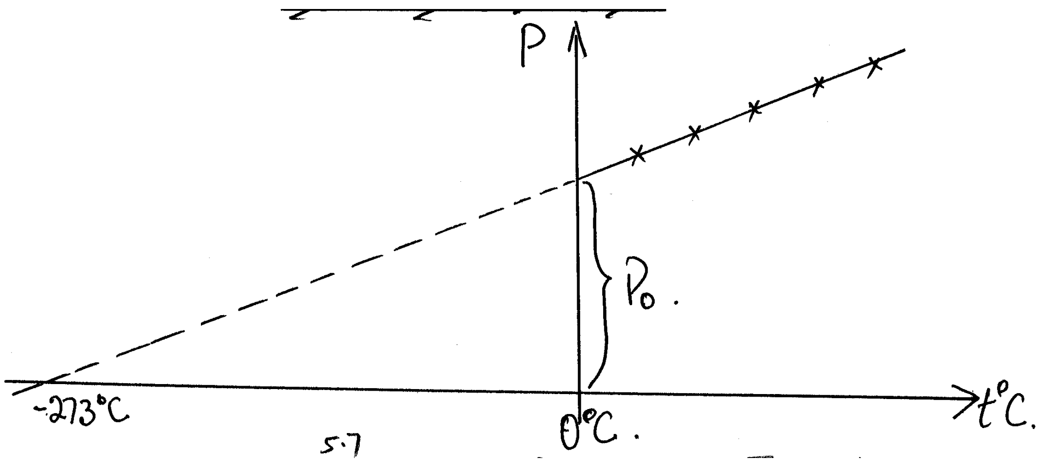


Fig. 5.7: Graph of Pressure versus Temperature

Our conclusion from this graph is that the pressure on the gas varies linearly with the temperature. When the graph is extrapolated, the pressure P_o at 0^oC can be read from the graph. Further extrapolation produces the absolute temperature which will be found to be approximately equal to -273^oC.

The slope of the graph $\beta = \frac{\Delta P}{P_o t}$

$$\therefore \quad \beta = \frac{P_t - P_o}{P_o t} = \frac{1}{273} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.14)$$

The following precautions are usually taken:

- the mercury level is changed gradually;
- the volume is maintained at constant value at A;

Sources of error could also be found in:

- the dead space of the capillary tube T and
- the expansion of the bulb which also introduces error.

If $\beta = \frac{1}{273}$

From, $\beta = \frac{\Delta P}{P_o t}$

$$\therefore \quad \beta = \frac{P_t - P_o}{P_o t}$$

$$\therefore \quad P_t - P_o = \beta Pot$$

$$\therefore \quad P_t = P_o + \beta Pot$$

$$P_t = P_o (1 + \beta t) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.15)$$

Put $\beta = \frac{1}{273}$ in Eq. (5.15), we get

$$= P_o \left(1 + \frac{1}{273} t\right)$$

$$P_t = P_o \left(\frac{273 + t}{273}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.16)$$

Using the absolute scale of temperature,

$$273 + t = T$$

$$\therefore \quad P_t = \frac{P_o}{273} \text{ x } T$$

$$= \frac{P_o T}{T_o}$$

$$\therefore \qquad \frac{P}{T} = \frac{P_o}{T_o} \quad = \quad K \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.17)$$

$$\therefore \quad P_t \quad = \quad KT$$

Hence, $P_t \quad \alpha \quad T \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.18)$

This means that the pressure of the gas is directly proportional to its absolute temperature provided the volume is kept constant. This is another consequence of the pressure law. The original law states that

That is for a fixed mass of any gas heated at constant volume, the pressure increase by $\frac{1}{273}$ of the pressure at $0^oC$for each Celsius degree rise in temperature. Whereas, $\frac{P}{T}$ = constant is the deduction or the consequence from the law. It states that the pressure of a given mass of gas is directly proportional to its absolute temperature.

SELF ASSESSMENT EXERCISE 3

The pressure in a diver's oxygen cylinder is $1.25 \times 10^6 \frac{N}{m^2}$ at 20^oC. Determine the pressure in the cylinder if it is lowered into water at 10^oC

3.5 Equation of State for Ideal Gases

In physics, two kinds of gases are usually discussed. They are real gases and ideal gases. You may like to know the difference between an ideal gas and a real gas. We shall now describe the properties of real gases and ideal gases.

3.5.1 Real Gases and Ideal Gases

At extremely low pressure, all gases closely obey Boyle's law. Thus if Boyle's law is obeyed, the volume coefficient r and the pressure coefficient β are the same for the same gas.

Starting with a fixed mass of a gas at 10^oC at pressure P_o and volume V_o, we then heat to a temperature of t^oC in two ways.

Firstly, we heat at constant pressure, then cool it down again to 0^oC and then repeat the process at constant volume.

(i) at constant pressure P_o the volume changes to V_t when heated to

temperature t^oC

$$\therefore \quad V_t \quad = \quad V_o(1 + rt)$$

(1) Starting point

$$\begin{tikzcd}
0^oC & P_o \arrow[d] & V_o \arrow[d] & \text{1st stage} \arrow[d] \\
 & t^oC & P_o & V_t = V_o(1 + rt)
\end{tikzcd}$$

(2) Heating the gas at Constant Pressure

$$\begin{tikzcd}
0^oC \arrow[d] & P_o \arrow[d] & V_o \arrow[d] & 2^{nd} \text{ stage} \\
P_o & P_{t =} P_o(1+ \beta t) & V_o &
\end{tikzcd}$$

(3) Heating the gas at Constant volume

At constant pressure the product PV = $P_oV_o(1 + rt)$ …..………….. (5.19)

(ii) At constant volume V_o, the pressure becomes pt = $P_o(1+ \beta t)$
The product PV then becomes $P_oV_o(1 + \beta t)$ ……………… (5.20)

If Boyle's law is obeyed, all values of the product PV at the same temperature t°C must be the same.

$$\therefore \quad P_oV_o(1 + rt) = P_oV_o(1 + \beta t) \quad …………..……………. (5.21)$$

Hence $r = \beta$

By plotting the values of r and β against pressure P, a graph shown in fig. 5.8 will be obtained.

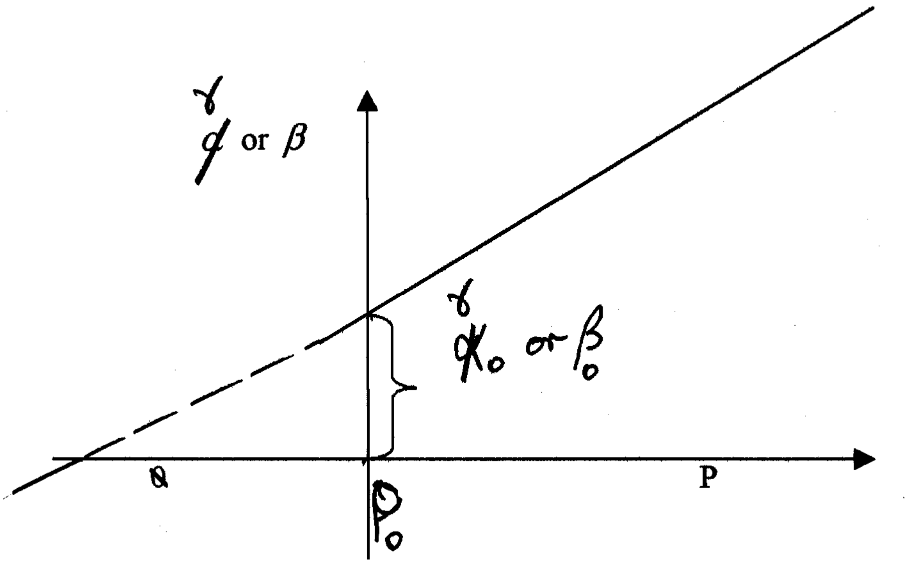


Fig. 5.8

By extrapolating to zero pressure, the pressure values of these coefficients at limiting conditions when Boyle's law is obeyed are found to be closely the same. (Table 5.1).

Table 5.1

Gas	**r**	$\boldsymbol{\beta}$
Helium	0.0036607	0.0036609
Hydrogen	0.0036611	0.0036610
Nitrogen	0.0036609	0.0036606

The table 5.1 shows that the pressure coefficient and the volume coefficient for each individual gas are very close to one another. Also the values for different gases are all close to the mean values of 0.0036608. Hence in the limiting case at extremely low density and pressure, when Boyle's law is obeyed closely, all gases have the same volume coefficient 0.036608 and the same pressure coefficient 0.0036608.

Real gases will behave in this way only at extremely low pressures. This behaviour is what is described as being ideal.

A gas, which would behave in this way at all pressure, is called an ideal or perfect gas. In practice real gases are not ideal but we consider some approximations to ideal gas under some specified conditions.

We are now in the position to produce the equation of state for an ideal gas. At constant pressure,

$$PV = P_oV_o(1 + rt) \quad \text{……………………….……………. (5.22)}$$

At constant pressure,

$$PV = P_oV_o(1 + \beta t) \quad \text{………………….…..…………………. (5.23)}$$

$$\text{But } r = \beta = 0.0036608 = \frac{1}{273} \quad \text{(nearly)}$$

$$\text{Thus } PV = P_oV_o\left(1+\frac{1}{273}t\right) \quad \text{………………….……….. (5.24)}$$

$$= P_oV_o\left(\frac{273+t}{273}\right)$$

$$\text{where } T_o = 273 \text{ and } 273 + t = T$$

$$\therefore PV = \left(\frac{P_oV_o}{T_o}T\right) \quad \text{…………………………...……….. (5.25)}$$

$$\therefore \frac{PV}{T} = \left(\frac{P_oV_o}{T_o}\right) = K = \text{Constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5.26)}$$

For a given mass of gas $PV = KT$

This is the equation of state for ideal gases.

3.5.2 Absolute Zero and Absolute Temperature

From the equation of state of an ideal gas we obtained,

$$PV = \frac{P_oV_o}{T_o}(273 + t)$$

$$\therefore PV = K(273 + t)$$

If the graph of PV is then plotted against temperature e t for a perfect gas, a graph as shown in fig. 5.9 is obtained.

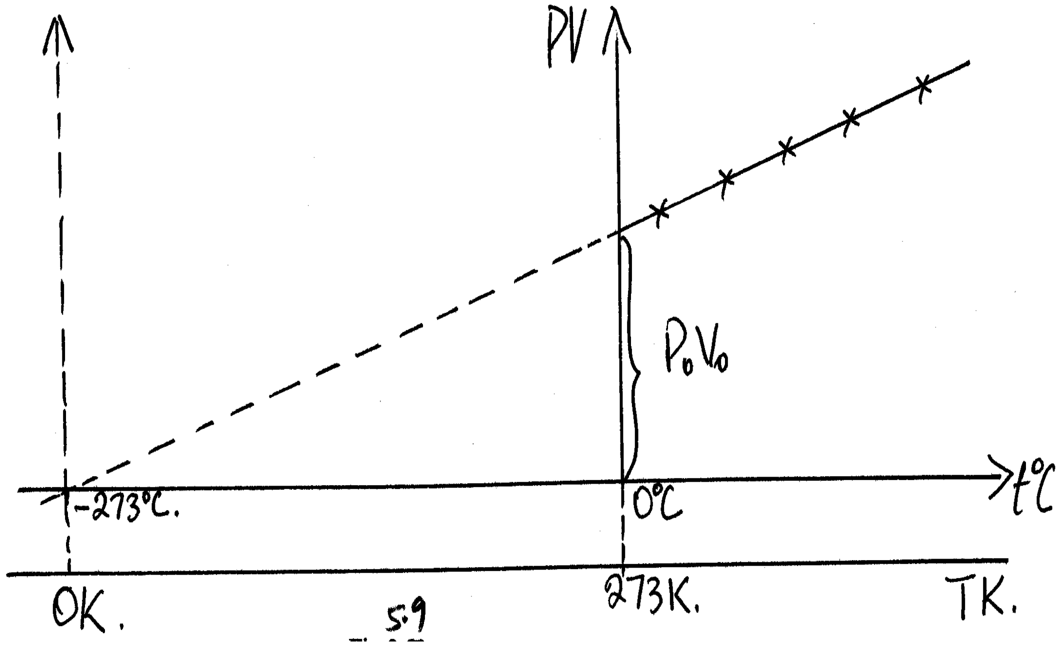


Fig. 5.9

The product PV has a value of zero when the temperature t is -273°C. At this point the product PV for a perfect gas would vanish completely. The point Z is therefore referred to as the absolute zero of temperature. If the origin of the graph 0 is now transferred to Z, then the new scale is denoted by T. This is what is referred to as the absolute scale of temperature, the SI unit of which is Kelvin. You will notice that both the Celsius scale and the Absolute scale arte related in such a way that

$$T = (273 + t)K$$

The unit of T is Kelvin while that of t is degree Celsius (^{o}C).

3.5.3 Universal gas Constant

From the Eq. (5.27) $PV = KT$

where $K = \frac{P_oV_o}{273} = \frac{P_oV_o}{T_o}$

The pressure P_o and 273 (T_o) fix the density of the gas when the volume V_o is proportional to the mass of the gas considered. Therefore K varies directly as the mass of the gas K is constant in the sense that it has a fixed value for a given mass of an ideal gas.

There are two kinds of units of mass: the gramme and the mole (the gramme molecular weight) – gramme – mole. If you consider one mole of a gas, R replaces the constant K.

$$\therefore \quad PV = RT$$

Hence the equation PV = RT is the ideal gas equation for one mole of the gas. Generally for n mole of a gas we would write:

$$PV = nRT \quad \text{……………...………………………….} \quad (5.28)$$

Where n = molar fraction
R is the Universal gas constant for a mole of a gas.

It has been experimentally found that under standard temperature (273K) and pressure (76mm of Hg) one mole of gas occupies approximately 22.4 litres.

$$1 \text{ litre} = 1000cm^3$$
$$1 \ cm^3 = 10^{-6}m^3$$
$$\therefore \quad 1000cm^3 = 1000 \times 10^{-6}m^3$$
$$\therefore \quad 22.4 \text{ litres} = 22.4 \times 10^{-3}m^3$$

The number of molecules in a mole of nay gas is

$6.03 \times 10^{23} = N =$ Avogadro's number.

If m is the mass of gas in gramme and M is the molecular weight of the gas, the number of moles of the gas is given as:

$$n = \frac{m}{M} \quad \text{………………………………………..….. (5.29)}$$

$$\therefore \quad PV = \frac{mRT}{M} \quad \text{………………...………………….. (5.30)}$$

The value of R for 1 mole of a gas

Let P be the pressure on the gas = 76cm of Hg = standard pressure.

$$\text{Put} \quad P = h\rho g$$

$$= \frac{76}{100}\,\text{m x }13600\,\frac{\text{kg}}{\text{m}^3}\text{ x }9.8\,\frac{\text{m}}{\text{s}^2}$$

$$= 1.0129 \text{ x } 10^5 \text{Nm}^{-2}$$

For a standard temperature T = 273K and standard volume V = 22.4 litres or 22.4 x 10^3m^{3}and for one mole of the gas, R can be obtained by suing the Eq. (5.28). The value of R

$$PV = nRT$$

$$\therefore \quad R = \frac{PV}{nT} \quad \text{where } n = 1$$

$$\therefore R = \frac{1.0129 \text{ x } 10^5 \dfrac{\text{N}}{\text{m}^2} \text{ x } 22.4 \text{ x } 10^{-10}\text{m}^3}{1 \text{ mole x } 273\text{K}}$$

$$= 8.31 \frac{\text{J}}{\text{mole K}}$$

R = 8.31 J mole^{-1} K^{-1} is the molar gas constant and it is the same for all gases.

SELF ASSESSMENT EXERCISE 4

Some hydrogen collected at 25^{o}C and 740mmHg has a volume of 550cm^3. What will be its volume at standard temperature and pressure (s.t.p.) i.e. 0^{o}C and 760mmHg?

3.5.5 Real Gas Equation

An ideal gas will obey Boyle's law at any temperature. However, real gases such as air, oxygen, nitrogen and other permanent gases will obey Boyle's law within less than one part in a thousand at ordinary pressures and temperatures. At higher pressures and lower temperature, the deviations are more pronounced. In other words, the relation PV is no longer valid.

Kinetic theory of gases suggest that Boyle's law should be obeyed if the molecules are themselves infinitesimally small and if they do not attract each other at all. These assumptions are not true for any real gas. Thus PV = nRT cannot be used for real gases.

In order to account for the difference between the behaviour of a gas and that of an ideal gas, we have to allow for the molecular attractions which converts the pressure P to $(P + \chi)$ and the finite volume occupies by the molecules which reduces the volume V of the gas to $(V - \gamma)$. These corrections therefore enable us to re-express PV = nRT as:

$$(P + \chi)(V - \gamma) \quad = \quad nRT \text{ ……………………………………. (5.31)}$$

We therefore need to find suitable expressions for χ and γ.

It was Van der Waal (1910) a Dutch Professor of Physics who found these expression to be $\chi = \frac{a}{V^2}$ and $\gamma = b$ ……………………… (5.32)

Where, a and b are constants for a unit mass of a gas under consideration.

Consequently substituting the values of Eq. (5.32) in Eq. (5.31), we now have the equation:

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT \text{ ………………………………….. (5.33)}$$

This is the Van der Waal's equation of state for real gases. It is therefore known as the real equation of state.

4.0 CONCLUSION

In this unit, the three gas laws: Boyle's, Charles's and the Pressure laws have been established. We did this by observing the behaviour of the gas by using the following properties – pressure, volume, temperature and the amount of the gas in moles. Any two of these properties are held constant while we study the variation of the remaining two properties. The equation of state was also stated as PV = nRT.

Further more, we established what real and ideal gases are. We also showed how van der Waal's equation of state was used to correct for the interactive forces, which affect the pressure and the volume occupied by the molecules, which corrects for the volume of the gas. This was given as:

$$\left(P + \frac{a}{V^2}\right)(V - b) = nRT$$

5.0 SUMMARY

In this unit, you have learnt about:

- The gas laws such as:
  - (i) Boyle's law (PV = constant)
  - (ii) Charles's law ($\frac{V}{T}$ = constant)
  - (iii) Pressure law ($\frac{P}{T}$ = constant)
- The three laws were combined to form the equation of state for ideal gases, which is PV = nRT.
- Pressure, volume, temperature and the amount of the gas in moles have been used to describe the behaviour of gases.
- The equation of state for real gases is $\left(P + \frac{a}{V^2}\right)(V - b) = nRT$.

ANSWER SELF ASSESSMENT EXERCISE 1

First Condition

P_1 = $14Nm^{-2}$
V_1 = $1.5m^3$

Second Condition

P_2 = $10Nm^{-2}$
V_2 = ? (to be found)

Using Boyle's law, Eq. (5.6)

$$\therefore \quad V_2 = \frac{P_1 V}{P_2}$$

$$V_2 = \frac{14\frac{N}{m^2} \times 1.5m^3}{10\frac{N}{m^2}}$$

We get,
V2 = $2.1m^3$

Volume of the gas at $10Nm^{-2}$ is $2.1m^3$

ANSWER SELF ASSESSMENT EXERCISE 2

$V_1 = 200\text{cm}^3$ (given)

$T_1 = 15^\circ\text{C} = (273 + 15)\text{K}$ (given) $= 288\text{K}$

$V_2 = 150\text{cm}^3$

$T_2 = ?$

Using Charles's law Eq. (5.12),

$$\therefore \quad \frac{V_1}{T_1} = \frac{V_2}{T_2} = \text{Constant}$$

$$\therefore \quad T_2 = \frac{V_2 T_1}{V_1}$$

$$= \frac{150\text{cm}^3 \text{ x } 288\text{K}}{200\text{cm}^3}$$

$$T_2 = 216\text{K} = 273 + t$$

$$\therefore \quad t = 216 - 273$$

$$\mathbf{t = -57^\circ C}$$

ANSWER SELF ASSESSMENT EXERCISE 3

$P_1 = 1.25 \text{ x } 10^6\text{Nm}^{-2}$

$T_1 = 20^\circ\text{C} = (273 + 20)\text{K} = 293\text{k}$

$P_2 = ?$

$T_2 = 15^\circ\text{C} = (273 + 10)\text{K} = 283\text{K}$

Using the pressure law:

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\therefore \quad P_2 = \frac{P_1 T_2}{T_1}$$

$$= \frac{1.25 \text{ x } 10^6 \dfrac{\text{N}}{\text{m}^2} \text{ x } 283\text{K}}{293\text{K}}$$

$$\mathbf{= 1.21 \text{ x } 10^6 Nm^{-2}}$$

ANSWER SELF ASSESSMENT EXERCISE 4

P_1 = 740mmHg
V_1 = $550cm^3$
T_1 = 25^oC = (273 + 25)K
= 298K

P_2 = 740mmHg
V_2 = ?
T_2 = 0^oC = 273K
Using the Eq. given below, we get

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$$\therefore \quad V_2 = \frac{P_1V_1T_2}{P_2T_2}$$

$$= \frac{740mmHg \times 550cm^3 \times 273K}{760mmHg \times 298K}$$

$$= \mathbf{490.60cm^3}$$

6.0 TUTOR-MARKED ASSIGNMENT

1. The density of some air at a pressure of 7720mmHg is $1.26kgm^{-3}$. Determine its density at a pressure of 600mmHg.

2. A fixed mass of gas of volume $546cm^3$ at 0^oC is heated at constant pressure. Calculate the volume of the gas at 2^oC.

3. A bottle is corked when the air inside is at 2^o0C and the pressure is $1.0 \times 10^5Nm^{-2}$. If the cork blows out with a pressure of $3.0 \times 10^5Nm^{-2}$, calculate the temperature to which the bottle must be heated for this to happen. (Assume the bottle does not expand).

7.0 REFERENCES/FURTHER READING

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