MODULE 3

UNIT 1 ELASTIC PROPERTY OF MATTER

CONTENTS

1.0 INTRODUCTION

As you know, when some objects are stretched or compressed by applying a force, their shapes change. But when the determining force is removed from the objects, the objects regain their original form or shape. Such objects are known as elastic objects.

Matter as a whole may be subjected to lads and forces. We may be able to see the effects of such load or force on matter. The example of stretching of a rubber shows where the effect is very obvious. The bridges are subjected to heavy loads but the effects of such loads are not very obvious. Thus the steel with which most bridges are built is usually subjected to all kinds of research in order to know how it can withstand the forces. The study of such a research in order to know it can withstand the forces. The study of such a research is referred to as the study of elasticity of the material.

In the beginning of this unit, we shall discuss about the elasticity. Then you will read about Hooke's law. Finally, you will learn about the concept of elastic limit and related terms: in the next unit we will discuss moduli of elasticity.

2.0 OBJECTIVES

At the end of this unit you should be able to:

- explain the term elasticity
- explain the elastic limit of a metal in the form of a wire
- define stress and strain
- state Hooke's law of elasticity
- solve problems on Hooke's law.

3.0 MAIN CONTENT

3.1 Concept of Elasticity

Elasticity is the property of a material to regain its original shape or form after removing the deforming force, or load provided the elastic limit is not exceeded.

There are certain terms used in connection with elasticity; stress and strain. Let us discuss briefly about these two terms. **The stress on a n elastic material is defined as the force exerted on the material per unit area.** It can also be expressed as:

$$\text{Stress} = \frac{\text{Force}}{\text{Area}} = \frac{F}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11.1)$$

The force is measured in Newton's (N) and the area is in meter square (m^2) then the unit of stress is therefore Newton per meter square (Nm^{-2}).

If an object has an original length of ℓ, when it is stretch or compressed in such a way that it has an extension or compression value of e, then, the strain on the object is defined as

$$\text{Strain} = \frac{\text{Extension produced on the object}}{\text{The original length of the object}}$$

$$\text{Strain} = \frac{e}{\ell} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (11.2)$$

Stress is related to the force producing the deformation. Strain is related to the amount of deformation.

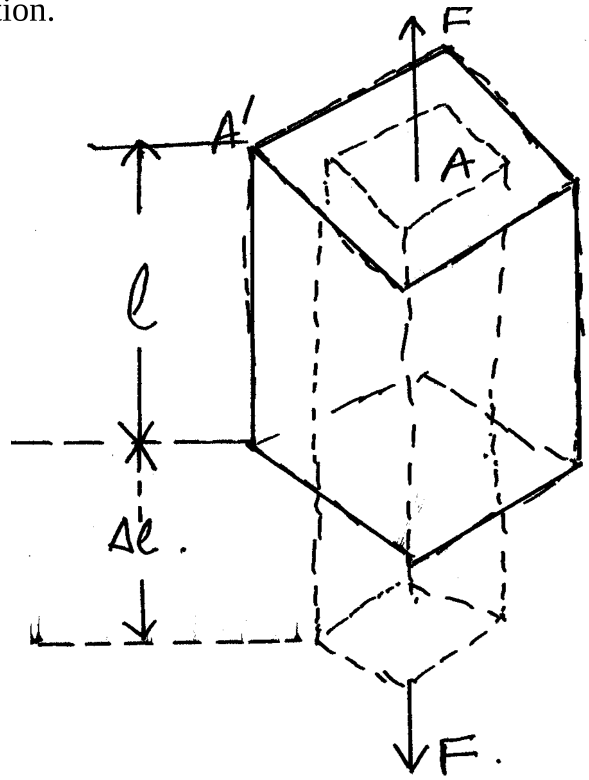


Fig. 11.1

Fig. 11.1 shows a typical deformation of a body subjected to a force **F** whose cross-sectional area A' changes to A and whose length is increases by $\Delta\ell$. Provided its elastic limit is not exceeded, the material goes back to the former state.

Materials such as Dough, Lead and Putty, for which elastic limit is small are called inelastic or plastic material. Steel is highly elastic because it recovers itself to its original position when the load or force is removed.

3.2 Hooke's Law

Consider a helical spring of original length ℓ_o (fig. 11.2 (i)). When a weight is attached to it, it will be observed that the spring will be stretched through a distance e. By using slotted weight, we can vary the force **F** acting on the spring and with the aid of a ruler we can measure the extension e for each corresponding weight.

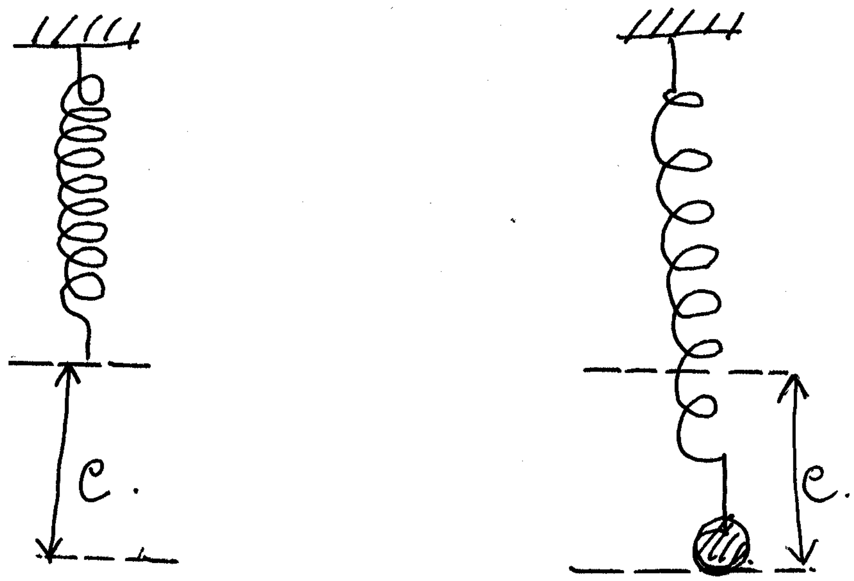


Fig. 11.2

When the various force (**F**) are plotted against their corresponding extensions (e) measured, a linear relationship is obtained as shown in fig. 11.3, that is, provided the elastic limit is not exceeded.

It is on the basis of this experiment that the British Scientist Robert Hooke formulated the law of elasticity which is known as Hooke's law.

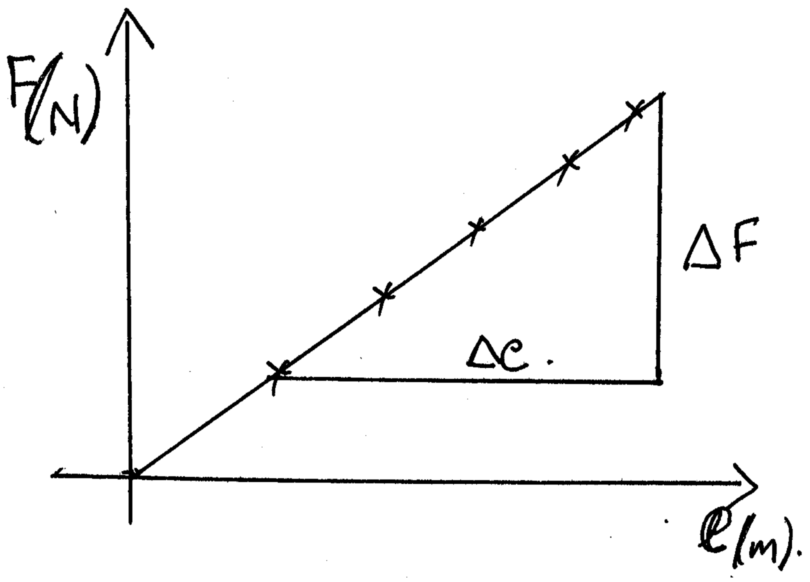


Fig. 11.3: A plot of Forces versus Extensions Measured

Hooke's law states that **the force applied on elastic material is directly proportional to the extension produced provided the elastic limit is not exceeded.**

$$F \quad \alpha \quad e$$
$$F = -ke$$

Where k is a constant of proportionality usually referred to as the spring constant. K is a property of elastic material. The negative sign is to show the restoring ability of the material. Later a modified form of this law was given. According to which "within elastic limit, the stress on an elastic material is directly proportional to the strain produced in the material".

Stress & strain
Stress = M strain

Where M is a constant. The constant is called a Modulus of elasticity of the material of the body.

SELF ASSESSMENT EXERCISE
State Hooke's law and describe an experiment to verify the law.

3.3 The Concept of Elastic Limit and Other Terms

We shall use the experimental set up in fig. 11.4 to show the concept of elastic limit, yield point and other terms used in the elastic property of matter. The apparatus is made up of two wires **P** and **Q**. Wire **P** is held taut by weight **A**. While Wire **P** carries a scale **M**. The second wire **Q** carry a vernier scale **V** is subjected to variation of forces **W**. The two wires **P** and **Q** are suspended from a common support **S**.

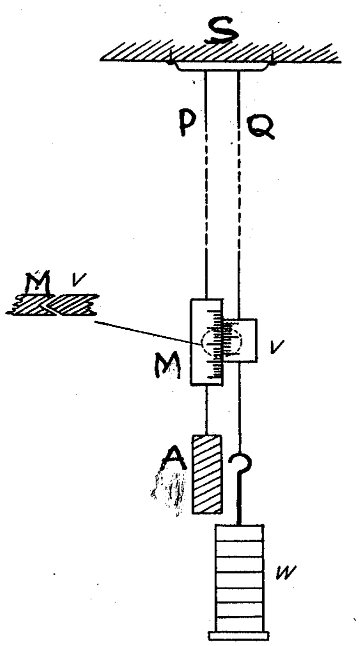


Fig. 11.4

If wire **Q** is loaded, the length of the wire increases. When it is off-loaded, it returns to its original position. If a material behaves this way, such a material is described as being elastic.

The load on wire **Q** is called the tensile force. When the wire is thus subjected to various tensile forces its corresponding extension (e) can then be observed.

The graph of the force (**F**) against the extension (e) can be plotted. A typical graph obtained from such an exercise is shown in fig. 11.5.

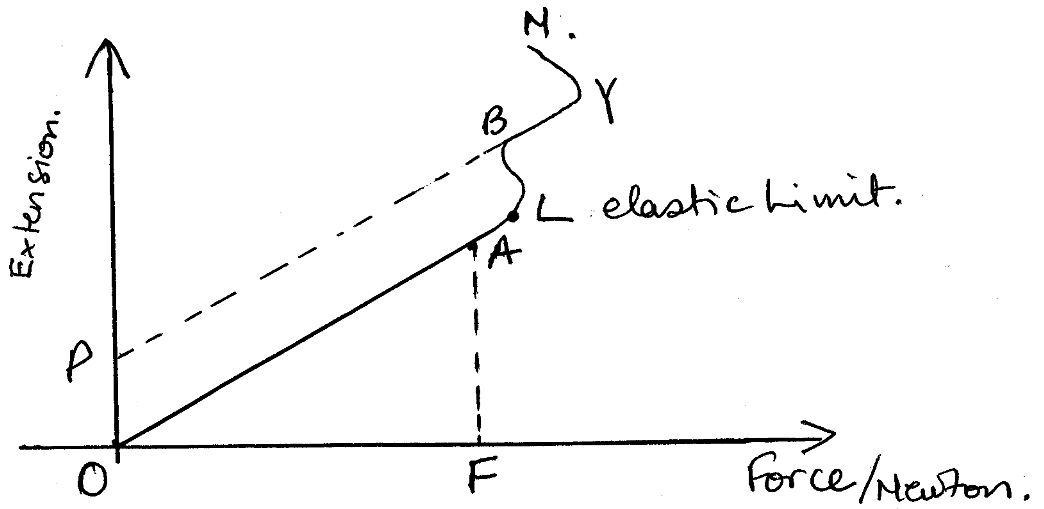


Fig. 11.5: Graph of Extension versus Force

Now let us discuss the shape of the graph obtained. It is made up of straight line **OA** followed by a curve **ABY** rising slowly at first and then sharply. Along and up to **L**, which is just beyond **A**, the wire is capable of returning to its original length when the load is removed. **A is** therefore the limit of proportionality. The force at **L** is the force of elastic limit.

Along **OL**, the metal is said to undergo changes called elastic deformation. Beyond **L**, the wire has a permanent extension as shown by line **OP**. When the force is at **B**.

Beyond **L**, the wire is no longer elastic. With more force the extension increases rapidly along the curve **ABY**. At **N**, the wire thins out and breaks.

The Yield Point

For mild steel and iron, the molecules of the wire begin to slide across each other soon after the load (force) has exceeded the elastic limit. At this stage, the metal is described as being plastic as shown in the slight kink at **B** beyond **L** (fig. 11.5). This is the yield point of the wire.

The change from elastic to plastic stage is shown by a sudden increase in the extension. In the plastic stage, the energy gained by the stretched wire is dissipated as heat. Unlike the elastic stage, the energy is not recoverable when the load is removed.

As the load is increased further, the extension increases rapidly along he curve **YN**. The wire then becomes narrower and finally breaks. The breaking stress of the wire is the corresponding force per unit area of the narrowest section of the wire.

Ductile Substances

Substances which lengthen considerably and undergo plastic deformation until they break are ductile substances. Examples of such substances are Lead (Pb), Copper (Cu) and Wrought Iron (Fe).

Brittle Substance

When substances break just after the elastic limit is attained, then, such substances are said to be Brittle. Glass, high carbon steel brass, bronze and other alloys have no yield point.

4.0 CONCLUSION

When a force is applied to a material it changes its shape. When the force is removed, the material regain its original sate, hen that object is said to be elastic. However, the amount of force applied to that object must be such that the elastic limit is not exceeded. If the force is applied in such a way that the object is permanently deformed, then we say that the elastic limit has been exceeded. Robert Hooke established the relationship between the force applied to a material and the extension produced within the elastic limit. The force applied per unit area of the material described the stress on the material. The ratio of the extension produced by the force and the original length of the material known as strain on the material. Substances are classified as being ductile or brittle depending on their behaviour when the elastic limit has been exceeded. A substance is said to be ductile if undergoes a plastic deformation and breaks. A substance is said to be brittle if it breaks just when it has exceeded its elastic limit.

5.0 SUMMARY

In this unit you have learnt that:

- A material is said to be elastic if when deformed by the application of a force and capable of regaining its original shape when the applied force is removed;
- The deformation can be in the form of change in length and shape;
- Robert Hooke showed that within the elastic limit, the force on a given length of wire is directly proportional to its extension (e);
- The yield point of an elastic material occurs when the load on it makes it to exceed the elastic limit and the material becomes permanently deformed or plastic;
- A material is described as being ductile when it undergoes a plastic deformation and then breaks;
- A material is described as being brittle if just when it is at its elastic limit any additional load makes it to break.

ANSWER TO SELF ASSESSMENT EXERCISE

Refer to Section 3.2 of Unit 11. To verify the law, you can take a metallic spring, a scale, and pan to attach with the spring. Also take some weight to put into pan and note the corresponding extensions from the lower scale. Draw a graph between load applied and the extension. Then conclude the result and verify the law.

6.0 TUTOR-MARKED ASSIGNMENT

The extension in length of a spiral spring varies as different weights are suspended from it. The result of the experiment with the spiral spring is as shown below in tabular form:

W(10^{-2}N)	5	10	20	30	40	50	60
Extension (cm)	0.9	1.8	3.6	5.4	7.2	9.0	10.8

Plot the graph of weight versus the extension produced, and deduce the relationship between the weight and the extension. Determine the spring constant of the spiral spring.

7.0 REFERENCES/FURTHER READING

Awe, O. and Okunola, O.O. (1986). *Comprehensive Certificate Physics*, Ibadan: University Press.

Ference, M. (Jnr.), Lemon, H.B. and Stephenson, RJ (1970). *Analytical Experimental Physics*, Chicago: The University of Chicago Press.

Feyman, R.P., Leighton, R.B. and Sands, M (1971). *The Feyman Lectures on Physics*, California: Addison-Wesley Publishing Company Limited.

Flowers, B.H. and Mendoza, E (1970). *Properties of Matter*, London: John Wiley and Sons Limited.

Noaks, M.A. (1963). *New Intermediate Physics*, Canada: Macmillan and company Limited.

Soars, F.W., Zemansky, M.W. and Young, H.D. (1980). *College Physic*s. London: Addison-Wesley Publishing Company.

Tyler, F. (1966). *Heat and Thermodynamics*. London: Edwards Arnold Publisher Limited.

Wilkinson, J. (1983). *Essentials of Physics*, Australia: Macmillan Education Publishers.

Zemansky, M.W. (1968). *Heat and Thermodynamics*. New York: McGraw-Hill Book Company.

UNIT 2 MODULI OF ELASTICITY

CONTENTS

1.0 INTRODUCTION

In the previous unit, you were introduced to the elastic properties of matter. You learnt about the law that governs the elastic nature of a material, most especially if it is a solid object. We have also observed that it is not only solid objects that are elastic or that can be subjected to stress and strain, what about liquids and gases?

Elastic nature of liquids is not very obvious as compared with that of gas. If you take a bicycle pump, close the end part with your finger very hard so that when you push down the piston air does not escape. Then push down the piston by applying force on the handle. The piston goes down through a considerable distance but when released the piston springs back to its original position. This shows the compressibility of the air molecules and their elastic nature.

Thus in considering the elastic nature of liquids and gases in this unit we shall embark on the other concepts of elasticity which involves the moduli of elasticity. These are the Young, Bulk and Shear moduli. Liquids and gases also experience stresses and strains when forces are applied on them. Besides, we would also consider the energy involved in the stretching of a wire.

2.0 OBJECTIVES

After studying this unit you should be able to:

- define Young's modulus of elasticity
- define Shear modulus of elasticity
- define Bulk modulus of elasticity
- relate Young's modulus with thermal expansion of a metal
- define energy stored in a stretched wire

- solve problems on moduli of elasticity.

For a given material, there can be different types of modulus of elasticity. Now in the subsequent sections, we will learn about these. It all depends upon the type of stress applied and the resulting strain produced. Let us discuss these facts.

3.0 MAIN CONTENT

3.1 Young's Modulus

When a force (F) is applied to the end of a wire of cross-sectional area A and having an initial length L, we define the tensile stress on the wire as the force per unit area.

$$\text{Tensile stress} = \frac{\text{Force}}{\text{Area}}$$

$$= \frac{F}{A} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12.1)}$$

Where F is measured in Newton's (N) and Area is in meter-square (m^2). Therefore the unit of tensile stress is Nm^{-2}.

The cross-sectional area A of the wire is expressed as,

$$A \quad = \quad \frac{\pi D^2}{4} \; \pi r^2$$

Where, r is the radius of the cross-section and
D is the diameter of the cross-section.

If on the application of the force F, the wire is extended by (e) then we would define its tensile strain as

$$\text{Tensile strain} = \frac{\text{extension}}{\text{initial length}}$$

$$= \frac{e}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(12.2)}$$

SELF ASSESSMENT EXERCISE 1

2kg mass is attached to the end of a vertical wire of length 2m with a diameter of 0.64mm and having an extension of 0.60m. Calculate

(i) the tensile stress
(ii) the tensile strain on the wire (take $g = 9.8m/s^2$)

Stress on a wire is plotted against the strain, a graph shown in fig. 12 is obtained.

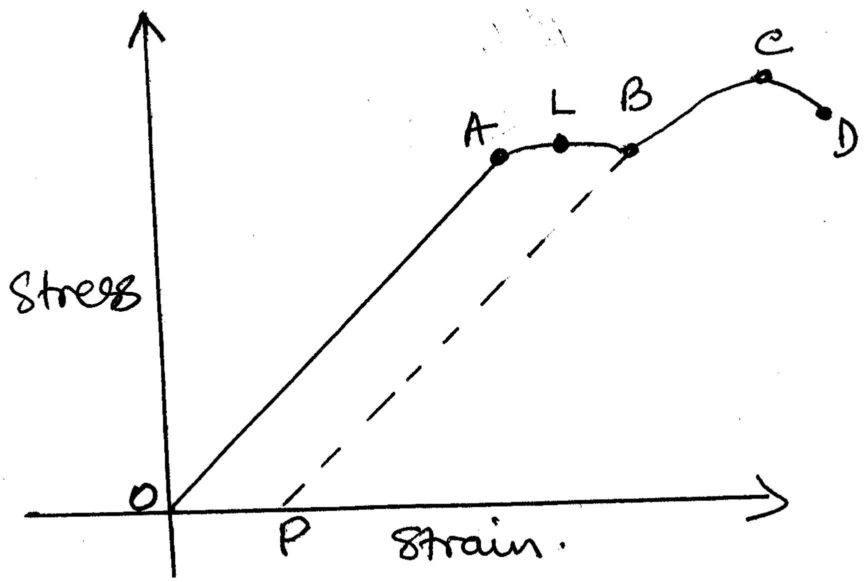


Fig. 12.1: Stress versus Strain

The points and region on the graphs shows

A	=	proportional limit
L	=	elastic limit
B	=	Yield point
C	=	Breaking Stress
D	=	wire breaks
OL	=	elastic deformation
BC	=	plastic deformation

From this graph, we shall define the Young's Modules of Elasticity symbolized as E.

Thus $$E = \frac{\text{Tensile stress}}{\text{Tensile Strain}}$$

$$E = \frac{F}{A} \div \frac{e}{\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (12.3)$$

Example 12.1

The Tensile stress and Tensile strain of a wire 6.09 x 10^7N)m^7 and 3 x 10^{-4} respectively. Determine Young's Modules of the wire.

Solution

$$E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}}$$

$$= \frac{6.09 \times 10^7 \frac{N}{m^2}}{3 \times 10^{-4}}$$

$$= 2.03 \times 10^{11} Nm/^2$$

The Young's Modulus = $2.03 \times 10^{11} Nm/^2$

3.1.1 Force in a Bar Due to Contraction or Expansion

When an iron bar is heated and is prevented from contracting as it cools, experiments have shown that a considerable force is exerted at the ends of the bar.

It is possible for us to obtain the force exerted with the knowledge of Young Modules E, cross-sectional area A, the coefficient of linear expansion α (linear expansivity) and the change in temperature due to the contraction or expansion, Δθ.

From Eq. (12.3), you know that the force F is given by

$$E = \frac{F}{A} \div \frac{e}{L}$$

$$\therefore \quad E = \frac{F}{A} \times \frac{L}{e}$$

$$\therefore \quad F = \frac{EAe}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (12.4)$$

Also, the coefficient of linear expansion is given as

$$\alpha \; \therefore \quad \frac{\Delta L}{L\Delta\theta} = \frac{e}{L\Delta\theta}$$

$$\therefore \quad e = \alpha L \Delta\theta \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (12.5)$$

Substituting Eq. (12.5) in Eq. (12.4), we get

Hence $F = \frac{EA}{L} \times \alpha L \Delta\theta$

$$F = EA\alpha\Delta\theta \; \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (12.6)$$

SELF ASSESSMENT EXERCISE 2

A steel rod of cross-sectional area $2.0cm^2$ heated to 100^oC and then prevented from contracting when it cooled to 10^oC. If linear expansivity of steel is $12 \times 10^{-6}\ k^{-1}$ and Young Modules = $2.0 \times 10^{11} N/m^2$. Calculate the force exerted on contraction.

3.1.2 Energy Stored in a Wire

As you know that work done is defined as the product of the force and the distance moved by the force

i.e. Work done = Force x distance

For a given wire, the work in stretching it is given by

Work done = Average force x extension

$$= \left(\frac{F}{2}\right) \text{ x e}$$

But now you may ask: why we are using the average force rather than absolute force?

This is because of the fact that the force increases from zero to a given value, provided the elastic limit is not exceeded. Therefore the amount of work done in stretching a wire, or the amount of energy stored in the wire is given by

$$W = \frac{Fe}{2} \qquad \text{(12.7)}$$

The energy stored is the gain in molecular Potential Energy of the molecules due to their displacement from their mean positions.

The energy store could be expressed in terms of Young Modulus. Now, substitute the value of F from Eq. (12.4) in Eq. (12.7), we get

$$W = \frac{1}{2} \text{ x } \left(\frac{EAe}{L}\right) \text{ x e}$$

$$W = \frac{1}{2} \text{ x } \frac{EAe^2}{L} \qquad \text{(12.8)}$$

SELF ASSESSMENT EXERCISE 3

A vertical wire, suspended from one end is stretched by attaching a weight of 20N to the lower end. If the weight extends the wire by 1mm. calculate the energy gained.

3.2 Bulk Modulus of Elasticity

Meanwhile, you would have observed that we have always been talking about elasticity in terms of solids (i.e. wire). What about liquids and gases? While Young Modulus of elasticity is associated with solid objects, the bulk modulus deals with liquids and gases.

When a liquid or gas is subjected to an increase pressure, the substance contracts. A change in the bulk thus occurs. The bulk strain is defined as the change in volume (ΔV) of a gas (or liquid) to the original volume V.

$$\text{Bulk Strain} = \frac{\text{Change in volume of gas or liquid}}{\text{Original volume of gas or liquid}}$$

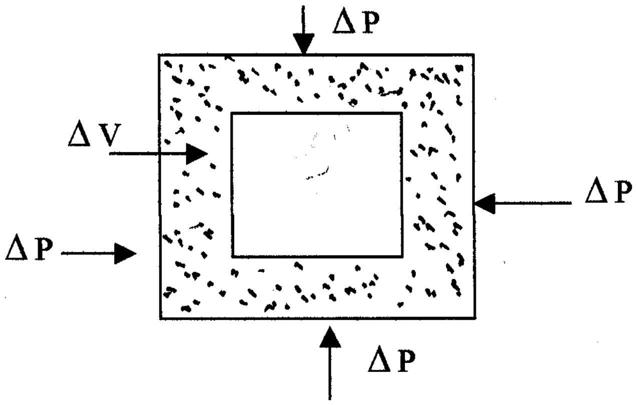


Fig. 12.2: Bulk Stress & Bulk Strain

Since the change in volume is $-\Delta V$

And the original volume is V,

Therefore the, Bulk Strain = $\frac{-\Delta V}{V}$ …………...…………….. (12.9)

The negative sign indicates that the volume decreases. Bulk stress is defined as the increase in force per unit area.

$\therefore$ Bulk Stress = $\frac{\text{Increase in force}}{\text{Area}} = \frac{\Delta F}{A}$ …………………..…. (12.10)

Then Bulk Modulus of elasticity K is defined as

$$K = \frac{\text{Bulk Stress}}{\text{Bulk Strain}}$$

$$= \Delta P \div \frac{-\Delta V}{V}$$

$$K = \frac{-\Delta VP}{\Delta V}$$ …………………...………. (12.11)

The limit of $\frac{\Delta P}{\Delta V}$ as ΔP and ΔV tend to zero is $\frac{dP}{dV}$

$$\text{Limit} \quad \frac{\Delta P}{\Delta V} = \frac{dP}{dV}$$
$$\Delta P, \Delta V \rightarrow 0$$

$$\therefore \quad k = \frac{-VdP}{dV} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (12.12)$$

The reciprocal of bulk modulus of elasticity K is $\frac{dP}{dV}$ which is known as compressibility of the material.

$$\therefore \quad \frac{1}{K} = \frac{1}{V}\frac{dV}{dP} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (12.13)$$

3.2.1 Bulk Modulus of a Gas – Isothermal Bulk Modulus

When the pressure and the volume of gas changes at constant temperature, we have the expression

$$PV = \text{Constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (12.14)$$

It is also known as Boyle's Law. In addition, it is also associated with what we describe as an isothermal condition. On differentiating this expression with respect to volume V. we get

$$P\frac{dV}{dV} + V\frac{dP}{dV} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12.5)$$

$$\therefore \quad P + V\frac{dP}{dV} = 0$$

$$\therefore \quad P = -V\frac{dP}{dV} = K \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12.6)$$

What does this expression remind you of? You will recall that the bulk modulus of elasticity is given as

$$K = -V\frac{dP}{dV}$$

$$\therefore \quad P = -V\frac{dP}{dV} = k \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (12.17)$$

Thus the isothermal bulk modulus of elasticity is equal to the pressure P.

3.2.2 Bulk Modulus of a Gas – Adiabatic Modulus

By adiabatic, we mean a system whereby heat is not allowed to escape into it or get out of it. The pressure, and volume of a gas can be related in such a way that

$$PV^{¾} = \text{Constant} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12.18)$$

where $\gamma = \frac{C_P}{C_V}$

The symbol r is pronounced as gamma.

$\frac{C_P}{C_V}$ is a ratio of the molar heat capacities of a gas at constant pressure C_P and constant volume C_V. A state is described as adiabatic when no heat is allowed to leave the system or enter the system of a gas. The external work done is wholly at the expense of the internal energy of the gas. The consequence is that the gas cools down starting from PV^r = Constant

On differentiating the Eq. (12.18) with respect to V, we get

$$\therefore \quad P\frac{d(V^{\gamma})}{dV} + V^{\gamma}\frac{d(P)}{dV} = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots (12.19)$$

$$\therefore \quad P \times \gamma V^{\gamma - 1} + V^{\gamma}\frac{dP}{dV} = 0$$

$$\therefore \quad P\gamma = -\frac{V^{\gamma}}{V^{\gamma - 1}}\frac{dP}{dV}$$

On arranging the terms, we get

$$= -\frac{V^{\gamma}V}{V^{\gamma}}\frac{dP}{dV}$$

Note that $V^{\gamma - 1} = V^{\gamma}V^{-1} = \frac{V^{\gamma}}{V}$

$$\therefore \quad P\gamma = -V\frac{dP}{dV}$$

But $K = -V\frac{dP}{dV}$

$$\therefore \quad P\gamma = K \quad \ldots\ldots\ldots\ldots\ldots\ldots (12.20)$$

Hence the adiabatic Bulk Modulus is

$$K = P\gamma$$

The following are the values of K at isothermal and adiabatic conditions:

For air at normal pressure,

K = $1.0 \times 10^5 Nm^{-2}$ for isothermal bulk modulus, while
K = $1.4 \times 10^5 Nm^{-2}$ for adiabatic bulk modulus.

These values are of the order of 10^5 times smaller than liquids as gases are much more compressible.

3.3 Shear Modulus of Elasticity or Modulus of Rigidity

Consider a block ABCD such as block of rubber as shown in fig. 12.3

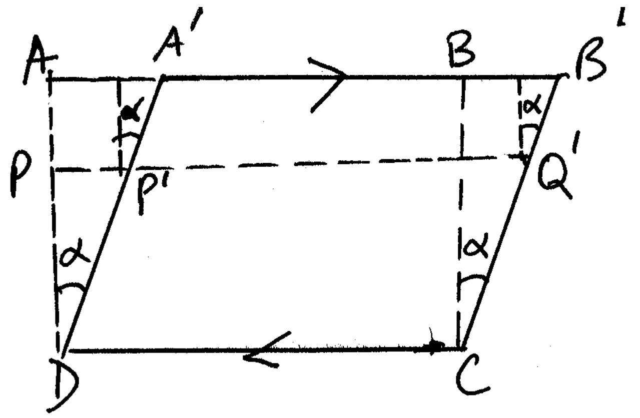


Fig. 12.3: Shear Stress and Shear Strain

The lower plane is kept fixed. A force F is applied to the upper side AB parallel to CD. The effect of this force on the material is a change in shape from ABCD to $A'B'CD$. There is a displacement of the planes in the body relative to the planes. The angular displacement α is defined as the shear strain. α is thus the angular displacement between any two planes of the material. For example planes $A'B'$ and CD or planes CD and $P'Q'$.

You should note that in shear modulus, no volume change occurs as in the Bulk modulus of elasticity. Since the force along CD is F in magnitude, it forms a couple with the force applied to the upper side AB.

The shear stress is defined as the shear force per unit area on the face AB or CD. The shear has a turning or displacement effect owing to the couple that exists. Besides, the solid does not collapse because in a strained equilibrium position such as $A'B'CD$, the external couple is acting on the solid due to the force F is balanced by an opposing couple due to stresses inside the material. If the elastic limit is not exceeded when a shear stress is applied, its original shape when the stress is removed, the modulus of rigidity or shear modulus G is defined as

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$G = \frac{\text{Force per unit area}}{\text{Angular displacement } \alpha}$$

Shear modulus is applicable to the helical spring. When the helical spring is stretched all parts become twisted. The applied force has thus developed a tensional or shear strain. The extension of the spring depends on its shear modulus in addition to its dimension. In sensitive, current measuring instruments, a weak control is needed for the rotation of the instrument coil. A long elastic wire of phosphor – bronze is used instead of a spring.

4.0 CONCLUSION

The ratio of stress and strain defines for us the modulus of elasticity. There are three moduli of elasticity:

- Young modulus
- Bulk modulus
- Shear modulus

When material expands or contracts an enormous force is involved. Besides, energy or work done in stretching or contracting an elastic material. Bulk modulus of elasticity may be considered under two conditions.

- Isothermal condition i.e. at constant temperature
- Adiabatic condition when there is no exchange of heat energy between the gas and its environment

5.0 SUMMARY

In this unit you have learnt that:

- A material is said to be elastic if it is capable of regaining its original shape when the applied force is removed.
- The deformation can be in form of change in length, volume (bulk) or shape.
- The stress on an elastic material is defined as the force per unit $\frac{F}{A}$. The unit of which is Nm^{-2}.

The strain of a given wire is defined as the ratio between the extension (e) and the original length (ℓ) of the wire i.e. $\frac{(e)}{\ell}$

- Young Modulus (E) of elasticity is defined as

$$E = \frac{\text{Tensile Stress}}{\text{Tensile Strain}}$$

$$= \frac{\text{Force per unit area}}{\text{extension / original length}}$$

- The force involved in the contraction or expansion of a wire is given as

$F = EA\,\alpha\Delta\theta$ where
E = Young Modulus
A = cross-sectional area
α = linear expansivity
$\Delta\theta$ = change in temperature

- The energy stored in a wire (W) is defined as

$$W = \frac{1}{2}\,\frac{EAe^2}{\ell}$$

E = Young Modulus
A = cross-sectional area
e = extension
ℓ = original length

- Bulk modulus of elasticity (K) is defined as

$$K = \frac{\text{Bulk Stress}}{\text{Bulk Strain}}$$

$$= -V\,\frac{dP}{dV}$$

Where V is the original volume and $\frac{dP}{dV}$ is differential rate of change of pressure with respect to volume.

- The compressibility of a material is the reciprocal of the Bulk modulus.

$$\text{Compressibility} = \frac{1}{K} = -V\,\frac{dP}{dV}$$

- Under isothermal condition, the bulk modulus of a gas is its pressure **P**.
- Under adiabatic condition, the bulk modulus of a a gas is rP where

$$\gamma = \frac{C_P}{C_V}$$

the ration of the molar thermal capacity of the gas at constant pressure C_P and the molar thermal capacity of the gas at constant volume C_V , and P the pressure.

- The shear modulus or the modulus of rigidity (G) is defined as

$$G = \frac{\text{Shear Stress}}{\text{Shear Strain}}$$

$$G = \frac{\text{Force per unit area}}{\text{Angular displacement } \alpha}$$

ANSWER TO SELF ASSESSMENT EXERCISE 1

(i) Tensile stress = $\frac{\text{Force}}{\text{Area}}$

$$= \frac{mg}{\pi D^2 / 4}$$

$$= \frac{2\text{kg x } 9.8 \frac{m}{s^2}}{\frac{\pi\left(\frac{0.64}{1000}\right)^2}{4} \quad m^2}$$

$$= \frac{4 \text{ x } 2 \text{ x } 9.8\text{N}}{1.287 \text{ x } 10^{-6} \text{ m}^2}$$

$= \quad 6.09 \text{ x } 10^7 \text{N/m}^2$

(ii) Tensile Stress = $\frac{e}{L}$

$$= \frac{\left(\frac{0.60}{1000}\right)m}{2m}$$

$$= \frac{0.6}{2} \text{ x } 10^{-4}$$

$$= 3 \text{ x } 10^{-4}$$

ANSWER TO SELF ASSESSMENT EXERCISE 2

Cross-sectional Area of rod = 2cm2

= $2 \times 10^{-4}m^2$

Change in temperature $\Delta\theta$ = $(100 - 10)^oC$

= $90\ ^oC$

From F = EA α $\Delta\theta$

$\therefore$ Force = $2 \times 10^{11}N \times 2 \times 10^{-4}m^2 \times 12 \times 10^{-6}C^{-1} \times 90^oC$

= $2 \times 2 \times 12 \times 90 \times 10^{11} \times 10^{-4} \times 10^{-6}$ N

= $4321 \times 10^{1}N$

= 43.20N

= $\underline{4.32 \times 10^{4}N}$

ANSWER TO SELF ASSESSMENT EXERCISE 3

Energy gained = ½ Fe

= ½ x 20N x $(1 \times 10^{-3}m)$

= $10^{-2}J$

= 0.01J

Gravitational Energy lost = mgh

= Wh

= $20N \times 1 \times 10^{-3}m$

= 0.02J

The excess energy is dissipated as heat when the weight comes to rest after vibrating at the end of the wire.

6.0 TUTOR-MARKED ASSIGNMENT

1. Calculate the maximum load, which may be placed on a steel wire of diameter 1.0mm if the permitted strain must not exceed 1/1000 and the Young modulus for steel is $2.0 \times 10^{11}N/m^2$.

2. An aluminum rood 300cm long, 0.580cm in diameter would normally contract 1.32cm in cooling from 225^oC to 25^oC. Calculate the force which would be required to prevent this contraction? (Young modulus for aluminum = 7.0 x $10N/m^2$).

7.0 REFERENCES/FURTHER READING

Awe, O. and Okunola, O.O. (1986). *Comprehensive Certificate Physics*, Ibadan: University Press.

Ference, M. (Jnr.), Lemon, H.B. and Stephenson, RJ (1970). *Analytical Experimental Physics*, Chicago: The University of Chicago Press.

Feyman, R.P., Leighton, R.B. and Sands, M (1971). *The Feyman Lectures on Physics*, California: Addison-Wesley Publishing Company.

Flowers, B.H. and Mendoza, E (1970). *Properties of Matter*, London: John Wiley and Sons Limited.

Noaks, M.A. (1963). *New Intermediate Physics*, Canada: Macmillan and company Limited.

Soars, F.W., Zemansky, M.W. and Young, H.D. (1980). *College Physics*. London: Addison-Wesley Publishing Company.

Tyler, F. (1966). *Heat and Thermodynamics*. London: Edwards Arnold Publisher Limited.

Wilkinson, J. (1983). *Essentials of Physics*, Australia: Macmillan Education Publishers.

Zemansky, M.W. (1968). *Heat and Thermodynamics*. New York: McGraw-Hill Book Company.

UNIT 3 KINETIC THEORY OF GASES AND ITS APPLICATION

CONTENTS

1.0 INTRODUCTION

In the previous unit, you have studied that liquid and gases experience stresses and strains when forces are applied on them. The substances are made up of molecules which exists in free state and possess all the basic properties of the substance. The state (solid, liquid or gas0 of a substance is determined by the interplay of thermal energy and intermolecular forces. This gives rise to molecular theory. Later it was supplemented by the laws of mechanics for individual molecules leads to kinetic theory.

In this unit, you will learn to derive an expression for pressure exerted by a gas and deal with kinetic theory of gases and see how it can explain all the gas laws of an ideal gas. You will also learn about the root mean square velocity of the molecules of a gas.

Further more the theory enables us to:

(i) Determine the root -mean -square velocity (r,m.s.) of the molecules of a given mass of gas.
(ii) Show variation of the r.m.s speed.
(iii) Show the distribution of molecular speeds.
(iv) Derive the internal energy of a gas.

2.0 OBJECTIVES

At the end of this module, you should be able to:

- state the assumption of the kinetic theory for an ideal gas
- derive an expression for the pressure of a gas in an enclosed system
- explain and derive the root-mean-square velocity (r.m.s.) of gas molecules
- derive the gas laws from the kinetic theory of gases
- show and explain the variation of r.m.s. speed
- explain Boltzmann constant
- derive the internal energy of a gas.

3.0 MAIN CONTENT

3.1 Kinetic Theory of Gases and its Applications

In this section, we shall explain the gas laws and other behaviour of gases by considering the motion of their molecules. That is, we will derive the gas laws through the Kinetic theory of gases. To do this, we use the following assumptions about the kinetic theory of gases:

(i) the gas consists of a very large number of exceedingly small - particles - identifiable with chemical molecules -which are in I incessant, rapid, haphazard motion. All direction of motions are equally probable.

(ii) for a given mass of gas, all these particles are identical and the smallest volume of gas consists of large number of molecules (i.e. in a $1mm^3$, under standard condition of temperature and pressure, there are nearly 3×10^{16} molecules).

(iii) the average distance between the molecules (called the mean free path) is so large compared with their linear dimensions that the particles may be considered to be of negligible size, i.e. the volume occupied by the molecules is negligible compared with the volume occupied by the entire gas.

(iv) the molecules exert no appreciable force on each other so that between collisions, they travel in straight paths, i.e. the attractive forces between molecules are negligible - there is no force of attraction among the molecules;

(v) collisions molecules between and with the walls are perfectly elastic. There is thus no loss of Kinetic energy during a collision process and a molecule rebounds from the wall with a velocity equal to that with which it strikes it;

(vi) the duration of a collision is negligible compared with the time between collisions.

We shall now use the above assumptions to derive an expression for the pressure exerted by a given mass of gas enclosed in a cubic container of length 1.

3.2 Pressure Exerted By Gas

Consider that there are N molecules of gas per cubic metre each of mass m as shown in fig. 13.1.

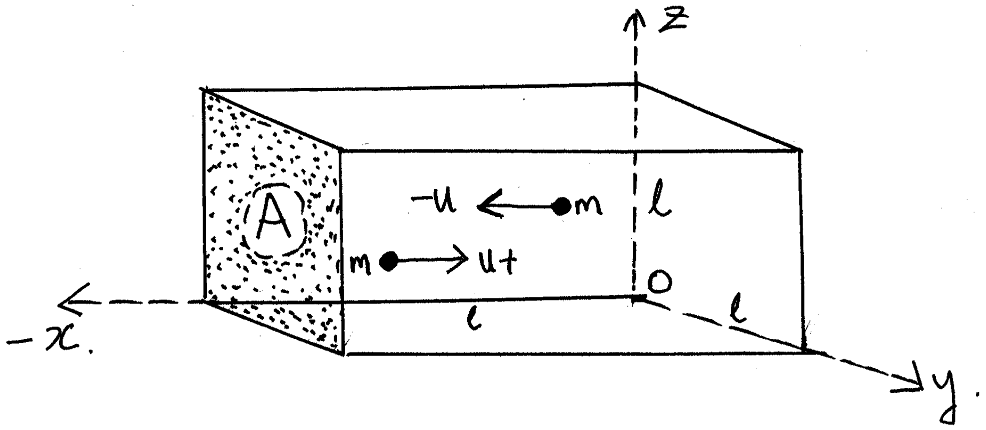


Fig. 13.1

Generally, the molecules move in different directions with different velocities C_1, $C_2 \ldots C_N$. Let us consider the molecules with particular velocity, say C_1. This C_1 has velocity components in three directions, O_x,O_y,O_z parallel to the three edges of the cube passing through the corner O. Suppose these velocity components are u^2, v^2, w^2 in the directions respectively. Then one can write,

$$C\frac{2}{1} = u^2 + v^2 + w^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (13.1)$$

Since u, v, w, are at 90^o to each other.

Let us now consider a molecule moving in the direction O_x normal to one face of the cube. The incident velocity - u and the rebound velocity after collision is +u. Using the Ox axis as being negative for a molecule of mass m and initial velocity -u, its initial momentum is -mu. On rebound its finial velocity is +u hence its finial momentum is +mu]

Time taken by the molecule to travel across to the opposite face and back. If the total distance covered is 2ℓ and the molecule has a velocity u, the time taken = $\frac{2\ell}{u}$ before it makes another impact again. Hence this molecule makes $\frac{u}{2\ell}$ collision per second.

Hence the change in momentum per second

= number of collisions per second x change in momentum

$$= \frac{mu^2}{\ell} = F \ldots\ldots (13.3)$$

But according to Newton's second law of motion, the change in momentum per second is force. So the magnitude of the force on the face of the cube

$$= \frac{mu^2}{\ell} = F \ldots\ldots (13.4)$$

This is the force due to the component u of velocity C_1. Adding the similar components of the velocity of all the other molecules, thus the total force on this face is

$$F = \frac{m}{\ell}(u_1^2 + u_2^2 + \ldots\ldots + u_N^2) \ldots\ldots (13.5)$$

The square of velocity u components is the average of all the square values of

$$(u_1^2 + u_2^2 + \ldots\ldots + u_N^2)$$

$$\therefore \overline{u^2} = \frac{u_1^2 + u_2^2 + \ldots\ldots + u_N^2}{N}$$

$$\therefore N\overline{u^2} = u_1^2 + u_2^2 + \ldots\ldots + u_N^2 \ldots\ldots (13.6)$$

Substituting Eq. (13.6) in Eq. (13.5), we get

Hence the total magnitude of the force = $\frac{Nm\overline{u^2}}{\ell} = F \ldots\ldots (13.7)$

For a large number of molecules in the cube, the mean square of the components along must be equal.

$$\therefore = \overline{u^2} + \overline{v^2} + \overline{w^2} \ldots\ldots (13.8)$$

$$\text{But} = \overline{u^2} + \overline{v^2} + \overline{w^2} = \overline{C^2} \ldots\ldots (13.9)$$

Where $\overline{C^2}$ is the mean square velocity of all the molecules. When the gas is in equilibrium there will be no preferred direction of motion i.e. all direction of velocity will be equally probable and therefore

$$\therefore \overline{u^2} = \frac{\overline{C^2}}{3} \ldots\ldots (13.10)$$

$$\therefore F = Nm\frac{\overline{C^2}}{3\ell} \ldots\ldots (13.11)$$

From the concept of Pressure, P, Pressure is defined as the force per unit area.

$$\therefore \quad P = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{\ell^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.12)$$

Substituting for F in Eq. (13.11), we get

$$\therefore \quad P = Nm\frac{\overline{C^2}}{3\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.13)$$

But as you know that ℓ^3 = V, the volume of the cube

$$\therefore \quad P = Nm\frac{\overline{C^2}}{3V}$$

$$\therefore \quad PV = \frac{1}{3}Nm\overline{C^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.14)$$

Since the Nm is the total mass of N molecule which is M and

$\frac{M}{V}$ =, ρ the density of the gas

$$\therefore \quad P = Nm\frac{\overline{C^2}}{3V}$$

$$\therefore \quad P = \frac{1}{3}\rho\overline{C^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.15)$$

This is an important result. It relates ρ with P and C. We have derived this result by assuming a cube.

3.2.1 Root Mean Square Velocity (R.M.S) of Gas Molecules

From Eq. (13.15), you know that the expression for pressure can be written as, $\frac{1}{3}\rho\overline{C^2}$

Therefore the root-mean-square velocity of all the gas molecules can be defined as

$$\therefore \quad \overline{C^2} = \frac{3P}{\rho}$$

$$\therefore \quad \sqrt{\overline{C^2}} = \sqrt{\frac{3P}{\rho}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13.6)$$

Consequently, if we know the pressure (P) and its density ρ of the gas. We can calculate the r.m.s. velocity of the gas molecules.

SELF ASSESSMENT EXERCISE 1

Given that the pressure of 1.013 x 10^5Pa (Nm^{-2}) is exerted on a given mass of hydrogen gas which has density 0.09kgm^{-3} at a temperature of 0^{-o}C. Calculate its r.m.s. velocity.

3.3 Deduction of Gas Laws from Kinetic Theory of Gases

Now in the subsequent section, we will discuss about the gas laws which were derived using the Eq. (13.15).

It has been shown earlier that for N molecules, each mass m, the mass M of a gas is Nm.

$$PV = \frac{1}{3}Nm\overline{C^2} \quad \text{..................................} \quad (13.17)$$

By multiplying and dividing by 2 of the R.H.S of Eq. 13.17

$$PV = \frac{2}{3}\left(\frac{1}{2}M\overline{C^2}\right) \quad \text{..................................} \quad (13.18)$$

But $\frac{1}{2}M\overline{C^2}$ in Eq. (13.18) is the average Kinetic energy of translation of the gas molecules.

(Rotational energy of the gas molecules is neglected). Now, heat is a form of energy and it is reasonable to assume that the gas molecules have greater energy of movement when its temperature is higher. We can therefore make the assumption that $\frac{1}{2}M\overline{C^2}$ is proportional to its absolute temperature T.

Hence, $\frac{1}{2}M\overline{C^2} \propto T$

$$\therefore \quad \frac{1}{2}M\overline{C^2} = KT \quad \text{..................................} \quad (13.19)$$

Where, K is a constant of proportionality.

Now putting Eq. (13.18) in Eq. (13.19) as given below,

$$PV = \frac{2}{3}\left(\frac{1}{2}M\overline{C^2}\right)$$

We get,

$$\therefore \quad PV = \frac{2}{3}KT \quad \text{..................................} \quad (13.20)$$

$$\therefore \quad PV = \text{Constant x } T$$

$$PV = RT \quad \text{..................................} \quad (13.21)$$

Where R is the constant for 1 mole of gas

Furthermore, we can also say that

$$PV = \frac{1}{3}M\overline{C^2} = RT \quad \text{……..………… (13.22)}$$

On dividing by ½ both sides, we get

$$\therefore \quad \frac{1}{2}M\overline{C^2} = \frac{3}{2} RT$$

$$\text{But,} \quad \frac{1}{2}M\overline{C^2} = \text{K.E. of the gas molecules}$$

$$\therefore \quad K.E. = \frac{3}{2} RT \quad \text{………………………………… (13.23)}$$

From, PV = RT
at constant temperature

$$\therefore \quad PV = \text{Constant}$$

This is Boyle's law

$$\frac{PV}{T} = R \quad \text{(The general gas law) ………..……………. (13.24)}$$

3.4 Variation of the Root Mean Square Speed

Consider the Eq. (13.22),

$$PV = \frac{1}{3}M\overline{C^2} = RT$$

$$\text{Then} \quad \frac{1}{3}M\overline{C^2} = RT$$

On rearranging the terms, we get

$$\therefore \quad \overline{C^2} = \frac{3RT}{M}$$

$$\therefore \quad \sqrt{\overline{C^2}} = \sqrt{\frac{3RT}{M}} \quad \text{……………………………………… (13.25)}$$

Since R is a constant, hence for a given temperature T, it can be concluded that r.m.s. velocity is inversely proportional to the molar mass (M) of the gas.

$$\text{r.m.s. velocity} \propto \frac{1}{\sqrt{M}}$$

This relationship can be sued to compare the r.m.s. velocities of two gases such as oxygen and hydrogen.

$$\frac{\text{r.m.s. velocity of oxygen}}{\text{r.m.s. velocity of hydrogen}} = \sqrt{\frac{MH_2}{MO_2}}$$

Where, MH_2 = molar mass of hydrogen = 2 and,
MO_2 =molar mass of oxygen = 32

However from, $\sqrt{\overline{C^2}} = \sqrt{\frac{3RT}{M}}$

For a given mass of gas at different temperatures
r.m.s. velocity $\sqrt{\overline{C^2}} \propto \sqrt{T}$

i.e. the root-mean-square velocity is directly proportional to the square root of its absolute temperature.

SELF ASSESSMENT EXERCISE 2

The r.m.s. velocity of hydrogen is $1800ms^{-1}$ at a given temperature, calculate the r.m.s. velocity of oxygen at the same temperature. (Molar mass of hydrogen = 2, molar mass of oxygen = 32).

SELF ASSESSMENT EXERCISE 3

The r.m.s. velocity of oxygen molecules is $400ms^{-1}$ at 0^oC. Calculate its r.m.s. velocity at 100^oC.

3.5 Distribution of Molecular Speeds

In this section we are connected about how the speeds of the molecules are distributed in a given closed system at a particular temperature. You will recall the assumption that not all molecules move with the same speed. So now you may ask: what is the distribution of molecules speed? So far, we have discussed what is meant by the root-mean-square speed and the mean speed of molecules in a given gas. The actual speeds vary from low to high values. At a given temperature, the variation follows what is known as Maxwellian distribution as shown in fig. 13.2.

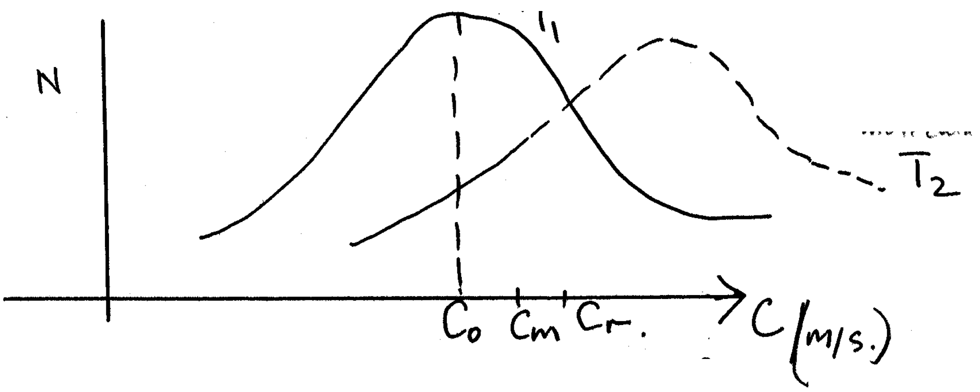


Fig. 13.2: Velocity Distribution of Gas Molecules at Different Temperatures

The vertical represents the number N of molecules which have speeds within a given range about a particular value C. The probable speed C_o is the maximum value. The mean speed C_m in the graph is greater than the probable speed C_o by about 13%. While the root-mean-square C_r is greater than the probable speed C_o by about 23%. These variations are shown with the curve $T_{1.}$ At higher temperatures as shown in the fig. 13.2, by the curve T_2, the speed of the molecules increases. You will note that as the temperature increases, the curve becomes broader and flattened.

As you will recall that in diffusion we learnt about the average speed $\overline{C}$ of the individual molecules. The greater the value of $\overline{C}$, the faster the rate of diffusion. However, when it comes to the pressure of a gas, its value is associated with the mean square $\overline{C^2}$.

3.6 Boltzmann Constant

It will be recalled that when we derived the expression for pressure for a given mass of gas, we obtained the expression

$$PV = \frac{1}{3}\frac{N}{V}m\overline{C^2}$$

$$\text{Or} \quad PV = \frac{1}{3}Nm\overline{C^2}$$

$$PV = \frac{1}{3}\left(\frac{1}{2}m\overline{C^2}\right)$$

Where $\frac{1}{2}m\overline{C^2}$ is the average Kinetic energy of the gas.

For one mole of a monatomic gas, translational K.E. can be related with its temperature such that

$$\frac{1}{2}m\overline{C^2} = \frac{3}{2}RT$$

If the member of molecule in one mole of a gas is N_A, the Avogadro constant which is 6.02×10^{23} $mole^{-1}$

$$\text{The average K.E. per mole} = \frac{3}{2}\frac{RT}{N_A} = \frac{3}{2}RT \quad \ldots\ldots\ldots\ldots\ldots (13.26)$$

Where K is the Boltzmann constant, thus $K = \frac{R}{N_A}$, where $R = 8.3\ JK^{-1}mol^{-1}$

$$\therefore \quad K = \frac{R}{N_A} = 1.38 \times 10^{-23} JK^{-1}$$

The value fro Boltzman Constant is $1.38 \times 10^{-23}JK^{-1}$.

3.7 Internal Energy of a Gas

In this section we shall consider two types of gas

(i) monatomic gas
(ii) diatomic gas

No we will discuss the internal energy and the degree of freedom possessed by each type of gas.

Monatomic Molecule

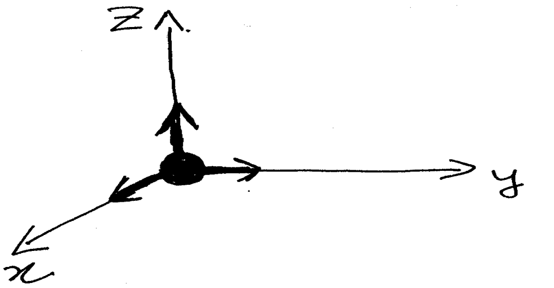


Fig. 13.3: Degree of Freedom of a Gas Molecule

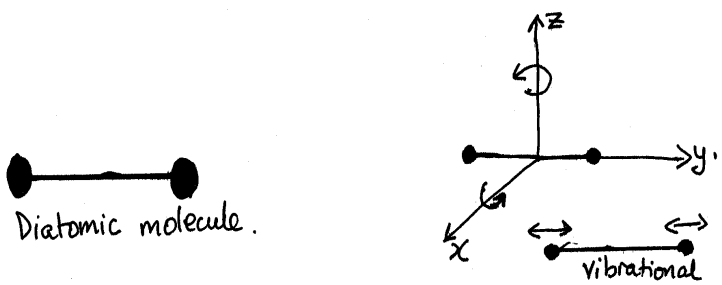
Consider a monatomic molecule of a gas (like helium, argon and krypton) at the point o in fig. 13.3. It is capable of moving in O_x, O_y and O_z directions. We therefore say that it has three translational degrees of freedom. As you know that the total translational Kinetic energy of a molecule is $\frac{3}{2}KT$, since all the three directions are equivalent. Hence the K.E. per degree of freedom of a molecule

$$= \frac{3}{2}KT \div 3 \text{ degrees} = \frac{1}{2}KT.$$

Therefore, the translational Kinetic energy associated with ach components of velocity is just one third of the total translational kinetic energy.

Monatomic Molecule

A diatomic molecule consists of two atoms with a bond between them Fig. 13.4(i)).



(i) **(ii)**

Like a dumbbell, it moves through the air with the following degrees of freedom:

(i) three translational degrees of freedom x, y and z
(ii) two rotational degrees of freedom. Since it rotates about two axes at right angles to the line joining them (x &z).
(iii) the vibrational energy is also there but negligible compared with others.

That is for a diatomic molecule there are five degrees of freedom. Hence the total energy of the diatomic molecule = 5 x $\frac{1}{2}$KT = $\frac{5}{2}$KT

Therefore, the molar heat capacity for 1 mole of the gas:

$$C_v = \frac{5}{2}R \text{ .... (13.27)}$$

$$\therefore \quad C_p = C_v + R = \frac{5}{2}R = R = \frac{7}{2}R \text{ .... (13.28)}$$

Therefore for monatomic molecule, the internal energy U is only translational K.E.

The molar heat capacity at constant volume C_v is the heat energy required to raise the temperature of 1 mole of gas by 1K. This is also the rise in internal energy of an ideal gas when its temperature rises by 1K.

$$\therefore \quad C_v = \Delta U = \frac{3}{2}R\,(T + 1) - \frac{3}{2}RT = \frac{3}{2}R$$

$$\Delta U = C_v = \frac{3}{2}R$$

The molar heat capacity at constant pressure C_p is given by

$$C_p - C_v = R$$

$$\therefore \quad C_p - C_v = R$$

$$\Delta U = C_p = \frac{5}{2}R$$

Therefore, the ratio of molar heat capacities is given as for monatomic molecules

$$\frac{C_P}{C_V} = \gamma = \frac{5}{3} = 1.67 \text{ .... (13.29)}$$

For diatomic molecule, the ratio of heat capacities can be obtained as:

$$\Delta U = C_v = \frac{5}{2}R$$

$$\Delta U = C_p = C_v + R = \frac{7}{2}R$$

$$\Delta U = C_p = \frac{7}{2}R$$

$$\frac{C_P}{C_V} = \gamma = \frac{7}{3} = 1.40 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(13.30)}$$

Here r is called (gamma), which is the ratio of heat capacities. This shows that the ratio of heat capacities decreases with increasing atomicity of gases.

4.0 CONCLUSION

Some assumptions have been given to the Kinetic theory of an ideal gas. By using these assumptions the expression for the pressure on a gas has been derived which is in consonance with Boyle's law (PV = K) and the equation of state (PV =nRT). The root-mean-square (r.m.s.) velocity of a gas molecules have been explained and derived. Finally, the internal energy of a gas has been derived.

5.0 SUMMARY

In this unit you have learnt:

- About the basic assumptions of the Kinetic Theory of gases;
- How these assumptions were used to derive the expression for the pressure exerted by a given mass of gas in a closed system.

  $$PV = \frac{1}{3}\frac{N}{V}m\overline{C^2} \text{ or}$$

  $$PV = \frac{1}{3}\frac{N}{V}m\overline{C^2} = RT$$

  The kinetic energy of translation was given as PV =

  $$\frac{1}{2}m\overline{C^2} = \frac{3}{2}RT$$
- That the ideal gas law PV = RT only follows from Kinetic Theory of gases by assuming that the kinetic energy of translation is proportional to the absolute temperature T;
- The root-mean-square velocity of the gas $\sqrt{\overline{C^2}} = \sqrt{\frac{3P}{\rho}}$ where P is the pressure of the gas and is the ρ density of the gas;
- The root-mean-square velocity of the gas is independent of the pressure, directly proportional to $\sqrt{T}$ (the absolute temperature) and inversely proportional to $\sqrt{M}$ where M is the mass of one mole of the gas;

  For a monatomic gas, the molar heat capacity at constant volume $C_v = \frac{3}{2}R$; the molar heat capacity at constant pressure $C_p = \frac{5}{2}R$;

the ratio $\frac{C_P}{C_V} = \gamma$ is just equal to 1.67; and

- For a diatomic gas, $C_v = \frac{5}{2}R$, $C_p = \frac{7}{2}R$ and $\frac{C_P}{C_V} = \gamma$ is just equal to 1.40.

ANSWER TO SELF ASSESSMENT EXERCISE 1

From $\sqrt{\overline{C^2}} = \sqrt{\frac{3P}{\rho}}$

$\therefore$ r.m.s. velocity $= \sqrt{\overline{C^2}} = \sqrt{\frac{3 \text{ x } 1.013 \text{ x } 10^5}{0.09}}$

$= 1840 \frac{m}{s}$

$= 1.84kms^{-1}$

r.m.s. velocity $= 1.84kms^{-1}$

ANSWER TO SELF ASSESSMENT EXERCISE 2

Using the relation (13.25), we can write

$$\frac{\sqrt{\overline{C^2O_2}}}{\sqrt{\overline{C^2H_2}}} = \frac{\sqrt{MH_2}}{\sqrt{MO_2}}$$

Substituting the values,

$$\therefore \quad \frac{\sqrt{\overline{C^2O_2}}}{1800 \frac{m}{s}} = \frac{\sqrt{2}}{\sqrt{32}} = \frac{1}{4}$$

We get

$$\therefore \quad \sqrt{\overline{C^2O_2}} = 1800 \frac{m}{s} \text{ x } \frac{1}{4} = 450 \frac{m}{s}$$

Hence, the r.m.s. velocity of oxygen is $450ms^{-1}$

ANSWER TO SELF ASSESSMENT EXERCISE 3

We know that

$$\sqrt{\overline{C^2}} \propto \sqrt{T}$$

Therefore,

$$\frac{\sqrt{\overline{C^2T_1}}}{\sqrt{\overline{C^2T_2}}} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$

Substituting the value in this equation:

$$\frac{\sqrt{\overline{C^2 0^o C}}}{\sqrt{\overline{C^2 100^o C}}} = \frac{\sqrt{273K}}{\sqrt{273 + 100K}}$$

$$\therefore \quad \frac{400ms^{-1}}{\sqrt{\overline{C^2 100^o C}}} = \sqrt{\frac{273}{373}}$$

We get,

$$\therefore \quad \sqrt{\overline{C^2 100^o C}} = 400\frac{m}{s} \text{ x } \sqrt{\frac{373}{273}} = 468ms^{-1}$$

Thus the r.m.s. velocity of oxygen molecules at 100oC is $468ms^{-1}$.

6.0 TUTOR MARKED ASSIGNMENT

1. Assuming air has 2% oxygen molecules and 80% nitrogen molecules of relative molar masses 32 and 28 respectively. Calculate

   (a) the ratio of the r.m.s velocity of oxygen to that of nitrogen; and
   (b) the ratio of the partial pressure of oxygen to that of nitrogen in air.

2. In a vessel, hydrogen gas contains 1.0 x 10^{24} molecules per m^3 which have a toot-mean-square velocity of $180ms^{-1}$. Calculate the pressure of gas assuming that the Avogadro constant = 6 x 10^{23} $mole^{-1}$ and the relative molar mass of hydrogen is 2.

3. Given that r.m.s. velocity of hydrogen is $1800ms^{-1}$ at a given temperature. Calculate the r.m.s. of nitrogen at the same temperature.

4. The r.m.s. velocity of oxygen molecules is $400ms^{-1}$ at 0^oC. Determine its r.m.s. velocity at 150^oC.

7.0 REFERENCES/FURTHER READING

Awe, O. and Okunola, O.O. (1986). *Comprehensive Certificate Physics*, Ibadan: University Press.

Ference, M. (Jnr.), Lemon, H.B. and Stephenson, RJ (1970). *Analytical Experimental Physics*, Chicago: The University of Chicago Press.

Feyman, R.P., Leighton, R.B. and Sands, M (1971). *The Feyman Lectures on Physics*, California: Addison-Wesley Publishing Company.

Flowers, B.H. and Mendoza, E (1970). *Properties of Matter*, London: John Wiley and Sons Limited.

Noaks, M.A. (1963). *New Intermediate Physics*, Canada: Macmillan and company Limited.

Soars, F.W., Zemansky, M.W. and Young, H.D. (1980). *College Physics*. London: Addison-Wesley Publishing Company.

Tyler, F. (1966). *Heat and Thermodynamics*. London: Edwards Arnold Publisher Limited.

Wilkinson, J. (1983). *Essentials of Physics*, Australia: Macmillan Education Publishers.

Zemansky, M.W. (1968). *Heat and Thermodynamics*. New York: McGraw-Hill Book Company.

UNIT 4 DENSITY

CONTENTS

1.0 INTRODUCTION

Matter is a substance which has certain mass and occupies some volume. But you must have observed that some substances are denser than others. It means that the individual particles of a substance are more closely packed together.

Suppose you are asked this question: "Which is heavier, one kilogram of copper or one kilogram of feathers?" What would be your answer?

This question has elicited many answers from students. Some students would say the feathers are heavier. But what is your answer? Think very well before giving your own solution.

However, the obvious answer is "Neither is heavier." The two objects are the same mass of one kilogram. But on a second thought, about these objects, something seems to make a difference. And what is that difference?

The difference is in terms of the space (volume) occupied by both objects. Besides, when the metal copper is placed in water it sinks while the feathers will float. The idea that relates the mass with volume and whether an object will sink or not is known as density. **Therefore, density is a physical property of matter that describes the degree of compactness of a substance.**

Thus in this unit we shall deal with the concept of density and its measurement.

2.0 OBJECTIVES

At the end of his unit you should be able:

- explain the meaning of density
- define the concept of density
- describe an experiment on how to determine the density of:
  - (i) A regular shaped object
  - (ii) An irregular shaped solid
  - (iii) A liquid, by method of displacement
- explain the meaning of relative density
- solve problems on density.

3.0 MAIN CONTENT

3.1 Concept of Density

The density of an object refers to how heavy or light that object is relative to water or air.

Density is defined as **"The mass of the body per unit volume."** Density = $\frac{\text{Mass}}{\text{Volume}}$ That is, the density of a body is the mass of that body in kilograms per one meter cube of its volume.

Thus in order to determine the density of a body, the following factors must be known: its mass and its volume.

Wha***t then is the mass of a body? What is the volume of the body?*** The answers to these two questions enable us to define the density of the body.

3.1.1 Mass of a Body

The quantity of matter in a body defines its mass. It is the degree of inertia i.e. its reluctance to motion. It is measured in kilogram (kg) using the S.I. unit system. Sometimes we use the smaller units such as (g) or milligram (mg). ***The question you may ask now: How do we measure the mass of an object?***

The beam balance or the scale may be used to measure the mass of a body. The mass of a body is compare with the standard mass of one kilogram of Platinum-Iridium in Paris. Thus by the definition of density, we need to determine the mass of the object using the beam balance. Having done this, we then determine its volume.

3.1.2 Volume of a Body

The volume of a body is the amount of space it occupies – its capacity. It is measured in cubic (m^3) of cubic centimeter (cm^3).

Observe these various conversations

If $1cm = \frac{1}{100} m$

$\therefore$ $1cm^3 = \frac{1}{100} m \ x \ \frac{1}{100} m \ x \ \frac{1}{100} m$

$= \frac{1}{10^6} m^3$

$\therefore$ $1cm^3 = 1 \ x \ 10^{-6} m^3$

or $1m^3 = 1x \ 10^6 cm^3$

The one cubic centimeter (cm^3) is also equal to one milliliter. $1cm^3 = 1ml$.

The problem now is ***"How* is volume of a body determined?"** From experience you will realize that this can be done in two ways:

- The use of regular method for regular objects
- The use of displacement method for irregular objects.

Now in the next section, we will discuss about the methods of determination of volume of regular objects.

3.1.3 Volume of Regular Objects

Regular objects include thing like a cube, a cuboid, a cylinder, a sphere, and a cone. Their shapes are very distinct and unique. Their volume can then be determined with the knowledge of their dimensions such as length, breadth, height, radius or diameter.

A Cube

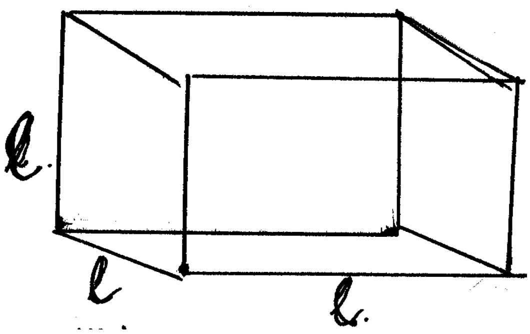


Fig. 14.1: A Cube

The length, breadth and height of a cube are of the same length ℓ. Therefore, by definition, the volume of a cube.

$$V = \ell \ x \ \ell \ x \ \ell = \ell^3 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14.1)$$

A Cuboid

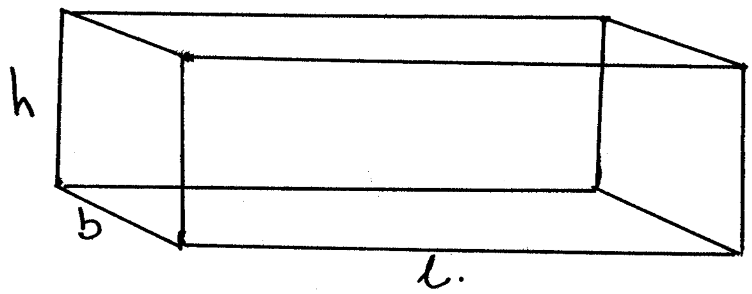


Fig. 14.2: A Cuboid

A cuboid has a height (h), a breadth (b) and a length (ℓ). Hence by definition the volume of cuboid is

$$V = h \times b \times \ell \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14.2)$$

A Sphere

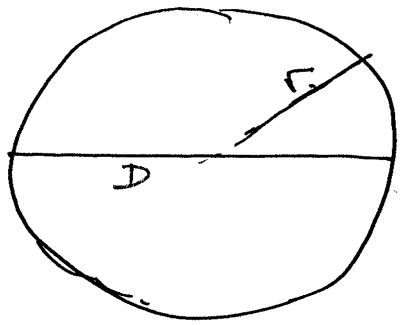


Fig. 14.3: A Sphere

A sphere is like a ball with a constant radius r or diameter D. BY definition, the volume of a sphere is given by

$$V = \frac{4}{3}\pi r^3 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14.3)$$

Where, the diameter is given as D, and where D = 2r

$$\therefore \qquad r = \frac{D}{2}$$

Hence, in terms of the diameter D the volume of the sphere can be expressed as

$$V = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3$$

$$V = \frac{4}{3}\pi\,\frac{D^3}{8} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (14.4)$$

Cylinder

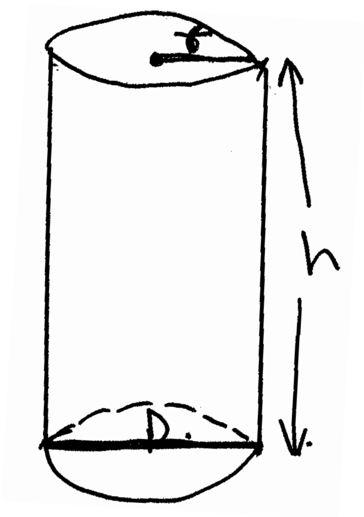


Fig. 14.4: A Cylinder

A cylinder is like a drum or tin of milk with a height h and circular cross-section of radius r (See fig. 14.4).
By definition, the volume of a cylinder is given as

$$V = \pi r^2 h \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(14.5)}$$

This relation can also be expressed in terms of Diameter D as

$$= \pi \left(\frac{D}{2}\right)^3 h$$

$$V = \pi \frac{D^2}{4} h \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(14.6)}$$

A Cone

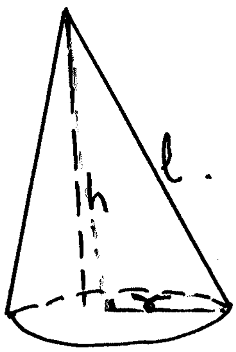


Fig. 14.5(i): A Cone

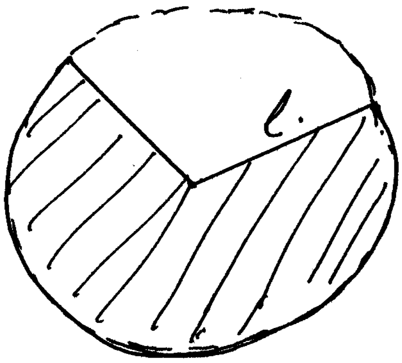


Fig. 14.5(ii): A Sector

A cone is like a toy top with a circular surface, which tapers to a vertex (fig. 14.5(i)). It is usually made from a sector of a circular sheet. The vertical height of the cone is h with the circular base having a radius r. By definition the volume of a cone is given as

$$V = \frac{4}{3}\pi r^2 h \text{ .... } (14.7)$$

Note that any object may be in form of a combination of the above objects. For example, we can have a combination of a cone and a cylinder as shown in fig. 14.6.

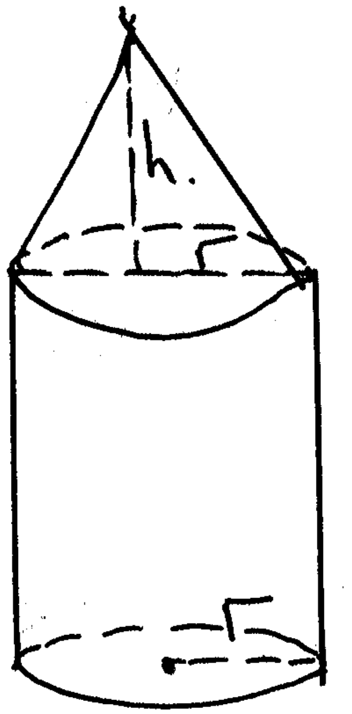


Fig. 14.6

Once you can identify the shape of the regular object, you can then determine its volume using the appropriate formula. But,

How do we determine the volume of an irregular object like you? Can you use your height, breadth and length? No. We shall find out how to determine the volume of an irregular object in the next section.

3.1.4 Volume of Irregular Objects

Irregular objects, in this case, are those objects that do not have regular shapes. These objects include stoned or any solid object that does not conform with any of the regular objects. We use indirect method of determining their volumes. The method is described as the displacement method.

Method 1

In this method, we use a measuring cylinder as shown in fig. 14.7 (i) & (ii).

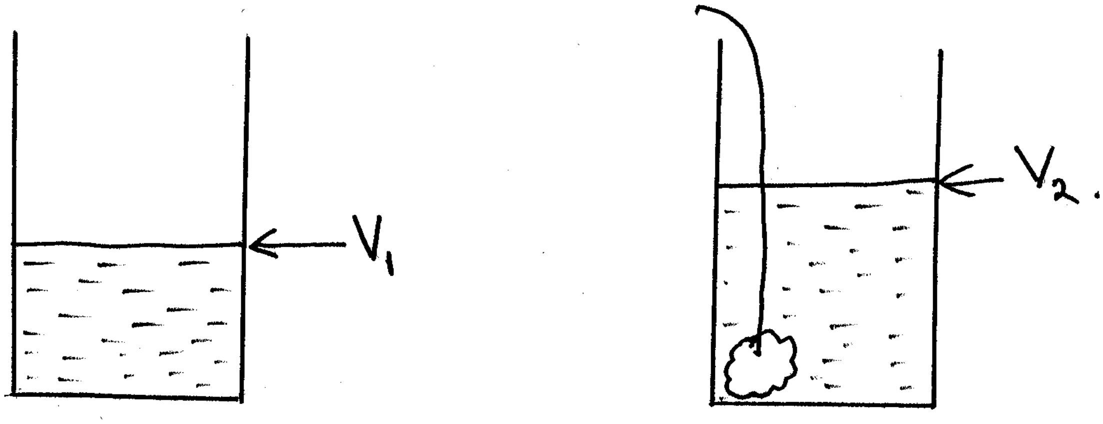


Fig. 14.7

First, you will pour some water into the cylinder and then observe the initial level of the water inside the cylinder (V_1) as shown in fig. 14.7.

Then, you tie a string on the stone, the irregular object and carefully lower the stone into the water with the aid of the string making sure there is no splash. But it is to be assume that the irregular object does not dissolve or react with the water. You will observe that the level of the water rises for the object has displaced its own volume of water. Then the new level of water in the cylinder is read as V_2. Thus the volume of the stone, which is the volume of water displaced, is given as

$$V = V_2 - V_1$$

Method 2

The second method uses the Eureka can otherwise known as the displacement can (fig. 14.8).

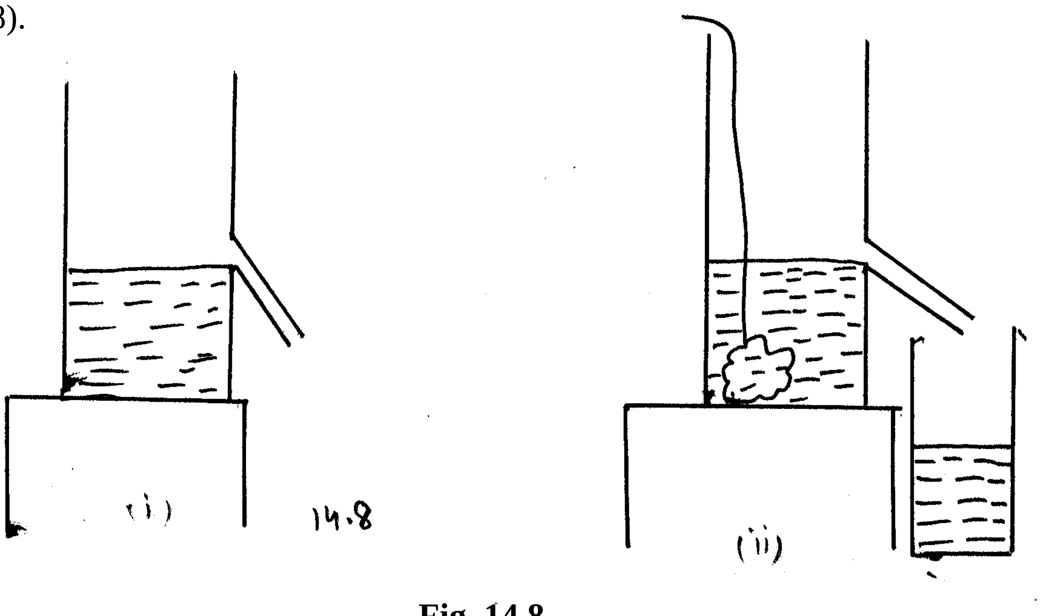


Fig. 14.8

First the spout of the can is closed with a finger. Then water is poured into the can until the level is above the spout. The finger is then removed thus allowing the excess water above the spout to flow out fig. 14.8(i).

The stone, whose volume is to be measured, is then carefully lowered into the can with the aid of a thread tied to it. It will be observed that the stone will displace water, which gradually flows out of the spout. It flows out until it has displaced the volume of water equal to its own volume (fig. 14.8(ii)).

A measuring cylinder meanwhile has been placed beneath this spout to collect the volume of water that has been displaced thus giving us the reading of the volume of water displaced.

SELF ASSESSMENT EXERCISE 1

Suppose you are given a light floating irregular object like a cork (wooden). How would you determine its volume?

Table 14.1: Density of Some Substances

Substance	Density (∫) (gcm^{-3})
Air	0.0013
Wood	0.6 – 0.9
Water	1.00
Bricks	1.84
Steel	7.80
Gold	19.30

3.2 Determination of the Density of an Object

You will recall, we defined density as

$$\text{Density} = \frac{\text{Mass of the Object}}{\text{Volume of teh Object}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (14.8)$$

By using the appropriate scale we can determine the mass of an object and using the appropriate method also, we can determine the volume of the object. Thus by dividing the mass of the object by its volume the density of the object can be determined. Therefore, the common units used to express density are leg^{-3}, gcm^{-3} and g/m/. Here first, do some exercises for clear the concept of density.

The density of some substances is given below in tabular 14.1.

SELF ASSESSMENT EXERCISE 2

A piece of stone has a mass of 300kg and a volume of $0.12m^3$. What is its density?

SELF ASSESSMENT EXERCISE 3

Find the volume of a bottle, which will just hold 63g of seawater of density $1.05g/cm^3$?

3.3 Density of Mixtures

In this section, we shall consider the problems concerned with mixtures of objects such as alloys of metals or mixtures of liquids (water and alcohol). Since different substances have different densities. Other mixtures will include acid solutions that is acid and water, water and milk etc. In science, there is usually the need to have such mixtures and thus determine the densities of such mixtures are useful.

Suppose there are given two substances A and B with the following properties:

Substance A

Mass of substance A = M_1
Volume of substance A = V_1
Density of substance A = ρ_1

Substance B

Mass of substance B = M_2
Volume of substance B = V_2
Density of substance B = ρ_2

Now the question is: ***Suppose the two substances are mixed together how shall we define the density of the mixture (ρ)?***

The density of the mixture can be represented as

$$\rho = \frac{\text{Mass of A} + \text{mass of B}}{\text{Volume of A} + \text{volume of B}}$$

In terms of the symbols,

$$\therefore \quad \rho = \frac{M_1 + M_2}{V_1 + V_2}$$

But $M_1 = \rho_1 V_1$ and $M_2 = \rho_2 V_2$

$$\therefore \quad \rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2} \quad \text{.................................... (14.9)}$$

Thus knowing the values of ρ_1, V_1, ρ_2 and V_2, we are in the position to determine the density of the mixture ρ.

SELF ASSESSMENT EXERCISE 4

What is the density of a mixture of $1.5m^3$ of water and $0.5m^3$ of alcohol of density $800kg/m^3$? (Density of water = $1000kg/m^3$).s

4.0 CONCLUSION

The density of an object is defined as the mass per unit volume of the properties of matter. It tells us how heavy or light an object is relative to water or air. If it sinks in water we say it is heavy, it has a density greater than water. If it floats in water we say it is light with a density lower than that of water. The knowledge of the mass and the volume of the object allow us to operationally define the density of a substance. We have also discussed the density of alloys (mixture of metals) and of solutions (mixture of liquids). We would now consider the concept of buoyancy, Archimedes Principle and the law of flotation in the next unit.

5.0 SUMMARY

In this unit, you have learnt the following:

- The concept of density as a property of matter;
- Density is defined as the mass per unit volume of a substance;
- The density of a substance can be determined from the knowledge of its mass and volume;
- The mass of the object may be determined by weighing on a balance or scale;
- The volume of the object may also be determined by using the regular formulae of calculating volumes or by the method of water displacement;
- From the knowledge of mass and volume, the density of the object can be determined;
- How to determine the density of the mixture of two substances.

ANSWER TO SELF ASSESSMENT EXERCISE 1

You must first realize that the object will not sink in water. It will float. ***So what would you do?***

So, to determine the volume of a light, floating object such as a cork, you will use a sinker (a heavy object), which will assist you in the method of displacement. First, you will get our usual measuring cylinder and fill it with water to give an initial volume of water V_1 as shown in fig. 14.9.

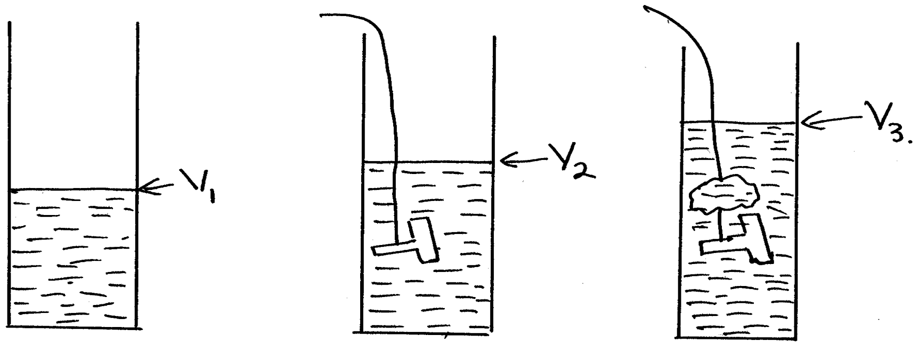


Fig. 14.9

Then you will determine the volume of the sinker by immersing it into the water suing a thread. This gives a new volume of water V_2 (fig. 14.9(ii)). ***Now what would you consider to be the volume of the sinker alone?***

It is $(V_2 – V_1)$

We bring out the sinker without losing any water. We tie the cork and the sinker together and then immerse the two into the measuring cylinder to produce another volume V_3 (fig. 14.9(iii)).

Thus what would you consider as the volume of the cork?

The volume of the cork = $V_3 - (V_2 – V_1)$

Note that the unit of volume is cm^3 or m^3.

ANSWER TO SELF ASSESSMENT EXERCISE 2

Using the Eq. (14.8), we get

$$\text{Density of stone} = \frac{\text{Mass of Stone}}{\text{Volume of Stone}}$$

$$= \frac{300\text{kg}}{0.12\text{m}^3}$$

$$\int = 2500\text{kgm}^{-3}$$

Note the unit of density as kg/m^3 or kgm^{-3}.

ANSWER TO SELF ASSESSMENT EXERCISE 3

Density of Sea water $= \dfrac{\text{Mass of Sea water}}{\text{Volume of Sea water}}$

$\therefore$ Volume of stone $= \dfrac{\text{Mass of Stone}}{\text{Density of Stone}}$

$$= \frac{63\text{g}}{1.05\dfrac{\text{g}}{\text{cm}^3}}$$

$= 60\text{cm}^3$

But the volume of seawater is equal to the volume of the bottle. Therefore, the volume of the bottle is 60cm^3.

ANSWER TO SELF ASSESSMENT EXERCISE 4

The two liquids are water and alcohol.

Where, Density of water = ρ_1 = 100kg/m^3
Volume of water = V_1 = 1.5m^3
Density of alcohol =ρ_2 = 800kg/m^3
Volume of alcohol = V_2= 0.5m^3

$\therefore$ Density of mixture = $\rho = \dfrac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$

$$= \frac{1000\dfrac{\text{kg}}{\text{m}^3} \text{ x } 1.5\text{m}^3 + 800\dfrac{\text{kg}}{\text{m}^3} \text{ x } 0.5\text{m}^3}{1.5\text{m}^3 + 0.5\text{m}^3}$$

$$= \frac{1500\text{kg} + 400\text{kg}}{2.0\text{m}^3}$$

$= 950\text{kgm}^{-3}$

The density of the mixture is 950kg/m^3 or 950kgm^{-3}

6.0 TUTOR - MARKED ASSIGNMENT

1. A bottle has a capacity of $60cm^3$. What mass of brine (density = $1.2g/cm^3$) will it hold?

2. Taking the density of air as $12kgm^{-3}$, find the mass of air that is in a room 6.0m long, 4.0m wide and 2.5m high.

3. Some zinc (density = $7100kgm^{-3}$) is mixed with twice its volume of copper (density = $8900kgm^{-3}$) to make brass. What is the density of brass?

4. What volume of water must be added to $40cm^3$ of glycerine (density = $1.3gcm^{-3}$) in order that the mixture shall have a density of $1.1gcm^{-3}$?

7.0 REFERENCES/FURTHER READING

Awe, O. and Okunola, O.O. (1986). *Comprehensive Certificate Physics*, Ibadan: University Press.

Ference, M. (Jnr.), Lemon, H.B. and Stephenson, RJ (1970). *Analytical Experimental Physics*, Chicago: The University of Chicago Press.

Feyman, R.P., Leighton, R.B. and Sands, M (1971). *The Feyman Lectures on Physics*, California: Addison-Wesley Publishing Company.

Flowers, B.H. and Mendoza, E (1970). *Properties of Matter*, London: John Wiley and Sons Limited.

Noaks, M.A. (1963). *New Intermediate Physics*, Canada: Macmillan and company Limited.

Soars, F.W., Zemansky, M.W. and Young, H.D. (1980). *College Physics*. London: Addison-Wesley Publishing Company.

Tyler, F. (1966). *Heat and Thermodynamics*. London: Edwards Arnold Publisher Limited.

Wilkinson, J. (1983). *Essentials of Physics*, Australia: Macmillan Education Publisher.

Zemansky, M.W. (1968). *Heat and Thermodynamics*. New York: McGraw-Hill Book Company.

UNIT 5 BUOYANCY – ARCHIMEDES PRINCIPLES

CONTENTS

1.0 INTRODUCTION

History has it that the concept of Buoyancy was developed by Archimedes (287 – 212 B.C.). When he solved a riddle for the King of Greece. What was the riddle? The king gave a piece of gold to his goldsmith to make a crown for him. The man came back with the well-made crown. The king was not only impressed by the beautiful crown made, rather, he became suspicious about the quality of the crown made from gold. So the King wanted to determine whether a goldsmith had defrauded him by replacing some of the gold in the crown with silver.

The king, therefore, called on his subjects, his philosophers to solve this problem. When Archimedes went into bathing pools experience the buoyant force and then came out of it rejoicing and shouting "Eureka! Eureka!! …." Meaning "I have found it".

What he found out then is what is described now as Archimedes' Principle, which enables us to determine the density of an object by immersing it in a fluid like water. It graduated into the law of floatation, which also help us to explain how huge ships or boats could float on water.

So in this unit, you will learn about the concept of buoyancy, Archimedes' principle and the law of floatation. Come along with me on the discussion.

2.0 OBJECTIVES

At the end of the unit you should be able to:

- explain the concept of buoyancy

- state Archimedes' Principle
- define relative density in terms of Archimedes' Principle
- state the law of floatation
- solve problems on Archimedes' Principle and the law of flotation.

3.0 MAIN CONTENT

3.1 Concepts of Buoyancy

Buoyancy is a concept that describes the floatation of an object in fluids generally. Buoyancy has therefore got to do with the density of the object. In simple words, the ability of an object to 'float' when it is placed in a fluid is called buoyant force. For example, a swimmer experiences this idea of buoyancy, floatation in water as he/she swims in it.

The water supports the body of the swimmer, which makes him appear to weigh less. This is the case with any liquid. This is because when you go into the water, you displace some liquid and the liquid exerts an upthrust on you.

This is also true of gases but because our body would displace a very smaller weight of air, the upthrust is much less. There are three possible situations for a body situated in a fluid as shown in fig. 15.1 below.

Let us see what you think will happen to the body in these three situations:

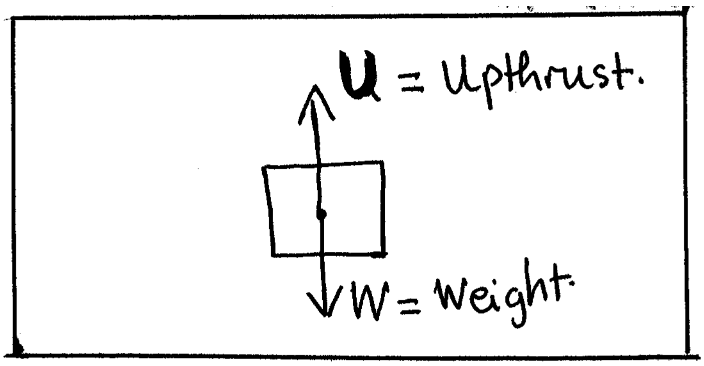


Fig. 15.1

- If its weight (W) is greater than the upthrust (U)

The body sinks. For example a stone placed in water.

$W > U \Rightarrow$ *SINKING OF OBJECT*

- Now, if the upthrust (U) is greater than the weight (W)

You are right if you say the body rises. For example, if a helium-filled balloon is left in air, the balloon rises

$U > W \Rightarrow$ *RISING OF OBJECT*

- If the weight (W) of the object balances if is equal to the upthrust (U)

The objects float in the fluid. For example, a boat or a ship floating on the sea.

W = U⇒ FLOATING OF OBJECT

These three conditions explain,

- Archimedes' Principle and
- The law of floatation.

3.2 Archimedes' Principle

"Archimedes' Principle states that when a body is either partially or totally immersed in a fluid, it will experience an upthrust equal to the weight of the fluid displaced".

Suppose the weight of the body in air is W, when the body is now immersed in a fluid partially or fully, it experiences an upthrust (U). The new weight registered by the scale (A) is now less than that of the air (W). A is called the apparent weight i.e. it appears to be the weight of the object.

The relationship between the apparent weight (A), the upthrust (U) and the actual weight in air (W) is show below.

Weight in air (W) = Apparent weight in fluid (A) + the upthrust (U) experienced.

$\therefore$ W = A + U

$\therefore$ *The upthrust U = Weight in air (W) – Weight in fluid (A)*

U = W - A …………………………….. (15.1)

Theoretically, the weight of a body in air is equal to its apparent weight because the displaced air will exert an upthrust but typically, this upthrust is so small compared with its weight that it can be ignored.

Note that the upthrust on the body is equal to the weight of the fluid displaced. This is the essential aspect of Archimedes' principle, which you must always remember.

3.2.1 Test of Archimedes' Principle

Is it possible to test if Archimedes' Principle is correct? Yes, it is possible to do so.

- You can weigh an object in air using a spring balance (fig. 15.2(i))
- You will read the scale on the spring balance, which will indicate the weight of the object in air. Record this weight as W_1 (fi. 15.2(ii))
- Immerse the object totally in the Eureka can as shown in fig 15.2(ii).

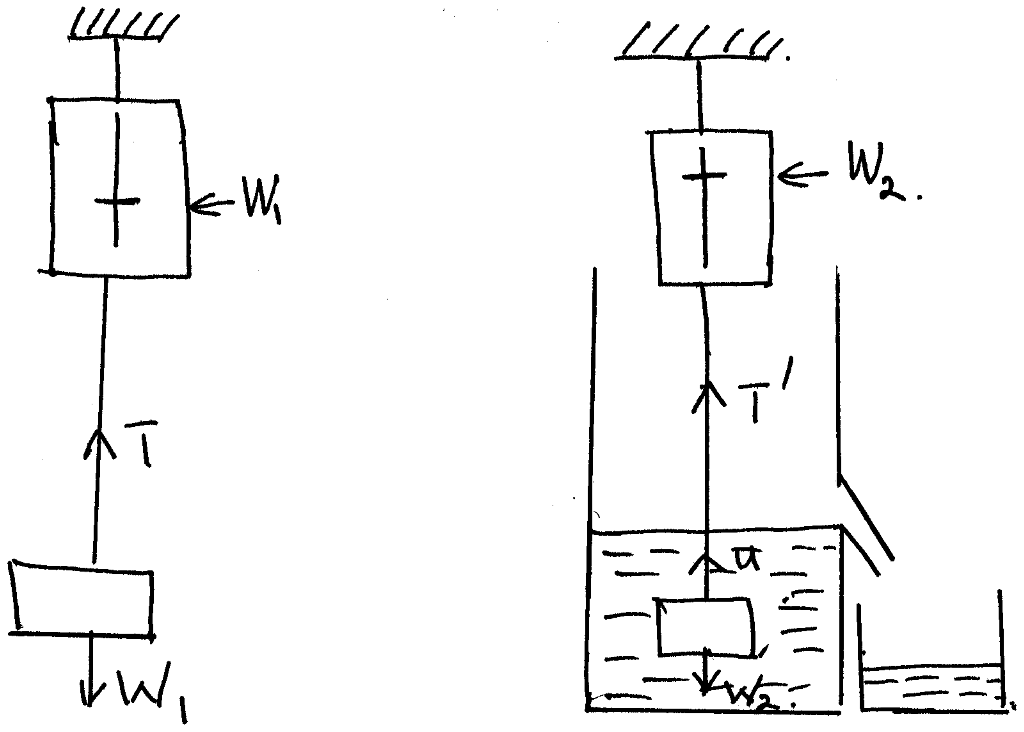


Fig. 15.2

- The water displaced is collected in the measuring cylinder as shown in fig. 15.2(ii)
- Read the new weight of the object when it is fully immersed in water and record it as W_2 (fig. 15.2(ii))
- Weigh the liquid displaced by using the chemical balance by finding the difference between the weight of cylinder and water and the weight of the empty cylinder. Suppose this weight is w.
- Compare the value of w with the difference between the weight of the object in air w_1 and the weight of the object w_2 when fully immersed in water
- You will observe that

$$W_1 - W_2 = w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15.2)$$

That is, the weight of water displaced W is equal to the difference between the weight in air of object and the apparent weight in air. That is the weight of displaced fluid is equal to the upthrust U.

SELF ASSESSMENT EXERCISE 1

A piece of metal has a volume $0.50m^3$. If it is suspended on a rope and immersed in water, find

(a) The volume of the displaced water
(b) Mass of the displaced water

(c) The weight of the displaced water
(d) The upward force (upthrust) on the metal due to the water.

3.2.2 Relative Density

As you have learnt in Table 14.1 of the preceding unit that then density of water is $1.00gcm^{-3}$. In this table, the densities or other substances are also given. Now we learn what the concept of relative density is.

When we compare the density of a substance with the density of water then we are talking about the relative density of that substance. In other words, we are saying how many times this substance is as heavy as water.

By definition,

$$\text{The Relative Density of a substance} = \frac{\text{The density of the substance}}{\text{The density of water}}$$

Let ρ_r = Te relative density of the substance
ρ = The density of the substance
ρ_w = the density of water

Therefore, the expression for relative density is

$$\therefore \quad \rho r = \frac{\rho}{\rho_w} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15.3)$$

$$\therefore \quad \rho = \rho_r \rho_w \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15.4)$$

That is the actual density of a substance is equal to the relative density of the substance times the density of water. It can be noted from the definition of relative density, that it has no unit since it is a ratio.

3.2.3 Application of Archimedes' Principle

From the definition of relative density,

$$\text{Relative Density} = \frac{\text{Density of the substance}}{\text{Density of water}}$$

$$= \frac{\dfrac{\text{Mass of substance}}{\text{Volume of substance}}}{\dfrac{\text{Mass of water displaced}}{\text{Volume of water displaced}}}$$

Let, m_1 = mass of substance
V_1 = volume of substance
M_2 = mass of water displaced
V_2 = volume of water displaced
ρ_r = relative density of substance

$$\therefore \quad \rho_r = \frac{m_1}{V_1} + \frac{m_2}{V_2}$$

$$\therefore \quad \rho_r = \frac{m_1}{V_1} \text{ x } \frac{V_2}{m_2}$$

$$= \frac{m_1}{m_2} + \frac{V_2}{V_1}$$

But the volume of water displaced V_2 is equal to the volume of the substance V_1.

$$\therefore \quad \rho_r = \frac{m_1}{m_2}$$

Hence the relative density of a substance can be defined as

$$\therefore \quad \rho_r = \frac{\text{Mass of the substance}}{\text{Mass of an equal volume of water}}$$

Hence $\rho_r = \frac{m_1}{m_2}$

Alternatively,

$$\rho_r = \frac{\text{Weight of the substance}}{\text{Weight of an equal volume of water}}$$

$$\rho_r = \frac{W_1}{W_2} = \frac{m_1 g}{m_2 g} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (15.5)$$

We apply the concept of relative density to derive the density of a solids or a liquid. The specific gravity bottle or relative density bottle is used in determining the relative densities of liquids (fig. 15.3).

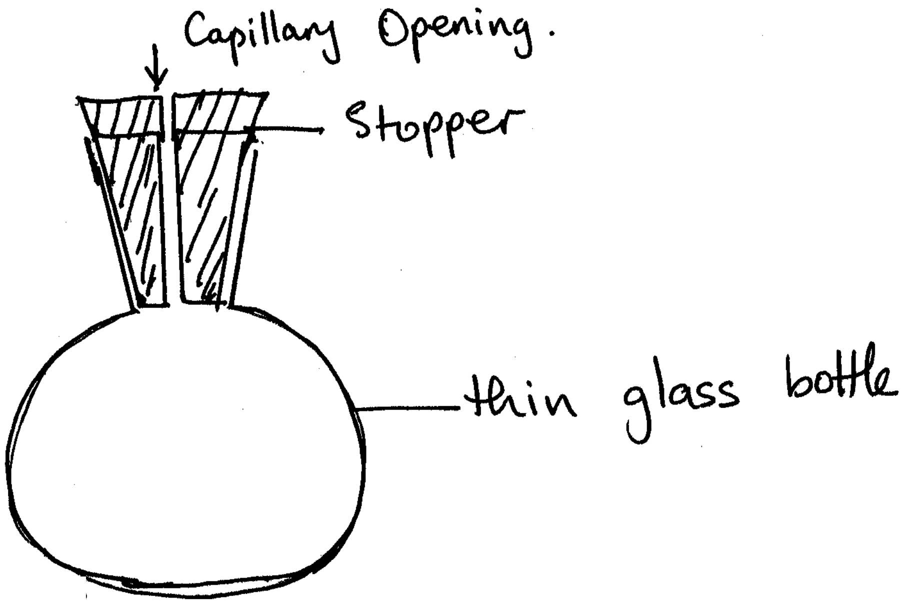


Fig. 15.3: Relative Density Bottle

The relative density bottle is a special stopper, which ensures constant volume of the liquid to be measured. The stopper has a narrow opening which allows excess liquid flow out when the bottle is closed. Thus the bottle will hold a specific volume of liquid at the time it is used. Usually the volume of such bottle varies. It could be $25cm^3$, $50cm^3$ or $75cm^3$ capacity. It is usually picked by the neck to avoid the expansion of the liquid because the glass bottle is so thin that it is sensitive to temperature changes, which, may cause the expansion of the liquid.

SELF ASSESSMENT EXERCISE 2

A stone hangs from a spring balance graduated in Newton's, and the reading is 30N. If the stone is immersed in water, the reading becomes 18N. Determine density of the stone?

The following example relates to the determination of the density of a liquid through the application of relative density.

SELF ASSESSMENT EXERCISE 3

A density bottle has a mass of 12.3g when empty 14.8g when filled with water and 32.1g when filled with turpentine. What is the density of turpentine?

3.3 Measuring the Relative Density of a Liquid Using Archimedes' Principle

In section 3.2.3, you have applied the concept of relative density to derive the density of a solid or a liquid. The specific gravity bottle is used in determining the relative densities of liquid. But can you determine the relative density of liquid by using the other methods? Let us discuss it; suppose you are asked to determine the relative density of liquid using the Archimedes' Principle, how would you do this?

To do this, you will require an object that sinks in water and the liquid, which is insoluble in both water, and the liquid (paraffin).

You will need a spiral spring balance to measure the weight of the solid object when immersed in water and the liquid.

- First weigh the solid in air using the spring balance
- Record this weight as W_1
- Then weigh the solid in the liquid using the spring balance
- Record this weight W_2
- Wipe the solid properly free from the liquid and then weigh it in the water using the spring balance
- Record the weight of the solid in water as W_3

Now determine what is the weight of the water displaced by the solid? The weight of liquid displaced by the solid = $W_1 - W_2$.
Again, what is the weight of the water displaced by the solid? Weight of water displaced by the solid = $W_1 - W_3$. Therefore, by definition, the relative density of the liquid,

$$= \frac{\text{Weight of the liquid displaced}}{\text{Weight of equal volumme of water}}$$

$$= \frac{W_1 - W_2}{W_1 - W_3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (15.6)$$

SELF ASSESSMENT EXERCISE 4

A block of aluminum of volume $0.25m^3$ hangs from a wire and the tension in the wire is 6500N. If the block is completely immersed in a liquid of density $1100kgm^{-2}$. Calculate the tension in the wire.

3.4 The Law of Floatation

What does the law of floatation states?

"The law of floatation states that a floating body displaces its own weight of the liquid in which it floats".

This is really a special case of Archimedes' Principle and it is the situation in which the weight of the body is exactly balanced by the upthrust from the fluid. This therefore implies that if a body is to float, it must displace its own weight of the fluid in which it is floating.

You should remember that if a body is floating in a liquid, the body is likely to float with certain amount of itself above the fluid or it is totally immersed. A floating ice block in water and a floating balloon in air are typical examples.

SELF ASSESSMENT EXERCISE 5

A boat of mass 650kg floats on water, what volume of water does it displace?

When an object at rest floats on a liquid, it is described to be in equilibrium under the action of two forces, namely:

- Its weight (W) and
- Its buoyant force or upthrust (U) which is equal to the weight of the liquid displaced (fig. 15.4)

In accordance with the Archimedes' Principle

Weight of substance = Weight of liquid displaced

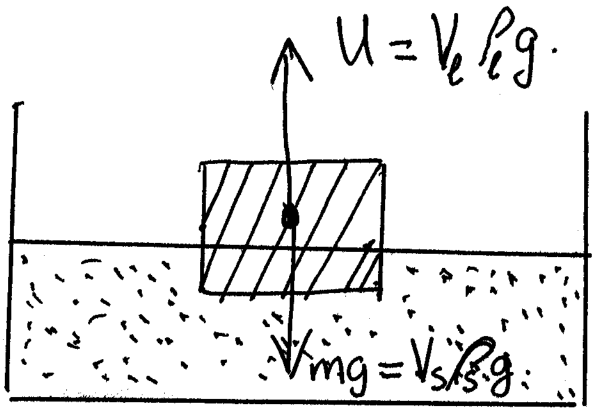


Fig. 15.4

Weight of object = Upthrust = weight of liquid displaced

$Mg = U$

$V_S\rho_S g = V_L\rho_L g$ ………………….……………. (15.7)

Where, V_S = volume of solid floating

ρ_S = density of solid floating

V_L = volume of liquid displaced

ρ_L = density of liquid displaced

g = acceleration due to gravity

$\therefore \quad V_S\rho_S = V_L\rho_L$

$$\frac{\rho S}{\rho L} = \frac{VL}{VS} = \rho_r$$

Volume of liquid displaced is equal to volume of solid

$$= \frac{\text{Mass of solid}}{\text{Mass of equal volume of liquid displaced}} \quad \text{………………….} (15.8)$$

$$= \frac{\text{Weight of solid}}{\text{Weight of equal volume of liquid displaced}} \ldots\ldots\ldots\ldots\ldots\ldots \quad (15.9)$$

SELF ASSESSMENT EXERCISE 6

An iceberg floats with 87% of its volume above the sea water, the density of which is $1.03g/cm^3$. Determine the density of the ice.

4.0 CONCLUSION

When a body is fully or partially immersed in a fluid, it experiences an upward force called the upthrust. This upthrsut is equal to the weight of the liquid displaced. There is a special case for this principle and that is when the entire body is made to float in the fluid. The law of floatation explains this, which states that the upthrust experienced by the floating object is equal to the entire weight of the floating objects. The Archimedes' principle and the law of floatation form the embodiment of the concept known as buoyancy.

5.0 SUMMARY

In this unit, you have learnt that:

- Buoyancy is the ability of an object to float, sink or rise in a fluid;
- Archimedes' Principle explains how a substance behaves when it is fully immersed in a fluid; and
- The law of floatation also explains how the big ships or boats can float on waters without sinking despite heir weight or mass.

ANSWER TO SELF ASSESSMENT EXERCISE 1

(a) Since the volume of the metal is $0.5m^3$, the volume of water displaced is $0.5m^3$. This is due to Archimedes' Principle.

(b) Mass of displaced water is equal to volume of water displaced times its density

m = Volume x density

= $0.5m^3 \times 1000kg/m^3$

= 500kg

(c) Weight of the displaced water

= mg

= $500kg \times 10m/s^2$

= $5000kgm/s^2$

= 5000N

(d) The upward force (upthrust)

= weight of water displaced

= **5000N**

ANSWER TO SELF ASSESSMENT EXERCISE 2

If weight of stone in air = W_1 = 30N
And weight of stone in water = W_2 = 18N
∴ The upthrust on the stone = $W_1 - W_2$
= 30N – 18N
= 12N

Which is equal to the weight of equal volume of water displaced by the stone.

$$\text{Relative density of stone} = \frac{\text{Weight in air}}{\text{Upthrust}}$$

$$= \frac{\text{Weight in air}}{\text{Weight of equal volume of water}}$$

$$= \frac{30}{12}$$

∴ $\rho_r = 2.5$

∴ ρ of stone = $\rho_r \rho_w$
= 2.5 x 1000kg/m^3
= 2500kg/m^3

Density of stone = **2500kg/m³**

ANSWER TO SELF ASSESSMENT EXERCISE 3

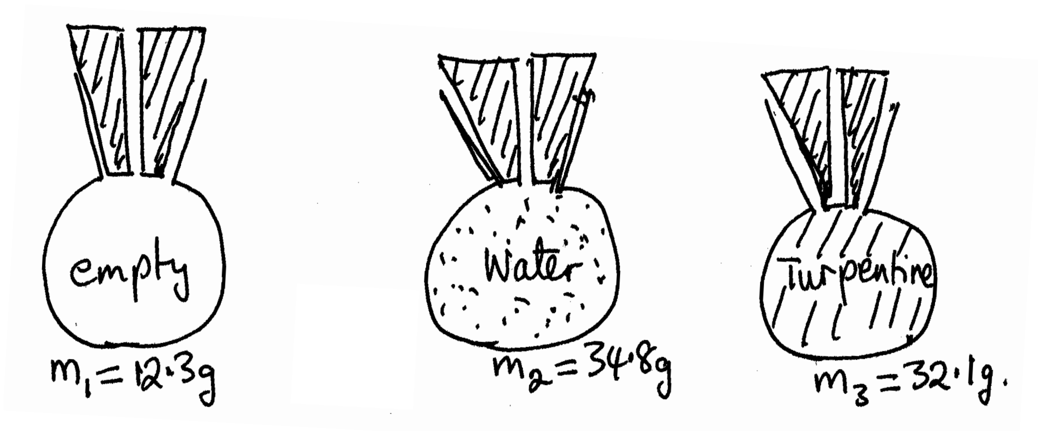


Fig. 15.5

Density of turpentine = relative density of turpentine x density of water

$$\text{Relative density of turpentine} = \frac{\text{Mass of Turpentine}}{\text{Mass of equal volume of water}}$$

Mass of turpentine = $m_3 - m_1$
Mass of equal volume of water = $m_2 - m_1$

$$\text{Relative density of turpentine} = \frac{m_3 - m_1}{m_2 - m_1}$$

$$= \frac{(32.1 - 12.3)g}{(34.8 - 12.3)g}$$

$$= \frac{19.8g}{0.88}$$

Density of turpentine = relative density of turpentine x density of water

(where, density of water = $1gcm^{-3}$)

$= \quad 0.88 \text{ x } 1gcm^3$

= $\mathbf{0.88g/cm^3}$

ANSWER TO SELF ASSESSMENT EXERCISE 4

If the volume of Aluminum is $0.25cm^3$ then it will displace this volume of the liquid of density $100kgm^{-3}$

∴ Mass of liquid displaced = volume x density
= $0.25m^3$ x $1100kgm^{-3}$
= 275kg

∴ Mass of liquid displaced = mg
= 275kg x $10ms^{-2}$
= 2750N

This is therefore the upthrust according to Archimedes' Principle

But, weight of object in air = Apparent weight + upthrust

∴ Apparent weight of aluminum = Tension in air – Upthrust
= 6500N – 2750N
= **3750N**

ANSWER TO SELF ASSESSMENT EXERCISE 5

Since it floats on water then its weight = upthrust

It will then displace the mass of water equal to its own mass

Thus the mass of water displaced = 650kg

If the density of water = $1000kg/m^3$

$$\therefore \quad \text{Volume of water displaced} = \frac{\text{Mass of water}}{\text{Density of water}}$$

$$= \frac{650kg}{1000\frac{kg}{m^3}}$$

$= 0.650m^3$

∴ Volume of water displaced = $0.650m^3$

ANSWER TO SELF ASSESSMENT EXERCISE 6

Let V be the volume of the iceberg

Let ρ be the density of the iceberg
Taking g as the acceleration due to gravity
Weight of iceberg = Vρg

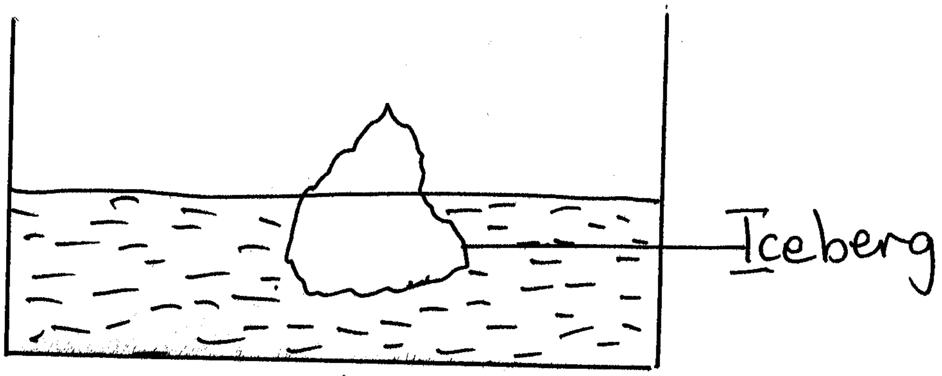


Fig. 15.6

The weight of water displaced = $\rho_w V_w g$
Where, ρ_w = density of sea water
V_w = volume of water displaced

We were told that 10.87% of the iceberg is above sea water

$$\therefore \text{ Volume of iceberg below sea} = \left(\frac{100-10.87}{100}\right)V$$

$$= \frac{89.13}{100}V$$

$$\therefore \quad V_w = \frac{89.13}{100}V$$

$$\therefore \quad \text{Volume of sea water displaced} = \frac{89.13}{100}V$$

$$\therefore \quad \text{Weight of liquid} = \rho_{sea} V_{sea} g$$

$$= 1.03\frac{g}{cm^3} \times \frac{89.13}{100}V\,g$$

But weight of ice = weight of sea water displaced

$$\therefore \quad \rho V g = 1.03\frac{g}{cm^3} \times \frac{89.13}{100}V\,g$$

$$\therefore \quad \rho = 1.03 \times \frac{89.13}{100}\frac{g}{cm^3}$$

$$\therefore \quad \rho = 1.918 g/cm^3$$

6.0 TUTOR-MARKED ASSIGNMENT

1. A piece of metal has a volume of $0.75m^3$. I fit is suspended on a rope and immersed in glycerine of density $1200kgm^{-3}$ find

   (a) The volume of glycerine displaced
   (b) The mass of the glycerine displaced
   (c) The weight of the displaced glycerine
   (d) The upward force (upthrust) on the metal due to the glycerine

2. A piece of rock slat weighs 3.3N in air and appears to weigh 1.5N when immersed in saturated salt solution of density $1200kgm^{-3}$, what is the density of the rock salt?

3. Full of water, a large flask has a mass of 3.5kg, and when full of the diesel oil, the mass is 3.2kg. What is the density of the diesel oil if the flask alone has a mass of 1.5kg?

4. A wooden block of volume $100cm^3$ and density $0.7gcm^{-3}$ is floating inbrine of density $1.2g/cm^3$. What is the mass of a body which when placed on top of the block will push it below the surface of the brine?

7.0 REFERENCES/FURTHER READING

Awe, O. and Okunola, O.O. (1986). *Comprehensive Certificate Physics*, Ibadan: University Press.

Ference, M. (Jnr.), Lemon, H.B. and Stephenson, RJ (1970). *Analytical Experimental Physics*, Chicago: The University of Chicago Press.

Feyman, R.P., Leighton, R.B. and Sands, M (1971). *The Feyman Lectures on Physics*, California: Addison-Wesley Publishing Company Limited.

Flowers, B.H. and Mendoza, E (1970). *Properties of Matter*, London: John Wiley and Sons Limited.

Noaks, M.A. (1963). *New Intermediate Physics*, Canada: Macmillan and company Limited.

Soars, F.W., Zemansky, M.W. and Young, H.D. (1980). *College Physics*. London: Addison-Wesley Publishing Company.

Tyler, F. (1966). *Heat and Thermodynamics*. London: Edwards Arnold Publisher Limited.