MODULE 1

UNIT 1 SCALAR AND VECTORS. REPRESENTATION OF VECTORS

CONTENT

1.0 INTRODUCTION

You are going to be introduced to a new way of representing quantities. In this case they have both magnitude and directions unlike other quantities.

Having seen the picture of what a vector is, you will then discuss its representation.

The study of Vectors greatly simplifies the study of mechanics.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- State the difference between vectors and scalar.
- Represent vectors.

3.0 MAIN CONTENT

3.1 Scalar and Vectors

You must be aware of the units of measurements for mass, temperature, density and others. You don't need to specify your temperature towards the North or East. It is just 98.4°F for a normal body.

These types of quantities are referred to as Scalar quantity.

3.1.1 Definition of Scalar

A Scalar is that quantity which possesses only magnitude

Example 1:

Apart from mass, temperature and density mentioned above, other examples of Scalar are energy, speed, length and time.

3.1.2 Displacement

Example 2:

Consider your movement from your house to the study center, all you need to give is the distance. But in corning to the center you must have passed through many routes involving turning from one direction to another. If you `turn' through two different corners, with the same magnitude of distance, you should be able to differentiate them by the different directions (routes) you have to turn.

In this situation, you will say your displacement is xkm in the direction θ° which is quite different from displacement of xkm in the direction a° if $a \neq \theta$ because it is a vector quantity.

Example 3:

Also imagine a car travelling at a regular `speed' of 60km/h round a circular track as in figure V1, the direction of PI is different from PZ or P3 or P4, that is, at every turn its `velocity' changes because it is a vector quantity.

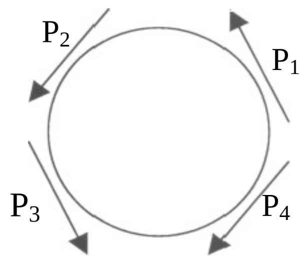


Fig. VI

3.1.3 Definition of Vectors

A vector quantity is that which has both magnitude as well as direction.

Example 4:

Vector quantities are force, momentum, velocity, acceleration, displacement, and electric field.

Exercise 1:

i. Which of these is a vector?
(a) Speed (b) Distance (c) Length (d) Force (e) Mass.

ii. Which of these is a scalar?
(a) Force (b) Velocity (c) Distance (d) Displacement (e) Weight.

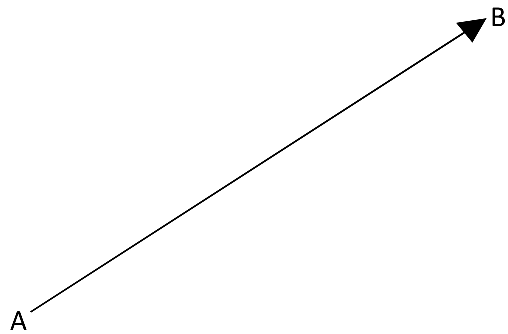
Solution

(i) (d) force (ii) (c) Distance.

3.2 Representations of Vectors.

3.2.1 Vector Specification

A directed line segment represents vector quantities with an arrowhead showing its direction V2. You can use capital letter **AB** with an arrow to denote the line segment.



You refer to **A** as the **initial point** or origin and **B** as its **terminal point.**

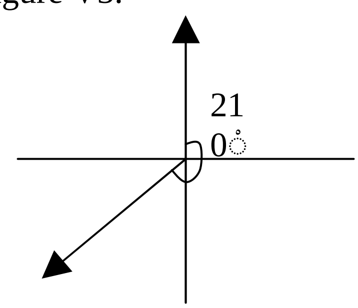
You denote **BA** as **-** **AB** to differentiate between the two directions; you could also represent vectors by small letters with a curl under them or bold e.g. u or **v**

You could also use angles (Bearing) measured from the North in a clockwise direction to represent the direction and a scale drawing of a line segment to represent the magnitude.

Example 5:
Draw a Vector **a** of magnitude 3 cm and direction 210°.

Solution:

See figure V3.



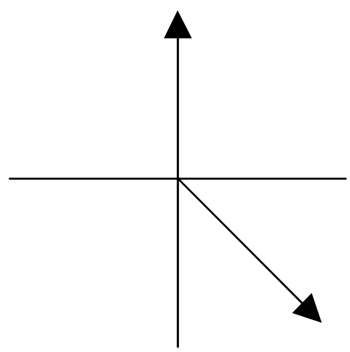


Fig. V3

3.2.2 Vector Routes from the Origin

Since you have been told that vector must originate from a point, you will now choose a specific origin (0, 0) on the coordinate axis. There, a unit movement on the x - axis is denoted i and on the y - axis **j**. **A unit vector** is a vector whose magnitude is one.

If the terminal point of your vector is at point (3, 4) while the initial point is on the origin (0, 0) then you can represent your vector

u = 3**i** + **4j** as simple as a column vector (34). Fig. V4.

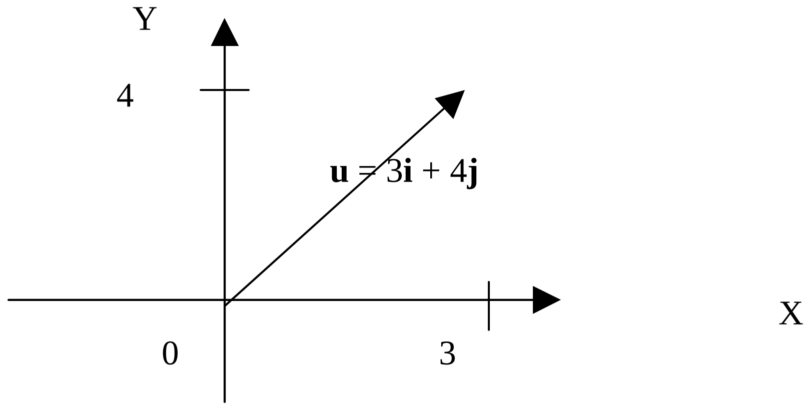


Fig. V4

SELF-ASSESSMENT EXERCISE 2

Represent the given velocity by vectors, using a convenient scale. (a)

120km/h in the direction 150°

Solution

Scale: 1 cm represents 30km/h. Length of line representing
The vector is $\frac{120}{30}$ = 4cm

v 4cm

Fig. V5

In figure V5,
v is the required velocity.

Example 6

Represent the following Vectors on the rectangular axis. (a) 5**i** – 4**j** (b) 4**i** + **5j** (c) (-3, -5)

Solution:

See figure V6 (on graph sheet),

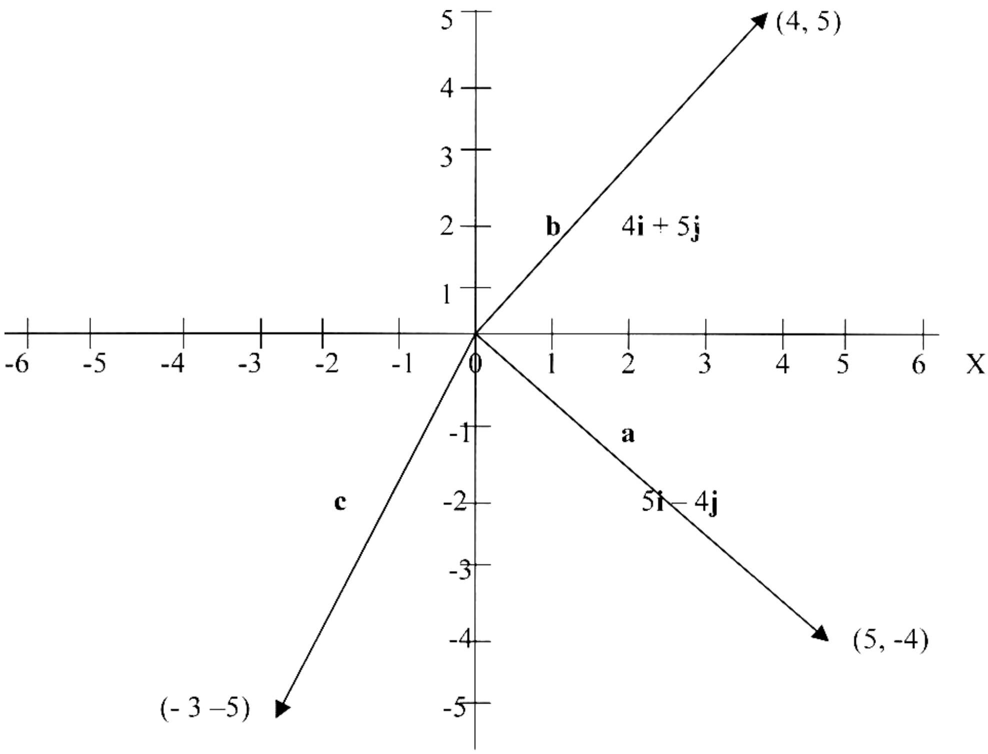


Fig. V6

4.0 CONCLUSION

You are now ready to work with vectors in the next unit, since you can now represent it.

You must remember to put the arrows on the vectors to show the direction.
You should also put a curl (~) under any letter used to represent a vector. Draw vectors on graphs as ordered pair of points on the x - y plane.

5.0 SUMMARY

In this unit you have been introduced to vectors and scalar, and you learnt that:

- Scalar are quantities with only magnitude like speed or mass.
- Vectors are quantities that must be specified with both magnitude as well as

directions e.g. velocity and weight.

- You can represent vectors by directed line segment AB or **BA, u** or ordered pairs of point (x, y) on the coordinate and this is denoted x**i** + y**j, i** and **j** are unit vectors in the direction of x and y - axis respectively.

6.0 TUTOR-MARKED ASSIGNMENT

1. State which of the following are scalars or vectors. (a) Weight
   (b) Specific heat
   (c) Density
   (d) Momentum
   (e) Distance
   (f) Displacement.
2. Represent graphically.
   (i) A force SN in a direction 120° (ii) **u** = 5**i** + 2**j**
   (iii) **v** = (3, -4)

7.0 REFERENCES/FURTHER READING

Egbe, E. Odili G. A., & Ugbebor, O. O. (2000) Further Mathematics Onitsha: Africana FEP

Keisler, H.J. (2005). Elementary Calculus. An Infinitesimal Approach, 559 Nathan Abbott, Stanford, Califonia, USA.

Wrede, R.C. & Spegel M. (2002). Schaum's and Problems of Advanced Calculus, McGraw – Hill N. Y.

UNIT 2 DEFINITION OF TERMS IN VECTOR ALGEBRA

CONTENTS

1.0 INTRODUCTION

It is very important to define terms in mathematics. As a language of science the major concepts involves using terms correctly.

This is why you must learn to use the definitions in this unit in other units in this course.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- define equal vectors, Negative vectors, and Null vectors
- recognize how scalar multiples of vectors give the definitions of Null vectors, parallel vectors and negative vectors.

3.1.1 Equal Vectors

You will say two vectors **u** and **v** are equal if they have the same magnitude and directions, regardless of the position of the vectors.

In figure V 1, AB = CD

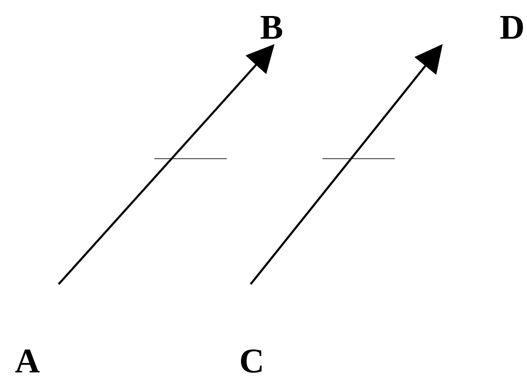
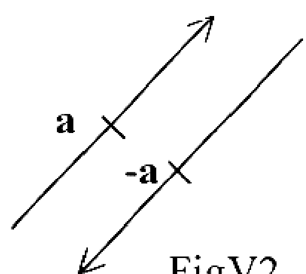


Fig. VI

3.1.2. Negative Vectors:

If you have a vector with opposite direction to another vector a, but having the same magnitude, you denote it –**a**



FigV2

3.1.3 The difference of vectors

The difference of two vectors **u** and **v,** you denote as **u-v**, and it is the sum of **u+ (-v).** That is, the sum of the vector **u**, and the (additive inverse of vector **v**) negative vector **v**.

3.2.1 Null vectors

In a particular case of **u+ (-v),** if **u**=**v** then their difference will give you the null vector or zero vector i.e**. u-u**=**0**. Note that the zero vector exists in this case. As it could mean a movement from A to B, and back to A from B; then the resultant will be zero.

Generally, you could then have **Null vector** or a **Proper vector**.

3.2.2 Scalar multiples of vectors

The Scalar product of u denoted m**u** where m is a scalar, is a vector having the same direction, but is multiplied by its magnitude. You should note that if m is 0, you have a null vector, if m > 0; you have a parallel vector **u** and if m
< 0, and if m = l, then you have a negative vector **u**, (**- u**).

Example 1:
Forces f_1, f_2, f_3, f_4, f_5; acted on a body, find the force needed to prevent the body from moving.

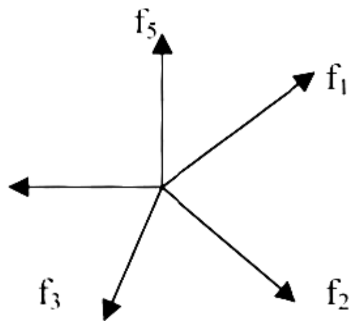


Fig. V3

Solution:
The force needed to prevent the body Q from moving is the negative resultant
-R = (F, + F_2 + F_3 + F_4 + F_5) which when added R + (-**R**) gives the zero vector **0**.
So you will have -R = **-** (F, + F_2 + F_3 + F_4 + F_5).

SELF-ASSESSMENT EXERCISE 1

Two vectors u and v have the following lengths and directions:

u with length 2cm and direction 180°, and **v** with length 3cm and direction 030°. Construct accurate diagrams to show:
(i) **u** (ii)-**u** (iii) **v** (iv) -2**v**

Solution (see fig. V4)

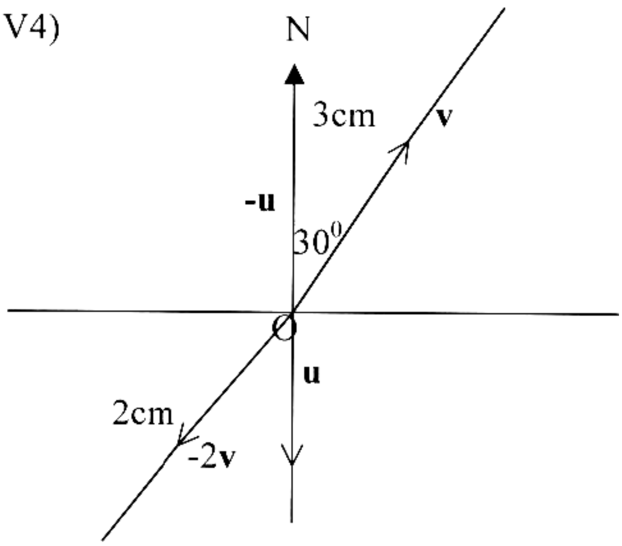


Fig V4

(ii) -u = 2cm direction 000° (iv) -2v = cm direction 2 10°

4.0 CONCLUSION

Vectors, although being specified by both magnitude and direction, still posses certain properties like any other number in the number system.

In order to appreciate these properties you need to be able to define certain terms in vector algebra.

Each of this special vectors will come in very useful in further studies of vector algebra.

5.0 SUMMARY

You have learnt in this unit, the following definitions:

1. Equal vectors - Vectors with same magnitude and directions.
2. Negative vectors - Vectors with the same magnitude, but in opposite directions.
3. Difference of vectors - The sum of a vector and the negative of another vector, **u+(-v)**.
4. Null vectors - The sum of a vector and it's negative, **u+(-u)= 0**.
5. Scalar multiples, m**u**, of vector **u**, where m is scalar - It is a vector with the same direction (except where m<0), but having a scalar multiple of its magnitude.

6. In scalar multiple, m**u**, of **u** - If m=0, you have a null vector; If m>0, you have parallel vector to **u**; and if m<0, then you have a negative vector **-u.**

6.0 TUTOR MARKED – ASSIGNMENT

If **u** is a vector of length 4cm in the direction 035°, **v** is a Vector of length 2cm in the direction 120° and **w** is a vector of length 3cm in the direction of 270°, draw an accurate diagram to show

(i) **u** (ii) **v** (iii) **w** (iv) -3**v**, -5/3**w**

7.0 REFERENCES/FURTHER READING

Keisler, H.J. (2005). Elementary Calculus. An Infinitesimal Approach, 559 Nathan Abbott, Stanford, Califonia, USA.

Wrede, R.C. and Spegel M. (2002). Schaum's and Problems of Advanced Calculus, McGraw – Hill N. Y.

UNIT 3 TRIANGLE AND PARALLELOGRAM LAW OF VECTOR ADDITION

CONTENTS

1.0 INTRODUCTION

The operations of addition, subtraction and multiplication formula in the algebra of numbers of scalars are, with suitable definition, capable of extension to algebra of vectors.

In this Unit you will be learning about these and the laws of vector Algebra.

2.0 OBJECTIVES

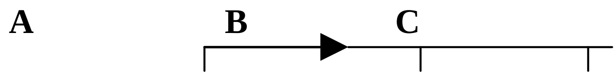
At the end of this unit, you should be able to:

- draw accurate diagrams to show the laws of vector algebra
- prove given statements using the triangle and parallelogram law of vector addition.

3.0 MAIN CONTENT

3.1.1 The Sum of or resultant of a Vector

Consider your movement from a point A to B to C,
The total distance of your journey will be AB + BC
And on a straight line



You could say the distance is AC

3.1.2 Triangle Law

Similarly if a vector $\overrightarrow{\mathbf{AB}}$ is followed by a vector $\overrightarrow{\mathbf{BC}}$ with the initial point **B** of $\overrightarrow{\mathbf{BC}}$, the terminal point of vector $\overrightarrow{\mathbf{AB}}$, then the sum.

$\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}}$ is, a vector $\overrightarrow{\mathbf{AC}}$ which will close the triangle ABC. This is the triangle law;

it gives the sum or resultant of two vectors.

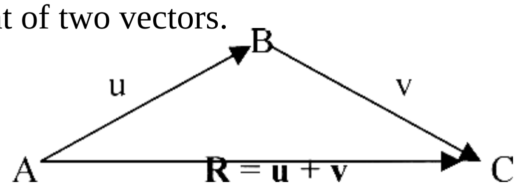


Fig. V6

3.1.3 The Parallelogram Law of Vector Addition.

Suppose the two vectors to be added up have the same initial point or origin, then their sum or resultant is the diagonal of the parallelogram formed, based on the two given vectors, where the three have the same initial point.

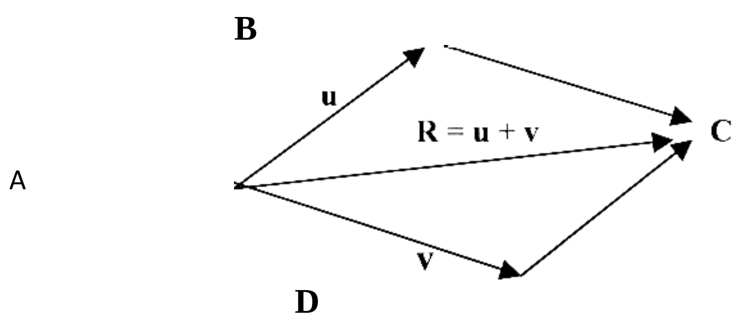


Fig. V7

From figure V7, you can see that the parallelogram law is an extension of the triangle law. Since $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{AD}} = \overrightarrow{\mathbf{AC}}$ for parallelogram law, but

$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ according to triangle law because $\overrightarrow{BC} = \overrightarrow{AD}$ (opposite sides of a parallelogram) you can extend these laws to three or more vectors, and the word displacement could be used for resultant.

Example I

An automobile travels 6km in the direction 000°, then l0km in the direction 045°. Represent these displacements graphically and determine the resultant displacement

(a) Graphically,
(b) Analytically.

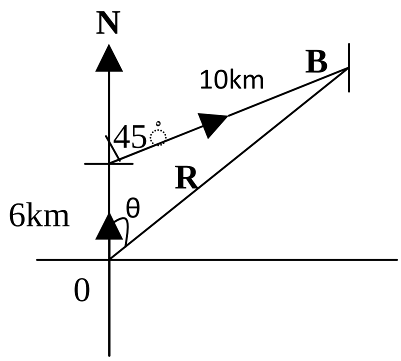


Fig. V8

Graphically-

Construct figure V8 and measure θ with a protractor for the direction of $\vec{R}$. Measure the distance OB. Multiply it by 2km the chosen scale, to give you the magnitude of the resultant $\vec{R}$.

From the measurement, the vector $\vec{R}$ has magnitude 7.4 x 2 = 14.8 km and direction 028°.

Analytically

Using cosine rule,

$$|\vec{R}|^2 = 6^2 + 10^2 - 2 \times 6 \times 10 [-\cos (180 - 45°)]$$

$$= 36 + 100 + 120 \cos 45°$$

$$|\vec{R}| = \sqrt{220.85}$$

$$= 14.86\text{km}$$

To calculate 0, use Sine Rule. Sin θ =

Sin (180 - 45)
10 14.85
. Sin Ɵ = 0.4758
.*. Ɵ = Sin-' 0.4758
= 28°
.'. You have the same result.

Example 2

Add the following Vectors.

$\overrightarrow{\mathbf{AB}} - \overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{BC}} - \overrightarrow{\mathbf{DC}} + \overrightarrow{\mathbf{DB}}$

Solution.

$\overrightarrow{\mathbf{AB}} - \overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{BC}} - \overrightarrow{\mathbf{DC}} + \overrightarrow{\mathbf{DB}}$

$= \overrightarrow{\mathbf{CA}} + \overrightarrow{\mathbf{BC}} + \overrightarrow{\mathbf{CD}} + \overrightarrow{\mathbf{DB}} + \overrightarrow{\mathbf{AB}}$

$= \overrightarrow{\mathbf{AC}} + \overrightarrow{\mathbf{CA}} + \overrightarrow{\mathbf{CB}}$

$= \overrightarrow{\mathbf{0}} + \overrightarrow{\mathbf{CB}}$

$= \overrightarrow{\mathbf{CB}}$

Note how you use the triangle law to add $\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{AC}}$ with B the middle letter canceling out.

3.2.1 Laws of Vectors Algebra

If **u, v** and **w** are vectors and m and n are scalar,
u + **v** = **v** + **u** (commutative law for addition)
u + **(v** + **w)** = **(u** + **v)** + **w** (associative law for addition).
m**u** = **u**m (commutative law for multiplication) m (n**u**) = (m**u**) **u** (Associative law for multiplication). (m + n) **u** = m**u** + n**u** (Distributive law)
m(**u** + **v**) = m**u** + m**v** (Distributive law).

You are now given the authority to treat vectors like any other number in the real number system.

You must remember, however, that multiplication, in this case is referring to scalar multiplication only.

You are going to learn later about products of vectors.

3.2.2 Proof of commutative law of addition

SELF-ASSESSMENT EXERCISE 3

Show that addition of vectors is commutative i.e. **u** + **v = v** + **u**

Solution:

Draw Fig. V9,

A parallelogram OABC

Formed by Vectors $\overrightarrow{\mathbf{OA}}$ = **u**, and $\overrightarrow{\mathbf{OC}}$ = **v**

u + **v** = $\overrightarrow{\mathbf{AO}}$ + $\overrightarrow{\mathbf{AB}}$ or $\overrightarrow{\mathbf{OC}}$ + $\overrightarrow{\mathbf{CB}}$

That is **u** + **v** = **v** + **u**

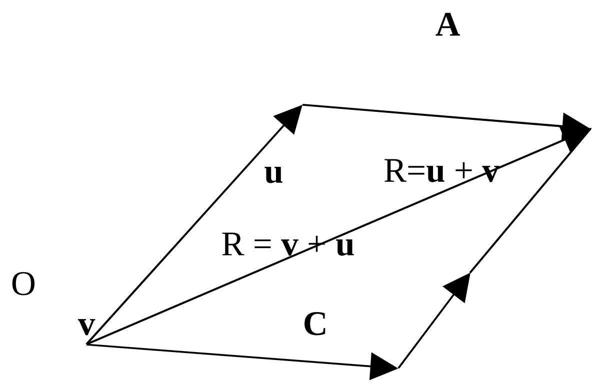


Fig. V9

SELF-ASSESSMENT EXERCISE 4

Show that the addition of vectors is associative i.e. **u + (v** + **w**) = (**u** + **v**) + **w**

Solution:

Let vector $\overrightarrow{\mathbf{OP}}$ = **u**, and $\overrightarrow{\mathbf{PQ}}$= **v** then $\overrightarrow{\mathbf{OP}}$ + $\overrightarrow{\mathbf{PQ}}$ = $\overrightarrow{\mathbf{PQ}}$ = **u** + **v**

I.e. **u** + **v** = $\overrightarrow{\mathbf{OQ}}$

See Fig. V 10

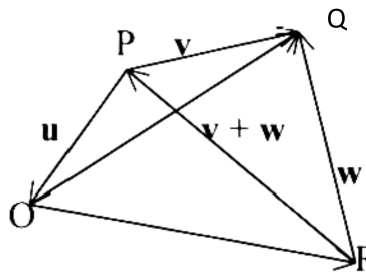


Fig V10

Also $\overrightarrow{PQ} + \overrightarrow{QR} = \overrightarrow{PR}$

$= \mathbf{v} + \mathbf{w} =$

$\overrightarrow{\mathbf{OP}} + \overrightarrow{\mathbf{PR}} = \overrightarrow{\mathbf{OR}}$

I.e. $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = \overrightarrow{\mathbf{OR}}$

Also $\overrightarrow{\mathbf{OQ}} + \overrightarrow{\mathbf{QR}} = \overrightarrow{\mathbf{OR}}$ i.e. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \overrightarrow{\mathbf{OR}}$

Since $\overrightarrow{\mathbf{OR}} = \overrightarrow{\mathbf{OR}}$, then $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$

4.0 CONCLUSION

You can treat vectors like any other number in the real number system.

You can add by using triangle or parallelogram law. You can subtract by using negative vectors. You can multiply by a scalar in this unit, and later by other vectors.

The vector algebra also obeys the laws of normal algebra since it is commutative, associative, and distributive.

5.0 SUMMARY

You have learnt in this unit the following:

l. Triangle law of vector addition which gives the resultant of two vectors

$\overrightarrow{\mathbf{AB}} + \overrightarrow{\mathbf{BC}} = \overrightarrow{\mathbf{AC}}$

2. The parallelogram law of vector addition gives the resultants of two vectors with the same initial point.

3. Laws of Vector Algebra.

u + **v**= **v** + **u**	(commutative)
(**u** + **v**) +**w** = **u** + (**v** + **w**)	(associative)
m**u** = (**u**)m	(commutative)
m(n**u**) = (mn) **u**	(associative)
(m + n) **u** = m**u** + n**u**	(Distributive)
m(**u** + **v**) = m**u** + m**v**	(Distributive)

m, n, ε R and **u**, **v**, and **w** Vectors.

6.0 TUTOR-MARKED – ASSIGNMENT

1. If **u** is a vector of length 4 cm in the direction 045°, **v** is a Vector of length 2cm in the direction 090° and **w** is a vector of length 3cm in the direction of 270°, Draw an accurate diagram (on graph sheet) to show
   (a) **u, v, w**
   (b) Draw diagrams to represents
   (i) **u** + **v**, (ii) 3(**u** + **w**) (iii) 2**u** + **v**.
2. A plane moves in a direction 315° (NW) at 125 km/h. relative to the ground, due to the fact there is a wind in the direction 270° (west) magnitude 50km/h relative to the ground. How fast and in what direction would the plane have traveled if there were no wind? (i.e. resultant).

7.0 REFERENCES/FURTHER READING

Keisler, H.J. (2005). Elementary Calculus. An Infinitesimal Approach, 559 Nathan Abbott, Stanford, Califonia, USA.

Wrede, R.C. and Spegel M. (2002). Schaum's and Problems of Advanced Calculus, McGraw – Hill N. Y.

UNIT 4 THE RECTANGLE UNIT VECTORS

CONTENTS

1.0 INTRODUCTION

In the last units you used the graphical methods to determine the resultant (sum) of two or more vectors.

There is therefore, from the graphs and construction, a need to find an easier way to do this.

In this unit, you will come across vectors, resolved into its components **i, j**, and **k**.

The sum therefore is just the sum of the coefficients of the components of the vectors.

From this a lot of other results will be arrived at, which you must take note of as you will need them in other units.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- Express vectors in terms of its components **i, j, k**
- Add vectors successfully using the components

- Calculate the magnitude of vectors successfully
- Use position vectors to calculate relative vectors of two given points
- Calculate correctly, the Unit vector of a given vector in its direction.

3.0 MAIN CONTENT

3.1.1 Unit Vectors

You refer to a vector with a unit magnitude (1 unit) as a unit vector. Therefore if a is the magnitude of vector u, then a **unit vector** in the direction of a is .u usually denoted $\mathbf{e}_u$ = u with u ≠ o

This implies you can represent a vector as its magnitude multiplied by its unit vector in its direction i.e. **u** = u$\mathbf{e}_u$

3.1.2 The rectangular Unit Vector 1, j, k. (1, 1, 1)

Consider the initial points of vector u, v and w, fixed at an origin represented by a corner of a cube as in fig. V 13.
You will have an important set of unit vectors, having their initial point at O and in the direction of the x, y, z and of a three dimensional rectangular co- ordinates system.

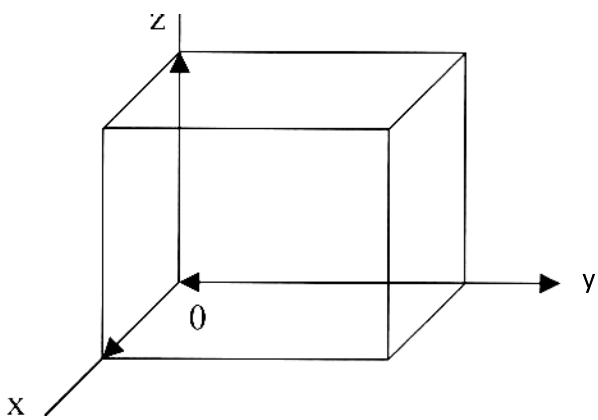
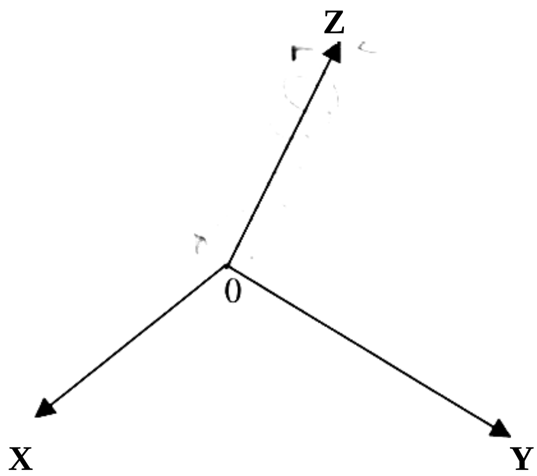


Fig. V13

These unit vectors, you denote i, j, k in the direction x, y, and z respectively.

3.1.3 The Right-handed (rule) Rectangular Co-ordinate System

As you must have observed from the cube in fig. V 13, any corner of the cube could have been chosen, and any side of the cube could be the x or y or z- axes.



To remove this confusion, you have a rule referred to as the right handed rule. This rule is assuming, first of all that you are a right handed person. Secondly that you have used the screwdriver before, with your right-hand. If you are to loosen a screw, think of the movement of your thumb, as the screw rotates anti-clockwise through 90° from x to y, y to z and z to x. Once you agree with this convention, it will be easy to carry out operations on unit vectors.

3.1.4 Position Vector or Radius Vector

u = x_1,**i** + y_1**j** + z_1**k** with x, y, z, the components of a in the x, y, and z direction and the magnitude **u** = $\sqrt{x^2 + y^2 + z^2}$

In particular, if you have **r** = **xi** + **yj** + z**k**, you referred to **r** as the **position vector** or **radius** vector, **r**, from point O (0, 0, 0) the origin to a point (x, y, z), a corner of the cube as in fig. V 14

r= | **r** | = $\sqrt{x^2 + y^2 + z^2}$

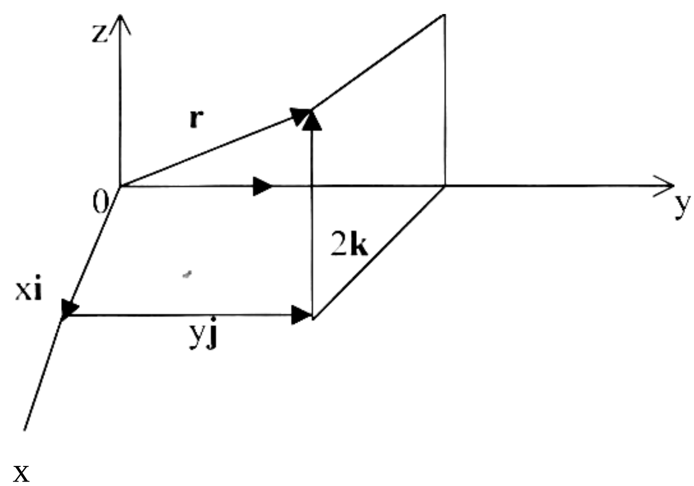


Fig V l 4

3.2.0 Scalar and vector field

Just as in the 2-dimensional space x - y-axis, you could plot points as you should have done while drawing graphs. In other words, you are aware of the ordered pairs of points (x, y) which is possible once you treat x, and y as variable and y as a **function** of x, y = f(x). This will make the next definition easy for you to understand.

3.2.1 Scalar field

If to each point (x, y, z) of a region R in space, there corresponds a number or scalar Φ (x, y, z), then you call Φ (a) **Scalar function of position or Scalar point function**, and you say that a scalar field Φ has been defined in R

Example:

$\Phi (x, y, z) = x^3y - z^2$ defines a scalar field.

3.2.2 Vector Field.

If to each point (x, y, z) of a region R in space there corresponds a vector **v** (x, y, z), then you call **v** a **vector function of position** or **vector point functions** and you say that a **vector field v** has been defined in R.

Example 7

(a) If the velocity at any point (x, y, z) within a moving fluid is known at a certain time, then a vector field is defined.

(b) $\mathbf{v} (x, y, z) = xy^2\mathbf{i} - 2yz^3\mathbf{j} + x^2z\mathbf{k}$ defines a vector field.
A vector field, which is independent of time, is called a stationary or steady state vector field.

Simply put the last two examples in section 3.2.1. and 3.2.2. is just an attempt to inform you that just as you can have scalar functions of (x, y, z), you can also have vector function of (x, y, z) in the scalar field (space) and vector field (space) respectively you should note that **i, j, k** are non-collinear vectors.

4.0 CONCLUSION

You can now add and subtract vectors easily.

All you need is what you have learnt in this unit - the components of the vector in the x, y, and z-axes.

Expressing vectors in terms of its components will help you carry out a lot of functions concerning vectors. You now know that there is the `vector field' as well the `scalar field'.

5.0 SUMMARY

In this Unit you have learnt the definition of:

(a) $\mathbf{e}_u$ = **u** the unit vector in the direction of **u**. u

(b) **i, j, k** are the unit vectors in the direction of x, y, and z-axes.

(c) The **right-handed** rule assumes, and expects movement in an anti clockwise direction from x - y, y - z and z - y and so on i.e. x ---> y---> z~ x--+ y... in choosing an origin and fixing the axes.

(d) The position vector or radius vector **r** = x**i** + **yj** + z**k** represents the components of vector **v** from point, **O** (0, 0, 0), the origin, to a point (x, y, z), a corner of a cube.

(e) The magnitude or modulus of r is **r** = $\sqrt{x^2+y^2+z^2}$.

(f) A scalar field or vector field, which is independent of time, is called a stationary or steady state scalar or vector field.

6.0 TUTOR-MARKED ASSIGNMENTS

1. Find the position vectors $\mathbf{r}_1$, and $\mathbf{r}_2$ for the points P (3, 4, 5) and Q(2,-5, 1)
2. Given the scalar field defined by ~(x, y, z)= 3x22-xy3+, find ~ at the points:
   (a) (0, 0, 0) (b) (2, -1, 2) (c) (-3, -1, -2)

7.0 REFERENCES/FURTHER READING

Keisler, H.J. (2005). Elementary Calculus. An Infinitesimal Approach, 559 Nathan Abbott, Stanford, Califonia, USA.

Wrede, R.C. & Spegel M. (2002). Schaum’s and Problems of Advanced Calculus, McGraw – Hill N. Y.

UNIT 5 COMPONENTS OF A VECTOR

CONTENTS

1.0 INTRODUCTION

Vector quantities have both magnitude and direction. For this reason, the method of addition or resultant needs to be thoroughly explored by you. It is for this reason that you are being presented with different ways of achieving this.

You already learnt the representation graphically of vectors, and its sum through such diagrams.

In unit 4, you were introduced to the definition of unit vectors especially **i, j**, and **k**.

You will, therefore, in this unit, deal with components of vectors, which are the smaller, parts whose sum represent the vectors.

From this concept, a lot of other results will be arrived at, which we must take note of.

2.0 OBJECTIVES

At the end of this unit you should be able to: Write vectors in terms of its components **i, j, k**.

3.0 MAIN CONTENT

3.1.1 Components of a Vector in terms of i, j, k

You have learnt in the previous section how all vectors can be represented as **u** = u$\mathbf{e}_U$. i.e. the magnitude of the vector multiplied by its unit vector.

You will now have $\mathbf{e}_U$ as a unit vector in the direction of the x, y, and z-axes called **i, j, k** respectively.

The implication of an origin like this is that of a vector **u**=x_1**i** + y_1**j** + z_1**k** with x_1, y_1, z_1, $\in R$.

x_1**i**, y_1**j** and z_1**k** are referred to as the **rectangular component vectors** or simply component vectors of **u** in the x_1, y_1, and z_1, directions.

x_1, y_1, and z_1, are referred to as the rectangular components or simply components of **u** in the x, y and z directions.

With these definitions or representation, you are now in a better position to deal with vector algebra.

The magnitude of **u** =x_1**i** + y_1**j** + z_1**k** is $\sqrt{x_1^2 + y_1^2 + z_1^2}$ without drawing the diagram as in unit 2.

3.1.2 Non - Collinear Vectors

Non-collinear vectors are vectors which are not parallel to the same line, and so when their initial points coincides, (in this case at (0, 0, 0). They determine the planes that make then up. i.e. x - y plane, y - z plane, and z - x plane, using the right - handed (thumb) rule.
'. you say vectors **i, j, k** are mutually perpendicular.

Example 2

Show that if x_1**u** + y_1**v** + z_1**w** = x_2**u** + y_2**v** + z_2**t**. Where **u**, **v**, and **w** are non - coplanar Then $x_1 = x_2$, $y_1 = y_2$, and $z_1 = z_2$.

Solution:

You can write the, equation as

$(x_1 - x_2)$ **u** + $(y_1 - y_2)$ **v**$_2$ + $(z_1 - z_2)$ **w** = 0.
The it follows that since a vector is either null or proper, and these are
Non-collinear vectors **- u, v**, and **w**, then $x_1 - x_2 = 0 <=> x, = x_2$

$y_1 - y_2 = 0 <=> y_1 = y_2$ and $Z_1 - Z_2 <=> z_1 = z_2$

3.1.3 Equation of a straight line passing through two given points

Find the equation of a straight line which passes through two given points A and B having **position vectors a** and **b** with respect to an origin O.

Solution:

Let **r** be the position vector of any point P on the line through A and B. Fig. V12.

From the figure V12.

$\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{AP}} = \overrightarrow{\mathbf{OP}}$ or

$\mathbf{a} + \overrightarrow{\mathbf{AP}} = \mathbf{r}$

I.e. $\overrightarrow{\mathbf{AP}} = \mathbf{r} - \mathbf{a}$

and you have

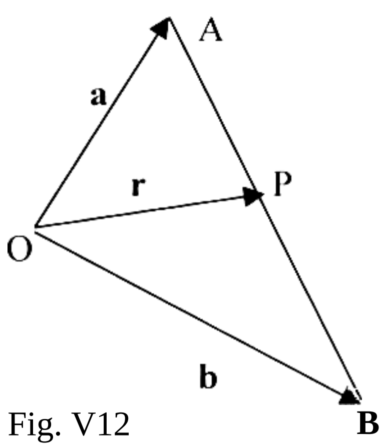


Fig. V12

$\overrightarrow{\mathbf{OA}} + \overrightarrow{\mathbf{AB}} = \overrightarrow{\mathbf{OB}}$

$\mathbf{a} + \overrightarrow{\mathbf{AB}} = b$ i.e. $\overrightarrow{\mathbf{AB}} = \mathbf{b} - \mathbf{a}$

Since $\overrightarrow{\mathbf{AP}}$ and $\overrightarrow{\mathbf{AB}}$ are collinear (lie on the same line or parallel to the same line).

$\overrightarrow{\mathbf{AP}} = t\,\overrightarrow{\mathbf{AB}}$ or $\mathbf{r} - \mathbf{a} = t\,(\mathbf{b} - \mathbf{a})$.

Then the required equation is $\mathbf{r} = \mathbf{a} + t\,(\mathbf{b} - \mathbf{a})$ or $\mathbf{r} = (1 - t)\,\mathbf{a} + t\mathbf{b}$ which compares to the general equation of a straight line $y = mx + c$. If the equation is written $(1 - t)\,\mathbf{a} + t\mathbf{b} - \mathbf{r} = 0$ the sum of the coefficient of **a**, **b** and **r** is $1 - t + t - 1 = 0$.

This implies that P is always on the line joining A and B does not depend on the choice of origin O.

Alternatively, you can solve this problem by the method below. Since $\overrightarrow{\mathbf{AP}}$ and $\overrightarrow{\mathbf{PB}}$ are collinear, then they are ratios of each other say AP:PB

$\overrightarrow{\mathbf{AP}} : \overrightarrow{\mathbf{PB}} = m : n$ or $n\overrightarrow{\mathbf{AP}} = m\overrightarrow{\mathbf{PB}}$ Using the **position vector** above. $\overrightarrow{\mathbf{AP}} = \mathbf{r} - \mathbf{a}$, $\overrightarrow{\mathbf{PB}} = \mathbf{b} - \mathbf{r}$

$\therefore\ n\,(\mathbf{r} - \mathbf{a}) = m(\mathbf{b} - \mathbf{r})$

which transforms to $\mathbf{r} = \frac{na + mb}{m + n}$

And you call this the symmetric form.

3.2.1 Relative Vectors

Please note the use of the position vector leads to what you will call relative vectors.

If **a** is the position vector of **A** and **b** the position vector of **B**, then the position vector **B relative to A** is vector $\overrightarrow{\mathbf{AB}} = \mathbf{b} - \mathbf{a}$. (note the order of the letters).

Example I

Write $\overrightarrow{\mathbf{AC}}$, $\overrightarrow{\mathbf{BD}}$, and $\overrightarrow{\mathbf{EF}}$ in terms of their relative vectors.

Solution:

$\overrightarrow{\mathbf{AC}} = \mathbf{c} - \mathbf{a}$, $\overrightarrow{\mathbf{BD}} = d - b$, $\overrightarrow{\mathbf{EF}} = \mathbf{f} - e$.

3.2.2 The Sum of Resultant of Vectors in Component Form

Example 1

(a) Find the position vectors of $\mathbf{r}_l$, and $\mathbf{r}_2$ for the point (2, 4, 3) and Q (1, -5, 2) of a rectangular coordinate system in terms of the unit vectors **i, j, k.**

(b) Determine analytically the resultant of $\mathbf{r}_1$, and r_2.

Solution

(a) P (2, 4, 3) and Q (1, -5, 2)
$\mathbf{r}_1 = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$ $\mathbf{r}_2 = \mathbf{i} - 5\mathbf{j} + 2\mathbf{k}$

(b) $\mathbf{r}_1 + \mathbf{r}_2 = (2 + 1)\mathbf{i} + (4 - 5)\mathbf{j} + (3 + 2)\mathbf{k}$
$= 3\mathbf{i} – \mathbf{j} + 5\mathbf{k}$

You should try the graphical method to appreciate this simpler way of adding vectors.

SELF-ASSESSMENT EXERCISE 2

Given $\mathbf{r}_1 = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$
$\mathbf{r}_2 = 2\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}$ $\mathbf{r}_3 = -\mathbf{I} + 2\mathbf{j} + 2\mathbf{k}$

Find the **magnitudes** of
(a) $\mathbf{r}_3$ (b) $\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$ (c) $2\mathbf{r}_1 - 3\mathbf{r}_2 - 5\mathbf{r}_3$

Solutions

(a) $|\mathbf{r}_3| = r_3 = |-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}|$
$= \sqrt{(1)^2 + (2)^2 + (2)^2}$
The square root of the sum of the squares of the coefficient of **i, j** and **k** respectively.
$\therefore \mathbf{r}_3 = \sqrt{9} = 3$

(b) $\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3 = (3 + 2 - 1)\mathbf{i} + (-2 + (-4) + 2)\mathbf{j} + (1 - 3 + 2)\mathbf{j}$

$= 4\mathbf{i} - 4\mathbf{j} + O\mathbf{k}$
$= 4\mathbf{i} – 4\mathbf{j}$
$\therefore |\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$ $4^2\sqrt{}$ = $\sqrt{4^2 + (-4^2)}$
$= \sqrt{32}$
$= 4\sqrt{2}$

(c) $2\mathbf{r}_1 - 3\mathbf{r}_2 - 5\mathbf{r}_3$ = [2x3+(-3)x2+(-5)x(-1)]**I** +[2x(-2)+(-3)x(-4)-5(2)] **j**+ (2x1+(-3) (-3) + (-5) x 2)**k**
= (6-6+5)**I** + (-4+12-10)**j** + (2+9-10)**k**
= 5**i** -2**j** + **k**
The magnitude, I 5**i** - 2**j** + **k** I
$=\sqrt{5^2 + (-2)^2 + 1^2}$
$=\sqrt{25 + 4 - 1}$
$=\sqrt{30}$
.'. The magnitude of $2\mathbf{r}_1 - 3\mathbf{r}_2 - 5\mathbf{r}_3$ is $\sqrt{30}$.

3.2.2 Centroid

If r_1, r_2 ... r_n are the position vectors of masses m_1, m_2 ... m_n respectively relative to an origin O.
Then the position vector of the Centroid can be proved as

$$\mathbf{r} = \frac{m_1 r_1 + m_2 r_2 + \dots m_n r_n}{m_1 + m_2 \dots m_n}$$

Simply put, the Centroid represents the weighed average or mean of several vectors.

4.0 CONCLUSION

Congratulations! You can now add and subtract vectors easily. All you need is their components on the x, y, z-axes.
Expressing vectors in terms of its components, will help you to calculate:

- Their resultant.
- Their magnitudes
- Their relative vectors
- The equation of the straight line passing through two given points

You will use these points in the next Unit in some calculations.

5.0 SUMMARY

In this unit you have learnt the following:

(a) A vector **u**= x**i** + y**j** + z**k** has three components, x**i**, y**j**, z**k** in the direction of the x, y, and z-axes.

(b) If $x_1\mathbf{u} + y_1\mathbf{v} + z_1\mathbf{w} = x_2\mathbf{u} + y_2\mathbf{v} + z_2\mathbf{w}$, where **u, v** and **w** are non-coplanar, then $x_1=x_2$; $y_1=y_2$; and $z_1=z_2$.

(c) The equation of a straight line passing through two given points A and B having position vectors **a** and **b** with respect to an origin O is **r** = a + t (**b**-**a**) or **r** = (1-t) a + t **b**.

(d) The symmetric form of the above equation is n (**r**-**a**) = m (**b**-**r**) and so $\mathbf{r} = \frac{n\mathbf{a} + m\mathbf{b}}{m + n}$, which is a condition for co linearity of points.
Where **a** is the position vector of **A**, **b** the position vector of **B** and **r** is the position vector of point $\vec{P}$, which divides $\overrightarrow{AB}$ in the ratio m:n.
As an extension, the relative vector **r** of position vector **B** relative to

A is **AB** = **b - a.**

(e) If $\mathbf{u} = x_1\mathbf{i} + y_l\mathbf{j} + z_1\mathbf{k}$, then
The resultant or sum of **u** and **v** is
$\mathbf{u} + \mathbf{v} = (x_1+x_2)\mathbf{I} + (y_1+y_2)\,\mathbf{j} + (z_I + z_2)\mathbf{k}$.

6.0 TUTOR - MARKED ASSIGNMENTS

I. If $\mathbf{r}_1 = 2\mathbf{i} - \mathbf{j} + \mathbf{k}$,
$\mathbf{r}_2 = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$,
$\mathbf{r}_3 = -2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and
$\mathbf{r}_4 = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

Find

(a) $\mathbf{r}_1 + \mathbf{r}_2$
(b) $\mathbf{r}_1 + \mathbf{r}_2 + \mathbf{r}_3$
(c) $2\mathbf{r}_1 + 3\mathbf{r}_2 + \mathbf{r}_3$
(d) $-2\mathbf{r}_1 + \mathbf{r}_2 - 3\mathbf{r}_3$

2. Find a Unit vector parallel to the resultant of vectors
$\mathbf{r}_1 = 2\mathbf{i} + 4\mathbf{j} - 5\mathbf{k}$,
$\mathbf{r}_2 = \mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$

3. The position vector of points P and Q are given by $\mathbf{r}_1 = 2\mathbf{i} + 3\,\mathbf{j} - \mathbf{k}$, $\mathbf{r}_2 = 4\mathbf{i} - 3\,\mathbf{j} + 2\mathbf{k}$ determine PQ in terms of **i, j, k** and find its magnitude.

7.0 REFERENCES/FURTHER READING

Keisler, H.J. (2005). Elementary Calculus. An Infinitesimal Approach, 559 Nathan Abbott, Stanford, Califonia, USA.

Wrede, R.C. and Spegel M. (2002). Schaum's and Problems of Advanced Calculus, McGraw – Hill N. Y.