MODULE1

Unit1 Prerequisites
Unit2 Mathematics of Counting
Unit 3 Permutation and Combination

UNIT1 PREREQUISITES

CONTENTS

1.0 INTRODUCTION

Themainprerequisitesforunderstandingthecontentofthisunitisaknowledgeof elementaryalgebra,settheory. In this unit, a set could be a well-definedlist, collection or class ofobjects.Theobjectivescomprisingthe setarecalledits elementsormembers. The object in a set could be anything such as set of numbers, set of letters of the alphabet, set of names of African countries etc. Note that set could be defined by listing its members, stating properties, etc. This unit will also discuss series.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state set and subsets of objects/numbers
- state and use sum of series
- use the binomial theorem to solve related problems
- usede Morgan's Law for two or more sets.

3.0 MAIN CONTENT

3.1 Set

AsetwillbedenotedbycapitallettersorsymbolssuchasX,Y,A,B,….anditselements willbedenotedbylowercaselettersx,y,a, b,…..

Example1.1 Tossacubicaldieonce.Therearesixpossiblenumbersthatcanappear.

We canwrite $\Omega=\{1,2,3,4,5,6\}$

WhereΩisasetconsistingofsixelements1,2,3,4,5,6 calledthe elementof thesetΩ.

Thereareessentiallytwowaysofspecifyaparticularset.Onewayifpossibleisbylistingitselementsasinexample1.1above.Theotherwayisbystatingpropertieswhichcharacterizetheelementsintheset.TheabovesetΩcanbewrittenas:

$\Omega=\{x\text{:isaninteger},1\leq x\leq 6\}$

IfxisanelementΩ,thenotation$x\in\Omega$meansthatxbelongstoΩ.Thenegationofthis assertioni.e.thestatementthatx doesnotbelongtoΩwillbedenotedby$x\in\Omega$

Thus,fortheabove example,$2\in\Omega$but$8\notin\Omega$.

Definition

Subset:AsetA isasubsetofasetif eachelementinAalsobelongsto Ω.

In example1 above,theset

$A = \{1, 3, 5\} =\{x:x\in \Omega$and xisodd$\}$ Is asubsetof Ω,thatis,eachelementof A isinΩ.

TwosetsA andBare calledequalif and onlyifthey containexactlythesameelements.

Throughoutthisbook,wheneverthewordsetisused,it willbeinterpretedtomeana subsetofagivensetdenotedbyΩ. ThesetwhichcontainsnoelementsiscalledtheNull set {} or Φ.

Definition

LetAbeaset.Theelementswhicharenot included inAalsoconstituteasubset.Thisis knownasthecomplementof A andisdenotedbyA^C.

As given in example1.1 above,ifA = {1,3,5}

$A^C=\{2,4,6\}=\{x:x\in \Omega$and xiseven$\}$.

Definition:

TwosetsA,Bdefinetworelatedsets.Oneoftheseisthesetofallelementswhich belongstoboth setsAand B.thisis calledtheintersectionof A andBandisdenotedby $A \cap B$.theotheristhe setof all theelementswhichoccurineitherA or Borboth.Thisis calledthe***union***ofA andBdenotedby$A \cup B$.

Example1.2

LetΩ={1, 2, 3, 4, 5, 6},A ={1, 3, 5}and B={2,3,5}

Then

$A \cap B$={3, 5},

$A \cup B$={1, 2, 3, 5}

AandBcontain3elementeachwhile$A \cap B$contains2elementsand$A \cup B$contains4 elements.

Notethatthenumberofelementsin$A \cap B$isnotthesumofthenumberofelementsinA andB.

Definition:*DifferenceofTwosets*

ThedifferentofAandBisthesetofelementswhichbelongtoAbutnottoBandis denoted byA/B

In example1.2 above A/B={x:x$\in$ A, x$\in$ B}$\in$ Ω

A/B={1}

B/A={2}

Theunionof two setsA Bcanbedividedintothreedisjointsets A/B,

$A \cap B$,B/A.

Thatis

$A \cap B$= (A/B)$\cap$(A$\cap$B)$\cap$(B/A)

Thenumberof elementinasetwillbedenotedbynA.Thus,

n(A$\cap$B)=n(A/B)+n(A$\cap$B)$\cap$ (B/A)

Since

(A/B),(A$\cap$B),(B/A)aredisjointsets

nA = n(A/B)+ n(A$\cap$B)

nB=n(B/A)+ n($A\cap B$),
n(A/B)= nA–n($A\cap B$),
n(B/A)=nB-n($A\cap B$),

hence,
n($A\cap B$)=nA - n($A\cap B$)+ n($A\cap B$)+nB- n($A\cap B$)=nA +nB–n($A\cap B$)
Note:A/B= $A\cap B^c$,B/A= $B\cap A^c$

For any setA,

$A\cap A^c = \Phi$,$A\cup A^c=\Omega$.

For anytwo setsA andB,wehavethefollowingdecomposition:

$B=B\cap\Omega= B\cap\ (A\cap A^c)=(B\cap A)\ \cap\ (B\cap A^c)$,

Since$A\cap B$and$A^c\cap B$aredisjoints,wehave

nB=n($A\cap B$)+n($A^c\cap B$)

deMorgan'sLaw

The laws are as follows :
For anytwo setsA andB,

i) $(A\cap B)^c=A^c\cup B^c$
ii). $(A\cup B)^c=A^c\cap B^c$

Proof by Examples:

1) Let Ω = {1, 2, 3, 4, 5, 6}, A = {2, 3} and B = {3, 4, 5}.
Show that $(A\cup B)^c = A^c\cap\ B^{c'}$.

Solution :
Ω = {1, 2, 3, 4, 5, 6}
A = {2, 3}

B = {3, 4, 5}

$A\cup B$ = {2, 3} ∪ {3, 4, 5}

= {2, 3, 4, 5}

∴ $(A \cup B)^c$ = {1, 6}

Also $A^c = \{1, 4, 5, 6\}$

$B^c = \{1, 2, 6\}$

$\therefore A^c \cap B^c = \{1, 4, 5, 6\} \cap \{1, 2, 6\}$

$= \{1, 6\}$

Hence $(A \cup B)^c = A^c \cap B^c$

2. If $\Omega = \{a,b,c,d,e\}$, $A = \{a,b,d\}$ and $B = \{b,d,e\}$. Prove De Morgan's law of intersection.

Solution :

$= \{a,b,c,d,e\}$

$A = \{a,b,d\}$

$B = \{b,d,e\}$

$(A \cap B) = \{a,b,d\} \cap \{b,d,e\}$

$(A \cap B) = \{b,d\}$

$\therefore (A \cap B)' = \{a, c,e\}$ ---------------------------------->(1)

$A' = \{c,e\}$ and $B' = \{a,c\}$

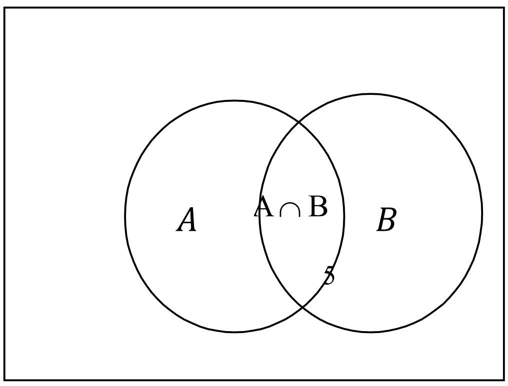
$\therefore A' \cup B' = \{c,e\} \cup \{a,c\}$

$A' \cup B' = \{a, c,e\}$ ----->(2)

From (1) and (2)

$(A \cap B)' = A' \cup B'$ (which is a De Morgan's law of intersection).

$A^c = (A \cap B)^c \cap (B/A)$,



B^c= $(A \cap B)^c \cap (A/B)$.

Thus,

$A^c \cap B^c = (A \cup B)^c$

SinceB/Aand A/Baredisjoint,Alternatively,

$(A \cup B)^c = \{ x : x \in A \text{ and } X \in B\} = A^c \cap B^c$

Ingeneral,if$A_1, A_2, \ldots A_n$are any nsets

Thus

$(A_1 \cap A_2 \cap \ldots \cap A_n)^c = A_1^c \cup A_2^c \ldots \cup A_n^c$.

3.2 Series

Asequenceisasetofnumbersoccurringin order,andthereisasimplerulebywhichthe termsareobtained.Forexample,1, x,x^2…isasequence.If thetermsof asequenceare consideredasasum,forinstance,$1+x+x^2$ +….Theexpressioniscalledaseries.A serieswitha finitenumberoftermsiscalledafiniteseriesotherwise itiscalledafinite seriesotherwiseitis calledaninfiniteseries.Thesummationisshown bythesymbol.

When the sumistakenfromthefirstterm (r =1) tothen[th]term (r =n).

Example

Theseries$1+x+x^2+\ldots+x^{10}$canbewrittenas $X^{r-1} = \frac{1-X^r}{1-X}$

Theabove seriesiscalledageometricserieswithcommon ratiox.thesum

$X^{r-1} = \frac{1-X^r}{1-X} = 1,$

1 – x

N, x = 1

Thecommon ratiois x.if–1<x<1,x^{n} → 0asn → α , thentheseriesconvergesto $\frac{1}{1-x}$

Theinfiniteseries

Example

Findthe sumof theseries

Solution

(1–p)$^{r-1}$=1+(1–p) +1 –p)^{2}+…….

Thecommonratiois1– p,from(1)letx=1–p,wehave

(1–p)$^{r-1}$= $\frac{1}{1-(1-p)} = \frac{1}{p}$

And

$$(1-p)^{r-1} = \frac{1-(1-p)^r}{1-(1-p)} = \frac{1-(1-p)^r}{p}$$

Exponentialseries

Thefunctiony=e^{x} iscalledanexponentialfunction.Thisfunctionisoneofthespecial functionsof analysisandcanbedefinedasthesumof an infiniteseries.Itis definedby

E^{x}= $1 + x + \frac{x^2}{2!} + \cdots + \frac{x}{n!} = \frac{x^r}{r!}$

Thenotationr! is calledfactorialrandisdefinedas R!=r(r – 1)(r-2) …..

For example
5! =5x4x3x2x1

Where0!=1and1!=1.R!definedabovehasnomeaningunlessritselfisapositive integer.

Thefactorialof any negativeintegerisinfinite.

GammaFunction

Itis easilyshownby directintegrationthatwhenmis aninteger

M!=

Providedthatm>-1.Itcanbeprovedthat(2)willalsohaveameaningforfractionm. for instance,itisknownthat

Sincem! =m(m-1)!.Bysettingm = 1wehave

2

3.3 TheBinomialTheorem

If nisapositiveinteger,

$(a+b)^2=a^n+{}^nC_1a^{n-1}B+{}^nC_2a^{n-2}b^2+\ldots.+b^n$

Where ${}^nC_r=\frac{n!}{(n-r)!r!}$. Using thesummationnotation,wehave

$(a+b)^n$using thesummationnotation,wehave

Example

Findthe sumof theseries

$S_n= P^n+{}^nC_{1p}^{n-1}(1-P)+{}^nC_{2p}^{n-2}(1-P)^2+\ldots(1-p)^n{}_nCrP+^{n-r}(1-P)^r$

Substitutingp and1-pfora,binthebinomialtheorem above,wehave

Thus,

GeneralizationoftheBinomialtheoremisthe***multinomial***theorem.Ifnisapositive integer

$(a_1+a_2+\ldots a_k)^n=\sum\ldots\ldots\sum\frac{n!}{N_1!\,n_2!\ldots.n_k!}a_1^{n1}a_2^{n2}\ldots a_k^{nk}$

Sumis overalln_1, $n_2\ldots..n_k$where$n_1+n_2+\ldots+n_k=n$.

3.4 ProductNotation

Theproductofthetermsofasequence$x_1,x_2\ldots,x_n$ canbewrittenas$x_1x_2x_3\ldots.x_n$.This productisshownbythe symbol.

Wheretheproductistakenfromx_1,(thefirstterm) tothenthterm, x_n.forexample,

$N_1!N_2!\ldots n_k!$and$a_{12}\ldots a_k$

Canbewrittenas

Thusthemultinomialtheorem canbewrittenas

Example

$(a_1+a_2+ a_3)^2 = \sum\sum \underline{2!}$
3

Wheren=2,k=3.Possiblevaluesofn1are

$n_1=2, n_2=0,n_3=0$

$n_1=0,n_2= 2,n_3=0$

$n_1=0,n2= 0,n_3=2$

$n_1=1,n2=1,n_3= 0$

$n_1=1, n_2==,n_3=1$

$n_1=0,n_2=1,n_3=$

hence,wehave

$(a_a+a_2+a_3)^2 = a_1^2 +a^2 + a^2_3+2a_1a_2+ 2a_{1a}3_{+2a}2_a3$

3.5 ExponentialFunctions

Thesearefunctioninwhichthevariableoccurintheindex,forexamplee$^{x}3^{2x}$ arecalled exponentialfunctions,Lety=e$^{h(x)}$then

For example,ify=e^{3x2} then

$\underline{dy}=6xe^{3x2}$
dx

Derivativeoflog$_e$h(x) Ify=log$_e$2x^3 then

$\underline{dy}=6x. \underline{1\ \ }= \quad \underline{3}$
dx2x^3 x^2

4.0 CONCLUSION

In this unit, we have stated the elements of set and subsets. We have also dealt with the use of de Morgan's Law for two sets. Also the sum of series were dealt with.

5.0 SUMMARY

In this unit, the following have been covered:

- Sets and subsets of objects/numbers
- Differences of two sets

- De Morgan's Law
- Series
- Exponential Series
- The Binomial Theorem
- Exponential functions
- Product Notation

6.0 TUTOR-MARKED ASSIGNMENT

1. Prove that if $A \subset B$ and $B \subset C$, then $A \subset C$
2. Prove de Morgan's first law using a Venn diagram
3. Let $\Omega = \{1,2,3,4,5,6,7,8\}$, A= $\{1,4,6\}$ and B= $\{4,6,7\}$ then find
   (i) $A \cap B$
   (ii) A^c
   (iii) $A \cup B^c$

7.0 REFERENCE/FURTHER READING

Harry Frank & Steven C. Althoen (1995). Statistics: Concepts and Applications. Cambridge University Press.

UNIT2 MATHEMATICS OF COUNTING

CONTENTS

1.0 INTRODUCTION

Probability haditsoriginin gamesof chancesuchasdiscandcardgames.The numberof definitionsyou canfindforitislimitedonlyby the numberof booksyoumaywishto consult.Probabilitycanbe definedas ameasureputonoccurrenceofarandom phenomenon. Probabilitytheoryisdevelopedasstudy oftheoutcomesof trailofan experiment.

2.0 OBJECTIVE

At the end of this unit, you should be able to:

- write down sample space of set of items
- state the 'event' in a set outcomes
- write down the possible outcomes in an event
- state the probability of a prime, even number or odd number when a die is rolled
- useclassical approach to calculate probability.

3.0 MAIN CONTENT

Definition
An experimentisaphenomenontobeobservedaccordingtoaclearlydefinedprocedure. Probabilities are numbers between 0 and 1, inclusive that reflect the chances of a particularphysicaleventsoccurring.Ifadieistossedonce,thepossibleoutcomesare1,2,3,4,5,6,let$\Omega=\{1,2,3,4,5,6\}$.ThenΩisasetconsistingofallpossibleoutcomeof tossingadieonce.Thissetisgivenaname,itiscalledasamplespace.
Example1.1

Writedownthesamplespaceforeachofthefollowingexperiments

(i) Tossacoin3timesandobservethetotalnumberofheads
(ii) Aboxcontains6itemsofwhich2aredefectives.Oneitemischosenoneafterthe other withoutreplacementuntil thelast defectiveitemsis chosen.We observe thetotalnumber ofitemsremovedfromthebox.

Solution

(i) Possibleoutcomesare:numberofheadsis0whenwe haveTTT,numberofheads is1whenwehaveHTTor THTorTTH,numberof headsis2whenwe have HHT,THH,HTHandnumberof headsare0,1,2,3.HencethesamplespaceΩ={0,1,2,3)

(ii) Totalnumberofitemsremovedis2ifwehave"thefirstisdefective(D)andthe secondisdefectiveD(Notethatthetotalnumberofitemsremovedcannotbe0or1)deno tedbyDD.Thetotalnumberofitemsremovedis3ifwehaveGDD orDGD (whereG denotedgooditem),andsoon.Thus,thesamplespace

Ω={2,3,4,5,6}

Definition1.2 Event

AnysubsetAof asamplespaceΩis calledanevent,whereA<Ω.

Example1.2

Thefollowingareexamplesof events

(i)Anoddnumber occurswhenadieisrolledonce

Ω={1,2,3,4,5,6},A={1,3,5}

(ii)Tossacoin3times,thetotalnumberofheadsobservediseven

Ω={0,1,2,3}A={0,2}

Definition1.3 TrialofanExperiment

A trialisasingleperformanceof anexperiment

Therearebasically twomethods forassigningprobabilitiestoevents.Theseare

(i) Relativefrequencyapproach
(ii) Classicalapproach

(i) RelativeFrequencyApproach

IfweareinterestedintheoccurrenceofaneventA,wecouldperformalarge numberof trialsanddefinetherelativefrequencyof A as

$$RF(a)= \frac{\text{Number oftrialsinwhichA occurs}}{\text{Totalnumberoftrials}}$$

TheratioRF(A)iscalledtheempiricalprobability.Ifthenumberoftrialsisverylarge RF(A)will(inmostcases0tendtoaparticularvaluecalledtheprobabilitythattheevent A willoccur,that isRF(A) is an estimateof P(A).theprobabilitythatA willoccur.

Example1.3

Supposeweareinterestedintheoccurrenceofa"6"whenwerollawellbalanceddie

(fair die) the die is then rolled 100 times and "6" appeared 15 times. The relativefrequencyof"6"isthus$\frac{15}{100}$whichisthe empiricalprobability of getting6

(ii)**ClassicalApproach:**

Intheaboveexample,therearesixwaysthediecanfallwhenitisrolledonce. Sowecandefinea theoreticalprobabilityby

$$P(6) = \frac{\text{number ofwaysofgettinga"6"whenrolledonce}}{\text{Totalnumberofpossibleoutcomeswhen rolledonce}} = \frac{1}{6}$$

Theclassicalapproachtoprobabilitygives

$$P(A)= \frac{nA}{n\Omega}$$

Thisdefinitionisvalidonly whenoutcomesareequallylikely.

Example1.4

A coinisrolledthrice.Thepossibleoutcomesassumedtobeequallylikelyare
Ω= {HHH, HHT,HTH,HTT,TTT,TTH,THT,THH}
LetAbetheeventthat2headsoccur.Then

A ={HHT,HTH,THH}
nΩ=8,nA= 3

hence

$$P(A)= \frac{3}{8}$$

Thisclassifiable approachwhenapplicable(thepossibleoutcomesareequally likely)has theadvantageof beingexact.Thustocomputeaprobabilityby usingtheabovedefinition onemustbeabletocount.

(i)nΩthetotalnumber ofpossibleoutcomesin thesamplespaceand

(ii)nA,thenumber ofwaysinwhicheventAcanoccur

equally likeoutcomesarealso calledequally probableoutcomes.If an eventcannot occur itsprobabilityis0,ifitmustoccuritsprobabilityis. thecomputationofnAandnΩiseasyifΩhasonlyafewpossibleoutcome,asthe number of possible outcomes becomes large, this method of counting all possible outcomesarecumbersomeandtimeconsuming.

Alternativemethodsofcountingmusttherefore bedeveloped.Forexampleifoneasks fortheprobabilityofgettingsumofnumbersshowingtobe30when7dicearerolledonemustd eterminehowmanydifferentwaysarepossibletogetthesumtobe30.Such possible waysinclude

(6,6,6,6,2,1,3) (2,5,2,6,6,3),(4,4,4,4,4,4,6)

Inthischapter weintroducenontechnical discussionoftechniquesofmathematics of countingfrequentlyneededinproblemsof findingnAandnΩ

SELF-ASSESSMENT EXERCISE1

1. A dieisrolledonce.Whataretheprobabilitiesof getting

(i) An evennumber
(ii) A primenumber
(iii) Anoffprimenumber
(iv) Anoddnumber
(v) An evennumber

FundamentalPrincipleofCounting

FirstLawofCounting

IfaneventA$_1$ canoccurinn$_1$ waysandthereafteraneventA$_2$ canoccurinn$_2$ ways,“bothA$_1$andA$_2$canoccurinthisorderinn$_1$n$_2$ways.

Example1.5

Rolladiefirstandthenacon

A$_1$canoccur in 6ways(1,2,3,4,5,or6)andA$_2$canoccurin2ways(H or T). Thus,bytheabovelaw,thereare

6x2=12

Possiblewaysfortheoutcomes

Theoutcomesare

(1,H) (1,T) (5,H) (5,T)
(2,H) (2,T) (6,H) (6,T)
(3,H) (3, T) (4,H) (4,T)

Ingeneral,ifaneventA $_1$ canoccurinn $_1$ indifferentwaysandiffollowingthisanevent

A $_2$ canoccurinn $_2$ differentways,andiffollowingthissecondevent,aneventA $_3$ canoccurinn $_3$ differentwaysandsoforth,thentheeventsA $_1$ andA $_2$ andA $_3$ ....AndA $_k$ can occurinthis orderinn $_1$ n $_2$ ....n $_k$ ways.

Example1.6

Ifadieisrolled10times.LetA $_1$ denotetheoutcomeoftheroll,i=1,2,....10.A $_1$ canoccurin6ways.

Thusthe numberofwaysA $_1$,A $_2$,....AndA $_{10}$ canoccuris 6^{10} possibleoutcome.

Thatis

$N\ \Omega=6^{10}$

SecondLawofCounting

Ifanevent A $_1$ canoccurinn $_1$ differentwaysandaneventA $_2$ canoccurinn $_2$ waysthen eitherA $_1$ or A $_2$ canoccurinn $_1$+n $_2$ differentways.

Example1.7

Letustossadieoracoinonce.leta $_1$ netheevent"thedieshowsanevennumberanda $_2$ betheevent"thecoinlandsheads".a $_1$ canoccurin3ways(2,4or6)anda $_2$ canoccur only once.
\thenumberof waysinwhichan evennumberorahead beobtainis

3+ 1=4

A $_1$ canoccur in 3waysA $_2$ canoccurin1way

ThereforeA $_1$ or A $_2$ canoccur in 4ways

Ingeneral,ifevents $A_1,A_2\ldots,A_k$ canoccurinn$_1\ldots n_k$ differentways,theneither orA_1

$+n_2+\ldots+n_k$differentways

Example1.8

Four peopleenterarestaurantfor lunchin whichtherearesixchars.In howmany ways cantheybeseated.

LetA_1,A_2,A_3,A_4denotetheevents“choiceofchairbythefourpeople”,thesuffix denotingorder of seating.Thefirst persontosit downhassixchoices.Hecandecidetositonanyofthesixvacantchairs.Therefore,thereare6differentwaysA_1 canoccur,afterthefirstpersonhasseated,thesecondpersoncansitonanyoftheremaining5chairs. Thereafterthethirdperson cansitontheremaining4chairs.
Thus,usingthenotationof lawofcounting

$N_1=6,n_2=5,n_3=4,n_4=3$
6x5x4x3=360ways

Theycanbeseated.

Example1.9

Howmany4digitsnumberscanbeformedfromthedigits0,1,2,3,4,5ifthefirstdigit mustnotbe0andrepetitionof digitsarenotallowed.

LetA_1,A_2,A_3, A_4,denotemevents:selectthefirstsecond, third andfourth digits respectively sincethefirstdigitscannot be0, thefirstdigitcaneeither1,2,3,4, or5 thereforeA_1 canoccurin5ways,havingchosenthefirstdigittheseconddigitcanbe selectedfrom theremaining5digits.therethefirstandseconddigitshavebeenchosen, thereremains4 digits.Thethird digitcan bechosenfromtheremaining4 digitsandthe fourthcanbechosenfromtheremaining3digits.Therefore, A_1,A_2,A_3 andA_4canoccur in5,5,4,3waysrespectively
Thusthere

5x5x4x3=300 numbers

Example1.10

Tencandidatesareeligibletofill4vacantposition.Howmanywaysarethereoffilling them?

LetA_1,A_2,A_3,A_4betheeventsdenotingcandidatesthatfillpositions1,2,3,4respectively.
Candidatefillingposition1canbeanyofthetendcandidates,thereforeA_1

canoccurin10ways.Followingthis,thenextposition2canbefilledby anyoftheremaining9 candidatessinceposition1hadbeenfilledbyonecandidate.ThereforeA2 canoccurin ways.Similarly,A3andA4canoccurin8 and7waysrespectively.Thus thereare10x9x8x7= 5040ways

SELF-ASSESSMENT EXERCISE2

1. Astudentistoanswerallthefivequestionsinanexamination.Itisbelievedthat thesequenceinwhichthequestionsareansweredmayhaveaconsiderableeffectonthe performanceofthestudent.Inhowmanydifferentordercanthequestion beanswered
2. If awomanhas 10blousesand6skirts, inhowmany wayscanshechooseadress assumingany combinationof blouseandskirtmatches
3. Ina studyof plants, fivecharacteristics are tobeexamined. If there are six recognizabledifferencesin eachof four characteristicsandeight,recognizable differencein theremainingcharacteristics.How many plantscanbedistinguished bythesefivecharacteristics?
4. A bus startswith6 peopleandstopsat10differentstops, howmanydifferent ways canthe6peopledepartif
   (i) Anypassengercandepartatany budstop
   (ii) Notwopassengerscanleaveathe samebus stop
5. Show that the number of ways of choosing r objects from n objects with replacementisgivenbyn”

4.0 CONCLUSION

In this unit, emphasis has been on the concept of probability, sample space, occurrence of events, laws of counting. Also, different ways of arranging objects.

5.0 SUMMARY

Two methods for assigning probabilities to events were discussed as relative and classical approaches. The fundamental principle of counting were discussed under the first and second laws. Permutation and combination of n distinct objects wherethe number of possible permutation of several objects gives the number of different ranking possible were also studied

6.0 TUTOR-MARKED ASSIGNMENTS

1. What is the probability of drawing either a heart or a face card (king, queen or jack) on one drawing form a 52-card deck?
2. Consider the following experiment and random variable: Roll an ordinary 6-sided die and count the spots on the uppermost face.
   (i) What is the value set for this random variable?

(ii) Let E_1 be "obtain a prime number." What is $P(E_1)$?
(iii) What is $P(E_2/E_1)$?
(iv) Are these two events independent? Why or why not?

7.0 REFERENCE/FURTHER READING

Harry Frank & Steven C. Althoen (1995). Statistics: Concepts and Applications. Cambridge University Press.

UNIT 3 PERMUTATION AND COMBINATION

CONTENT

1.0 INTRODUCTION

Apermutationisanarrangementofobjectsinadefiniteorder(calledorderedsample).Acombinationisaselectionof objectswithoutregardtoorder(unorderedsample)

A groupofobjects,withregardtopermutationandcombinationhasthreecharacteristics:

1. Theway theobjectsinagrouparearranged
2. Thekindofobjectsin thegroup
3. Thenumberof objectsof eachkindinthegroup

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state both the way objects are arranged and the kind of objects in a group
- define permutation and combination concepts
- state how many permutations can be formed in a given set
- state the distinct combinations of a given number of objects without replacement
- usebasic rules of permutations and combinations to solve related problems.

3.0 MAIN CONTENT

3.1 Permutations

Supposethatasetcontainnobjects.Weareofteninterestedinarrangingtheobjectsina definiteorder.

Twogroupscontainingnobjectaresaidtoformdifferentpermutationsiftheydifferin arrangements
Considergroups of lettersa,b,c, d.

(a, b,c,d):(b,c, d,a):(c,a, d,b)

Theyarealldifferentpermutationsbecausethearrangementoftheletterssisdifferentin eachgroup.

Example1.11

Howmanypermutationsoffour letterscanbeformedfrom thelettera,b,c, d. Toanswer this questionwereasonasfollows:

Sincearepermuting4 letters,foureventsareinvolved.LetA_1betheeventsdenotingthe lettertooccupytheithposition.A_1 canoccurin4ways,thatisthefirstlettercanbe eithera,b,c,ord.afterthiseventhastakenplace,A_2 canoccurinthreeways(thatis, afterhavingchosenthefirstletter,theletter occupyingthesecond position canbechosen from theremainingthreeletters.Afterthesecondevent,thereremainsonlytwoletters from whichoneisto bechosento occupythethirdposition.So A_3is2 anditremainsonly oneletter.

A_4canoccur onlyinoneway.Thus,thereare4x3x 2x1 =24 permutationofthefour letters.Permutationoffourobjectsfromfourobjectsiscalled permutation 4andis denoted by$^4P_4$

3.2 PermutationofnDistinctObjects

Considerasetconsistingofndistrictobjects.permutationofthissetconsistofnevents

A_1,A_2,….A_n,wheretheobjectoccupyingtheithpositionistheoutcomeofA_1,I=1,2,…..nA_1canoccurinn$_1$ways,A_2canoccurinn-1waysandsoon.

Thustherearen(n-1)(n-2)…x3x2x1permutationofthenobjects.Thenumberof permutationsof ndistinctobjectsis n!.

Example1.12

If n ballsaredistributedatrandomintorboxes, inhowmanywayscanthis bedoneif

(i) Eachof theballscangointo anyof therboxes

(ii) Noboxhasmore thanoneball.

Solution

(i)

LetA$_1$,A$_2$….A$_n$wheretheobjectoccupyingtheithpositionistheoutcomeof A$_1$I=1,2,…..n.A$_1$canoccurinn$_1$way, A$_2$canoccurinn-1ways andsoon.

(ii) Thefirstballcangointoanyoftherboxes,thesecondanyoneoftheremaining(r-1) boxesetc,soinallthererarer (r-1) (r-2)…(r-n+1)differentways(n≤r).

Example1.13

Howmanypermutationofthreeletterscanbeformedfromthelettersa,b,c,d,e.abc, bae,cba,cdb,…..are few of therequired permutation.Sincethepermutationconsistof 3 letters,3eventsareinvolved,A$_1$,A$_2$ A$_3$.Thefirstlettercanbeanyofthe5letters, thereforeA$_1$can occurin5ways.SimilarlyA$_3$canoccurin3ways. Thus,thereare 5x4x3=60

Permutationof threelettersfrom5letters.Thisis denotedby$^5P_3$(5permutation3).

Definition1.4

Thenumberofpermutation of r(r≤n)objects fromndistrict objectsiscalled ^{n}npermutationrand is denoted by^np_r
Itcaneasilybeshownthat

^nP_r=n(n-1)(n-2)…(n-r+1)

toseethiswecanargueasfollows.Sincewearepermutingrobjects,therearerevents

A$_1$,A$_2$…..A$_3$involved.
Thefirstpositioncanbeoccupiedby anyofthenobjects,followingthisthesecond positioncanbeoccupiedbyany of theremainingn-1objectsandsoon. ThereforeA$_1$, A$_2$,…A$_r$ can occur in n, n-1, n-2, …n-(r-1) ways respectively. thust h e numberof permutationis

But

n(n-1)(n-20…(n-r+1)

n!= n(n-1) (n-2)…(n-r+1)(n-r)(n-r-1)...1 (3.2.1)

(n-r)! =(n-r)(n-r-1)…2.1

$\frac{n!}{(n-r)!} = n(n-1)(n-2)\ldots(n-r+1).$

Therefore

$^nP_r=n(n-1)(n-2)\ldots(n-r+1)$

$^nP_r=\frac{n!}{(n-r)!}$

In example1.13above,n=5,r =3

$^5P_3=\frac{5!}{(5-3)}=\frac{5!}{2}= 5x4x3=60$

Example1.14

Agroupofstudentsconsistof5menand3women.Thestudentsarerankedaccordingto theirperformanceinaquizcompetition.Assuming notwostudentsobtainthesamescore

(i) Howmanydifferentranking arepossible?

(ii) Ifthemenarerankedjustamongthemselvesandthewomenamongthemselves, howmanydifferentrankingsarepossible?

Solution

(i) Apossiblerankingcorrespondsto apermutationofthestudents.Thenumberof possiblepermutationof the 8studentsgivesthe number of differentranking possible.Thustheansweris

$^8P_8= \frac{8!}{(8-8)!}=8!= 40320.$

(ii) Thereare $^5P_5$ =5!=120possiblerankingsofthemenand $^3P_3$=3!=6possible rankingsofthewomen.Itfollowsfrom thefundamentalprincipleofcounting thatthereare

$^5P_5 x\, ^3P_3$=5! X 3! =72- possiblerankings

Example1.15

Fourdigitsnumbersaretobeformedusinganyofthedigits1,2,3,4,5,6,(Norepetition of digitisallowed).

(i) Howmanyfour digitnumbercanbeformed

(ii) Howmany4digitnumbersgreaterthan3000canbeformed?

(iii) Howmany4-digitevennumberscanbeformed?

(iv) Howmanyof theseeven4-digitnumbersaregreaterthan3000?

Solution

(i) Thisispermutationof4digitsfrom 6digits.Thereforethedifferentpermutationis

$^6P_4 = \frac{6!}{2!} = 360.$

Alternatively,wecanreasonasfollow:

Thefirstdigittobeselectedcanbeanyofthesixgivendigits,son_1 =6.Thesecond digitstobeselectcanbeanyoftheremaining5digits(sincenoreparationisallowed)so n_2=5.Similarly,n_3=4 andn_4= 3.Thus,theansweris
N_1x n_2xn_3xn_4=6x5x4x3=360

(ii) Sincethenumbermustbegreaterthan3000,thefirstdigitmustbechosenroom3 ,4,5or6son_1=4.Theseconddigitcanbeanyoftheremaining5digits,son_2=5similarly, n_3=4,n_4=3.Thus,thereare

4x5x4x3= 240

4digitnumbers greaterthan3000thatcanbe formedusingthedigits1,2,3,4,5,6,

(iii) Anumbercanbedefinedtobeanevennumberofthelastdigitofthenumber iseven.Goingby thisdefinition,wesee thattherequirednumberisevenifthe lastdigitis2,4,or 6. Sincethereisarestrictionon thelastdigit.Wehaveto selectthelastdigitfirst.Thelastdigitcanbeselectedfrom2,4or6,sowe have3choices. Havingchosenthelastdigit,wecannowselectthefirst, secondandthirddigit.Afterchosenthelastdigitsthereremains5digits,son_1=5similarly,n_2= 4,n_3=3.Thus,theansweris 5x4x 3x3=180

(iv) Thelastdigitmustbe2,4or6 andthefirstdigitmustbe3,4,5or 6. Ifthelast digitsis2, thenthenumberofwaysofselectingthefirstdigitis4.However,if thelastdigitisselectedis2or6thenumberofwaysofselectingthefirstdigit is3(thatis3,5or6,3,4or 5)

Case1: n_4=1n_1=4n_2=4, n_3=3

Thenumberof 4 digitsnumbersthatcan beformedinthis caseis

4x4x3x1=48

CaseII: n_4=2, n_1=3n_2=4,n_3=3

Thenumberof 4 digitnumbersthatcanbeformedinthis caseis

3x4x3x2=72

Hence,thetotalnumbersofeven4digitnumbersgreaterthan3000thatcanbeformedis

48+72=120.

Example1.16

TheletterslettersA,B,C,DandEareplacedatrandomtoformafiveletterssword(without repetition).Howmanyways canaword beformedsuchthat

(i) D directlyfollowsA.

(ii) A and Dfollow eachother

(iii) A,D andEfolloweachother

Solution

(i) For Dto directlyfollowA,wemustalwayshaveADappearingin thewordso formedADcanberegardedasalettersothatwenowhavetheletttersB,CE andAD.Thenumberof waysofrearrangingtehsenewfour lettersis 4! Thus thereare4! =24waysof formingawordsuchthatD directlyfollowsA

(ii) AandDfolloweachother;eitherwehaveADorDaeachhaving4!Ways.Thus thereare2x4!Waysofformingawordsuch thatAand follow eachother

(iii) ADEcanberegardedasalettersothatwenowhavethelettersB.CADE.Thereare3!Permutationofthenew3letters.ADEcanalsobepermutedin3! Ways,thatis

ADE,AED,DAE,DEA,EAD,EDA.

Thusthereare

3! X 3! =36ways

3.3 PermutationofIndistinguishableObjects

Considernobjectswheren_1 areoftype1,oftype2,….$N_k$oftypek.,inshownmany ways canthen object bearranged.

For exampleinhowmany wayscanthelettersof thewordbook bearranged.heren=4,k

=3,n_1=1,n_2=2,n_3 =1.FirstgivethetwoO'ssuffixesb$o_1$$o_2$k.thentreatingtheO'sas different,the4lettersmaybearrangedin4!Ways.Ineverydistinctarrangement,the2O'smay berearrangedamongstthemselvesin2!Wayswithoutalteringthepermutation for instanceo$_1$bo$_2$karethesame whenthesuffixesareremoved.

Therefore,thenumberof permutationsof thelettersof thewordbookis

$$\frac{4!}{2!}=6$$

Book,ookb,oobk,obok,okob, koob.

Ingeneral,thenumberofwaysinwhichnobjectswheren$_1$areoftype1,n$_2$oftype2…, n$_k$of typek canbearrangedis givenby

$$\frac{n!}{n_1!n_2!...n_k!}:n+_n2+\dots+_nk$$

Example1.17

Howmanydistinctpermutationsarethereof thelettersof thewordTelevision?
Thetenletterstobepermutedconsistof2e's,2i's,IT,I,v,s,o,n.thusthenumberof distinctpermutationis

$$\frac{10!}{2!\,2!1!1!1!\,1!\,1!1!}=\frac{10!}{2!\,2!}$$

3.4 Combinations

Definition1.5

Twogroupsaresaidtoformdifferent"combination"iftheydifferinthenumberof any kindof object in the groups. Consider a group of 4 letters a, b, c, d. thecombinationabcd,bcda,cadbareidenticalcombinationseachofthem containsthe samenumberofa,b,c,d: oneb,onecandoned.

Thecombinationsof the4letterstaken3 ata timeare:

Abc,acd,abd,cbd

Therefore,thereare4distinctcombinationforthreelettersfromthefourletters.Each of tehsecombinationshas 3! =6 permutation.Forinstance.

abc	=	abc,	acb	cab	cba	bac	bca
acd	=	acd	adc	cad	cda	dac	dca
abd	=	abd	adb	bad	bda	dab	dba

cdb= cbdcdbbcdbdcdbcdcb

Thenumberof distinctpermutationis $^4P_3$=24

Thenumberof distinctcombinationsis 4. Therefore.

Numberofcombinations=$\frac{\text{numberofpermutations}}{3!}$

Thenumberofdistinctcombinationof4objectstaken3atatimeisdenotedby $^4C_3$. Thus,

$^4C_3 = {}^4P_3/3!$

Ingeneral,

$^nC_r = {}^nP_r/r! = \frac{n!}{(n-r)!r}$

Theorem1.1

Thenumberofdistinctcombinationsofnobjectstakenratatime(thatisthenumber ofwaysofchoosingrobjectsoutof n, disregardingorderand withoutreplacement)is givenby

Proof:

Thereare $^nP_r = \frac{n!}{(n-r)!}$permutationsofnobjectstakenratatime.Ifwedisregard order amongthe r objects, there are r! permutation thatwill give the same combination.Thereforethe number of combinationsisthenumberof permutation dividedbyr! thus.

$^nC_r = \frac{^nP_r}{r!} = \frac{n!}{(n-r)!r!}$ -

Example1.18

A clubconsistof15members.Inhowmanyways canacommitteeof3to bechosen?

Solution

Thiscan bedonein $^{15}C_3$ways

$^{15}C_3 = \frac{15!}{3!\ 12!} = \frac{15 x\ 7 x\ 13}{3}$=455ways

Example1.19

Aclubconsistof10menand5women,inhow manywayscanacommitteeof 6 consistingof4menand2womenbechosen.The4mencanbechosenfromthe10 menin ${}^{10}C_4$ways,the2womencanbechosenfromthe5womenin ${}^{5}C_2$ways.Hence thecommitteecanbechosenin(bythefundamentalprinciplef counting)

$${}^{10}C_4 \times {}^{5}C_2 = \frac{10!}{6!4!} X \frac{15!}{2!3!} = 2,100 \text{ways}$$

Exmaple1.20

Supposewehaveaboxcontainingnballsofwhichrareblackandtheremaining white.Arandom sampleofsizekisselectedwithoutreplacement.Inhowmany ways canthesamplebeselectedsuchthatit containsxblackballs

Thexblackballscanbeselectedin ${}^{r}C_x$waysandk-xwhiteballsnbeselectedfromn-r whiteballs.Thus,thereare

$${}^{r}C_x X\, {}^{n-r}C_{k-r}$$

ways of selectingasamplesuch thatitcontainsxblackballsand k-xwhiteballs

Example1.21

Acommitteeof4menand2womenisselectedfrom10menand5women.Iftwoof themenarefeudingandwillnotserveon thecommitteetogether,inhowmanyways canthecommitteebeselected

Solution

Thenumberof waysofselecting4menand2womenis

$${}^{10}C_4 \times {}^{5}C_2 = 2100$$

The numberof waysof selectingthecommitteesuchthatthe twomenare inthe committeeis

$${}^{8}C_2 \times {}^{5}C_2 = 280$$

Hence,thenumberofdifferentcommitteesthatcanbeformedsuchthatthetwomen arenotin thecommitteetogetheris

2100–280=1820

Anothermethodistoconsider3cases

CaseI: Thetwomensay,A andBarenotinthecommittee

$${}^{8}C_4 \times {}^{5}C_2 = 700$$

CaseII: A isthecommitteebutnotB:

$^{8}C_4 x\, ^{5}C_2$=560

CaseIII:B isthecommitteebutnotA

$^{8}C_4 x\, ^{5}C_2$= 560

Thus,thetotalnumberof waysthecommitteecan beselectedis

700+ 560+560= 1,820

Example1.22

Howmansubsetscanbeformed,containingatleastonememberfromasetof nelement

Solution

Thereare $^{n}C_{k}$subsetsofsizek thatcan beformed,thetotal numberofsubsets containingisleastonememberis

Example1.23

Astudentistoanswer5outof8questioninanexamination.Howmanyifhemust answerat least2 ofthefirstfor 4questions?

Solution

(i) Bythecombinationlawtheanswerof $^{8}C_5$= 56
(ii) Possiblechoicesare(2,3)(3,2)(4,1)where(2,3)meansanswer2questionsfrom thefirst4questionsand3fromtheremaining4 questions.Thenumber ofways of doingthisis $^{4}C_2 x\, ^{4}C_3$.

Thustheansweris

$(^{4}C_2 x\, ^{4}C_3)+ (^{4}C_3 x\, ^{4}C_2)+(^{4}C_4 x\, ^{4}C_1)$=52

3.5 Partitioning

Dividingapopulationorasampleofnobjectsintokorderedpartsofwhichthefirst

containrobject,thesecondr2 objectsandsooniscalled*orderedpartition.*Ifthe divisionisintok***unorderedparts,***thenthepartitionissaidtobeunordered.For examplesupposeaclasscontains15studentsandwewanttodividetheclassinto3 tutorialgroups of5 each. Threelecturersare availableandeachisto takeeachgroup. Inotherwise,wewanttodividethe15studentsinto3orderedgroups(A,B,C)thisis anorderedpartitionsincethereare
3! =6ways

Thelecturerscanbeassignedtotakeanypartition,forinstancegroups(A,B,C)can betakenby

(L_1,L_2,L_3)or (L_2,L_1,L_3)or(L_1,L_3,L_2)
Or (L_2,L_3,L_1)or (L_3,L_2,L_1)or (L_3,L_1,L_2)
WhereL_1meanslecturerIad(L_1,L_2,L_3)meansL_1takesgroupsA,L_2takesgroupB, L_3takesgroupCandsoon
Thereare $^{15}C_5$waysofselectingthosestudentsthosestudentstobeingroupA,followingthis,thereare10studentsleftandsothereare $^{20}C_5$selectingthosetobein thesecondgroup, thereforebythefundamentalprincipleofcounting,thereare

$$^{15}C_5 \times {}^{10}C_5 \times {}^{5}C_5 = \frac{15!}{10!\,5!} \times \frac{10!}{5!5!} \times \frac{5!}{5!} = \frac{15!}{5!\,5!\,5!}$$

Orderedpartitions.Thenumberofwaysinwhichnobjectcanbedividedinto***ordered parts***of whichthefistcontains$r_1$objectsthe second$r_2$objectsandsoonis

$$^{n}C_{r1} \cdot {}^{n-r1}C_{r2} \cdot {}^{n-r1-r2}C_{r3} \cdots {}^{n-r1-r2-\ldots-rk-1}C_{rk}$$

$$= \frac{n!}{(n-r_1)!\,r_1!} \cdot \frac{(n-r_2)!}{r_2!(n-r_1-r_2)!} \cdot \frac{(n-r_1-r_2)!}{(r_3!)n\,r_1-r_2-r_3)!} \cdots \frac{(n-r_1-r_2\ldots r_{k-1})!}{R_k!(n-r_1-r_2\ldots r_{k-1}-r_k)!} = \frac{n!}{r_1!r_2!r_3!\ldots r_k!}$$

Where$n=r_1+r_2+\ldots+r_k$

Example1.24

Inhowmanywayscanthreecommitteesoffive,threeandtwopersonsbeformedfrom10persons.

Weseekthenumberoforderedpartitionsof the10persons

Thisisgivenby

$$\frac{10!}{5!\,3!2!} = 2520$$

Example1.25

Inhowmany wayscan9toys bedividedamongthreechildrenifeachgets3toys

If thetoysarenumbered1through9, thepartition{(1, 2, 3), (4,5,6),(7,8,9})means childA getstoys1, 2,3childBgetstoys(4,5,6)whilechildCgetstoys8, 8, 9.We distinguishbetween{(1,2,3),(4,5,6)(7,8,9)}and{(4,5,6),(1,2,3),(7,8,9)so theseareorderedpartitions.

Thusthereare

$$\frac{9!}{3!\,3!3!}=1680\text{ways}$$

Whenr1=rjforI=j, wecandistinguishbetweenorderedandunorderedpartitionfor examplepartitionasetconsistingof8objectsnumbered1to8into3partsofwhich thefirst contains2objects,thesecond4 objectsand thethird2 objects.A partitionis

{(1,2),(3,4,5,6),(7,8)}.

In an orderedpartitionwedistinguishbetweenthepartition

{(1,2),(3,4,5,6),(7,8)} and{(7, 8),(3,4,5,6),(1,2)}
Buttheyarethe samefor unorderedpartition.Therefore,the number of unordered partitionis

$$\frac{8!}{2!\,4!2!}.\frac{1}{2!}=210$$

Example1.26

Inhowmanywayscanafamilyof9divideitselfinto3groupssothateachgroup contains3persons?

Solution

Weareseekingfor unorderedpartitions$r_1= 3$, $r_2=3$ $r_3=3$

Thenumberof unorderedpartitionsis

$$\frac{9!}{3!\,3!3!}\ \frac{1}{3!}=280$$

Sincethethreepartscontainthesamenumberof objects

Thesamerelationshipthatexistsbetweenpermutationandcombinationexistbetween orderedandunorderedpartitionofaset.

Example1.27

Inhowmanywayscanafamilyof10bedividedintothreegroups,onecontaining4 andtheothers3?

Solution

$R_1=4, r_2=, r_3=3$

Thenumberof orderedpartitionsis

$\frac{10!}{4!\,3!3!}=4,200$

Andthenumberofunorderedpartitionis

$\frac{10!}{4!\,3!3!}\frac{1}{2!}=2,100$

Sinceonlytwoofthethreepartscontainthe samenumberof objects

3.6 SelectionofNon distinctObjects

Thenumberofwaysrdistrictsballscanbedistributedintoncellsinr.thenumberof waysifaspecifiedcellcontainsexactlykballs(k=0,1,2…r)is $^{r}C_k(n-1)^{r-k}$

Thatistheballscanbechosenin $^{r}C_k$waysandtheremainingr-kballscanbepacedintotheremainingn-1cellsin$(n-1)^{r-k}$ ways

Now,supposetheballsarenon-district(indistinguishable)wecanonlytalkabout numberofballsintheithcells
LetX$_1$,x$_2$…, x$_n$denotethenumberofballsintheithcell,then

$x_1+x_2+\ldots+x_n=r$

Thenumberof distinctdistributioninwhichnocellremainsemptyis

$^{r-1}C_{n-1}$

toseethisletusassumethatthernon-distinctobjectsarelined by andbarsusedto dividetheminto groups.Ther ballsr-1spaceofwhichn-1aretobeoccupiedby bars. For exampleifr=9andn= 5,wehave

0∩0∩0∩0∩0∩0∩0∩0∩0

Thus0/00/0/000/00correspondsto
$X_1=1,x_2=2,x_3=1,x_4=3,x_5=2.$

Andthereare $^8C_4$possibledistributionofthebars.
anotherpossibledistributionis000/0/00/00
If $x_1 \geq 0$,thatis cellscanremainempty,thenumber of non-negativesolutionsof

$X_1+X_2+\ldots+x_n=r$

Is thesameasthenumber of positivesolutionsof

$Y_1+y_2+\ldots+y_n=r+n$

Wherey1= x_1+1.Thus,thereare

$$^{r+n-1}C_{n-1} \qquad (1.2)$$

Distinctsolutionsatisfying$x_1+x_2+\ldots+x_n=r$

Thenumberofdistinctdistributionisthenumberofwaysn–1spacescanbeselected outofthen+r-1spaces

ApplicationtoRuns

Definition

A runis any orderedsequenceof elementsof twokinds

For example, by a run of wins we mean a consecutive sequence of wins. The sequenceWWWLWWLLWLWWgives4 runs ofwins.Thefirstrun islength3,the secondrunoflength2,thethird oflength1 andthefourthof length2.

Suppose nowthatwe havetwoletters(W andL)nnon-distinctletters(L)andm non-districtletter(W).thetotalnumberofdistinctorderingsofWandLis$^{(n+m)}C_{n-}{}^{r}$ rruns ofWisequivalenttoarrangingtheletters Wintorcellsnoneofwhichisempty.If therearerruns onW,
ThenumberofLrunisnecessarilyr+1,r-1or.Thusfrom(1.1)wehave$(^{m-1)}C_{r-}$

1distinctwasofhavingr runsof W.hencethereare.

$^{(m-1)}C^{r-1}\ ^{(n-1)}C_r$waysofhavingrruns ofWand (r+ 1) runsofl… A sequencerepresentingr runsofWis

LL…	WW…W	LL…	WW…W	…	WW…	L..L
Y_1	X_1	Y_2	X_2		X_r	Y_{r+1}

Where$y_1+y_2+\ldots y_{r+1}=n, y_i\geq x_1+x+2++\ldots+x_r=m, x_i\geq 0$.

Let$y_1+1, y_i=y_i$,I =2,…,r,$y_{r+1}=y_{r+1}+1$

Thenumberof non-negativesolutionto

$Y_1y_2+\ldots+y_{1+1}=n$, $(y_i\geq 0)$

Is thesameas numberofpositivesolutionto

$Y_1+y_2+\ldots+y_{r+1}=n+2$

Thus,thenumberof outcomesthatresultnr runs is $^{n+1}C_r$

Hence,$^{n-1}C_r{}^{m-1C}{}_{r-1}$isthetotalnumberofwaysof havingrruns ofW

SELF-ASSESSMENT EXERCISE3

i. Howmany4digitsnumberscanbeformedfromthedigits0,1,2,3,4,5,6,7,8,9 ifthefirstdigitmustnotbe0andrepetitionsisallowed.howmanyofthese numberare(i)even(ii)less than5,000?
ii. A codeconsistof fivesymbols. Thefirstthreesymbolsarelettersandthelasttwo aredigits.Howmanycodescanbemadeifnoletternordigitisrepeatedinany codeword?
iii. Supposenobjectsarepermutedatrandomamongthemselves.Inhowwayscan this bedonesuch thatkspecifiedobjectsoccupykspecifiedpositions
iv. Howmanydistinct arrangementsarethereofthelettersofword(1)University(ii)biology (iii)Mississippi
v. Thenumber1,2,3…narearrangedinrandomorder.inhowmanywayscanthis bedonesuchthat
   (i) Q and 2follow eachother
   (ii) 2,3 and4followeachotherinthatorder
vi. Showthatthenumberofdistinctorderedsampleofsizerthatcanbedrawnfrom a populationwithnobjectsis
   (i) N^rifsamplingiswithreplacementand
   (ii) n1ifsamplingiswithoutreplacement

n –r!

vii. Atotalofnballsarerandomlyplacedintoncells.Inhowmany wayscanthisbe done.Inhowmanywaysif eachcellis occupied

viii. Awarehousehas6differentcontainerstobedistributedamong10retailers.In howmanywayscanthisbedone?Howmanywayifnoretailerreceivesmore than onecontainer?

ix. Provethefollowingidentities

a. $^{n}C_{x} = {}^{n}C_{n-x}$ (b) $^{m+1}C_{x} = {}^{n}C_{x} + {}^{n}C_{x-1}$

x. (a)In howmany wayscan4boysand2girlsbearrangedtositina row? (b) Inhowmany waysifonly theboysmustsittogether?

(c)In howmany waysiftheboyandnogirlmustsittogether? (d) Inhowmany waysifno boyandno girlmustsittogether?

xi. Inhow manywayscanafootballteambeselectedfrom15players.Inhowmany waysif6 particularplayersmustbeincludedin theteam?

xii. Fromaboxcontaining5red,4whiteand3blackmarbles,threemarblesare drawnoneaftertheotherwithoutreplacement,howmany ways canthis bedoneif

(i) allarewhite
(ii) 2rewhiteand1isred
(iii) atleastoneisblack

xiii. A pairof diceisrolledonce.In howmany wayscan

(i) Thesumofthetwonumbers appearingexceeds8
(ii) Themaximum of thetwonumbersisgreaterthan4
(iii) Theminimum of thetwo numbersis greater than4

xiv. Adisciplinarycommitteeoffouristobechosenfromsixmenandfivewomen. Oneparticularmanandoneparticularwomanrefusestoserveiftheotherperson is onthecommittee.Howmanycommitteesmay beformed

xv. Elevenpeopleareto travelintwocars-salonandstationwagon.Thesaloonhas4 setsandthestationgoing7seats.Inhowmany ways canpartybesplitup?

xvi. In how many ways can a committee of 6 composing of 3 full professors, 2 associateprofessorsand1 seniorlecturerbeselectedfrom5 fullprofessor,10 associateprofessorsand 20seniorslecturers

xvii. A boxcontain12ballslabeled1,2, 3… 12,supposearandom sampleofsize4is selected.Inhowmany wayscanthesamplebeselectedifballslabeled2,3,are among thefourselected.

xviii. Supposearandomsampleofsizerisdrawnfromapopulationofnobjects.In howmany wayscanthisbedone ifkgivenobjectmust be includedinthesample and(a)samplingiswithoutreplacement(b)Samplingiswith replacement

xix. I bought2 ticketstoalotteryforwhichnticketsweresoldand4 prizestobegiven inhowmany wayscantheticketsbedrawnsuchthat1winatleastaprize?

xx. Acommitteeof8istobeformedfrom 10couples(10menand10women).Inhow many wayscanthe committeebeformed ifnohusbandservesonitwithhiswife.

4.0CONCLUSION

The arrangement of objects in different ways had been stated in permutation and

combination. Permutation is characterized by the identity of the elements it includes and by the order in which they appear. It was deduced that by the extended principle of multiplication, the number of permutations of all N objects must be
$N(N - 1)(N - 2) \ldots 1 = N!$

5.0 SUMMARY

In this unit, we have discussed permutation and combination, basic rules of permutations and combinations and how to solve related problems using them.

We also studied a combination as a selection of distinct objects without regard to order. It was also noted that the total number of permutations is therefore equal to the number of combinations multiplied by the number of ways to order each combination.

6.0 TUTOR-MARKED ASSIGNMENT

1. Linesaredrawntopassthroughsixpoints.Inhowmany wayscanthisbedoneif eachlinepassesathroughonly twopoints?
2. Interchangesmay occurbetweenanytwo of then chromosomesof acell
3. (a) Inhowmany wayscanexactlyoneinterchangeoccur?
   (b) Inhowmany wayscanexactlykinterchangesoccur?
4. If n=5,inhowmanywayscanatmostthreeinterchangesoccur?
5. Provethat $^{(n1-n2)}C_k = {}^{n1}C_r \; {}^{n2}C_{k-r}$ where $k \le n_1, n_2$
   HINT: Selectkobjectsfrom$n_1 + n_2$objectsof$n_1$of type1 and$n_2$of type II
6. Acompanyisconsideringbuildingadditionalwarehouseatnewlocations.There aretensatisfactorylocationandthecompanymustdecidehowmany and which onestoselect.Howmany choicesarethere?
7. Aretheremoresamplesobtainableinfivedrawsfrom10objectswithreplacement than 12objectswithoutreplacement?
8. Eachoffiftyitemsistestedandfoundtobedefectiveornon-defective.Howmany possibleoutcomesarethere?

7.0 REFERENCE/FURTHER READING

Harry Frank & Steven C. Althoen (1995). Statistics: Concepts and Applications. Cambridge University Press.