

SOLUTION TO ACTIVITIES

Module 1, Unit 1, Activity 1

Let y , the solution be defined by

$$y = c_0 + c_1x + c_2x^2 + \dots = \sum_0^{\infty} c_m x^m$$

With y' and y'' given by $y' = c_1 + 2c_2x + 3c_3x^2 + \dots = \sum_1^{\infty} m(m-1)c_m x^{m-1}$;

$$y'' = 2c_2 + 3 \cdot 2c_3x + 4 \cdot 3c_4x^2 + \dots = \sum_2^{\infty} m(m-1)c_m x^{m-2}$$

Then substitute for y , y' and y'' in $y'' - 3y' + 2y = 0$ you obtain

$$\sum m(m-1)c_m x^{m-2} + 2 \sum c_m x^m = 0 \text{ sum of coefficients of } x_s \text{ is the same as } B_s \text{ as}$$

$$B_i = 0, i = 0, 1, 2, \dots$$

$$B_0: 2 \cdot 1c_2 - 3c_1 + 2c_0 = 0 \text{ up to } B_s \text{ which is } (s+2)(s+1)c_{s+2} - 3(s+1)$$

$$c_{s+1} + 2c_s = 0, s = 0, 1, 2, \dots \text{ taking } c_0 \text{ and } c_1 \text{ as arbitrary constants then the } B_0 \text{ to}$$

$$B_s \text{ becomes } c_2 = \frac{3}{2}c_1 - c_0, c_3 = \frac{7}{3 \cdot 2}c_1 - c_0 \text{ etc.}$$

Module 2, Unit 1, Activity 1

$$\beta(p, q) = 2 \int_0^{\frac{\pi}{2}} \sin^{2p-1} \theta \cos^{2q-1} \theta d\theta$$

$$\text{Let } 2p - 1 = \alpha \text{ then } p = \frac{\alpha+1}{2};$$

$$2q - 1 = 0 \text{ then } q = \frac{1}{2}$$

$$\therefore \beta\left(\frac{\alpha+1}{2}, \frac{1}{2}\right) = 2 \int_0^{\frac{\pi}{2}} \sin \alpha \theta d\theta$$

$$\text{That is } \int_0^{\frac{\pi}{2}} \sin \alpha \theta d\theta = \frac{1}{2} \beta\left(\frac{\alpha+1}{2}, \frac{1}{2}\right)$$

Module 2, Unit 2, Activity 2

General Solution for $8x(1-x)y'' + (4-14x)y' - y = 0$ is

$$A(1-x)^{-\frac{1}{4}} + B\sqrt{x} F\left(1, \frac{3}{4}, \frac{3}{2}; x\right)$$

Module 2, Unit 3, Activity 2

The prove of $\int_0^{\frac{\pi}{2}} J_0(z \cos \theta) \cos \theta d\theta = \frac{\sin z}{z}$ has been solved inside the manual, so check it up!

Module 3, Unit 2, Activity 2

From $L(f(\mu t)) = \frac{1}{\mu} F(s/\mu)$ and $L(\sin t) = \frac{1}{s^2+1}$

$$L(\sin \mu t) = \frac{1}{\mu} \left(\frac{1}{(s/\mu)^2 + 1} \right) = \frac{1}{\mu} \frac{z}{s^2 + \mu^2} = \frac{\mu}{s^2 + \mu^2}$$