MODULE 1 BASICS ISSUESIN MATHEMATICS EDUCATION

UNIT 1 AIMS AND OBJECTIVES OF TEACHING MATHEMATICS

CONTENTS

1.0 INTRODUCTION

Everyyear, atthebeginningofthesemester,Iaskmygroupofstudent-teachers“whydoweteachmathematics?Whatdowewanttoachievein our mathematics lessons?” These were the questions which I askedmyselfinjoiningtheprofessionasa mathematicsteacher.In fact,theyaregoodquestionswhichallteachersshouldaskthemselves fromtime totime in their daily practice. Different teachers may have different answerstothesequestions.Somepossibleanswerstothefirstquestionare “mathematics is important and useful in our daily life”; mathematics is the basis for other subjects such as science and engineering”;“mathematics helpsusdeveloplogicalthinking”and “mathematicshelps usfindtherightwaytosolveproblems”.Someeven say“Ilikemathematics,soI wouldliketohelpmy studentsappreciate the subject”. Each of these answers suggests a reason for the importanceof mathematicsorschoolmathematics intheteacher’smind. Nevertheless, each answer is only a partial answer to the question. Tolook for a comprehensive answer, we inevitably need to address the questionwhymathematicsisessentialinour world.

2.0 OBJECTIVES

Attheendofthisunit,you shouldbeableto:

- explain mathematics as an important part of understanding o u r world
- p rovethatthesubjectandits applications inscience,commerce and technology are important if students are to understand and appreciate the relationships and patterns of both number andspaceintheirdailylife
- expresswhatmathematicsisclearlyandconcisely
- explainthat mathematics also help students to develop

their capacity of reasoning so that they could think more logically and independently in making rational decisions.

3.0 MAINCONTENT

3.1 Importanceof Mathematics

In the past, teaching objectives in Mathematics were limited to having students memorisefacts and obtain skills in manipulating and calculatingnumbers.Memorisingofrulesandmechanicalmanipulationof numbers were considered sufficient. Today, we emphasize skill incompilation as well as skill in mastery of ideas and understanding ofoperations.Theapplicationofknowledgeandfactstonew situations is the best criterion of effective learning. Applications need clear understanding,closestudyandconcentratedattention.Hencethe teacherofmathematicshastodevelopallthesehabitsandattitudes in thepupils.

There shouldbenoinsistenceuponmemorizingfacts.Sothechiefvalue of mathematicsstudyisthatittrainsyouintheuseofreasoningpower. Henceinteaching,theteachershouldemphasizethinkingandreasoning, ratherthanmemoryworkandrotelearning.

Tothestudents,thesolving of a difficult problem is a discovery and constitutes training in suchwork. The chief aim of teaching mathematics is to develop thesefacultiesthatleadtodiscoveryandinventions.Thefamouspedagogue, Schultze, remarks that "mathematical study trains the students in systematic and orderly habits and the pleasure connected with the successful conquering of a difficulty stimulate will power". It alsocultivatesthepowerofattention, forinmathematics,theslightestslack in attention is ruinous. Mathematics makes constant demands upon imagination(Prakash,2011).

To enablestudents to cope confidently withthe mathematics needed in theirfuturestudies, workplacesor dailylifein a technological and information-rich society, thecurriculum should aimatdevelopinginthestudents:

- theability to conceptualise, inquire, reasonand communicate mathematically, andto use mathematics to formulate and solve problemsindailylifeas wellasinmathematicalcontexts
- theability to manipulate numbers, symbols and other mathematicalobjects
- thenumbersense, symbol sense, spatial sense and a sense of measurement as wellas the capabilityin appreciating structures andpatterns
- apositive attitude towardsmathematics and the capability of appreciatingthe aesthetic nature and cultural aspect of mathematics

Themaingoalsofteachingmathematicsattheprimarylevel(ages6to 12years) aretohelp studentstoacquire:

a) thebasic skills innumeracy
b) theabilitytousetheseskillstosolveproblems
c) theabilitytoestimateandmakeorcalculateapproximationsand
d) theabilitytointerpretgraphsandarrangementsofnumericaldata

More specifically, the curriculum s h o u l d be outlined so that students shouldbeableto:

a) masterthe skills in writing numbers, counting and stating place value
b) acquirethe basic skills in the four basic operations of adding, subtracting,multiplyinganddividing
c) acquirethe ability to measure, weigh, state time and specify the facevalueofcurrency
d) identifyand state the shapes of objects and able to know theproperties of square, rectangles, triangles, cuboids, cylinders,spheres,conesandpyramids
e) solveproblemsinvolvingnumbers, measurement,weight,money, distance,spaceandtime
f) estimateandcalculateapproximations
g) recordand read groups of data in the form of simple tables and graphs.

4.0 CONCLUSION

The study of mathematicscontributes to the development ofthe individualandfurtheringanation'sscientificemancipation.

5.0 SUMMARY

Mathematicsdevelops inthepupilstheabilitytoacquirebasicskillsin numeracyandusetheseskillstosolveproblems. Italsohelpsthem to estimate and make or calculate approximations and to interpret graphs andarrangementsofnumericaldata.

6.0 TUTOR-MARKEDASSIGNMENT

State fivereasonsfortheteachingofmathematicsinprimaryschools

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UNIT 2 FEATURES OF THE NEW 9-YEAR BASIC MATHEMATICSCURRICULUM

CONTENTS

1.0 INTRODUCTION

Theschoolcurriculumcomprisesalllearningandother experiencesthat each school plans for its pupils. The National Curriculum is an importantelementoftheschoolcurriculumandateacher'sawarenessofthisis vitalfor effectiveusageofthecurriculum.

2.0 OBJECTIVES

Attheendofthisunit,you shouldbeableto:

- explainmathematicsas aproblemsolvingactivities inallareasoflife
- explainmathematicsas awayofcommunication
- explainmathematicsas awayofreasoningindiverseareasoflife activities
- explainmathematicsas awayofconnectionsof ideas
- describethe formatofprimarymathematics curriculum.

3.0 MAINCONTENT

3.1 Introduction tothe New National Mathematics CurriculumforBasicEducationProgramme

Adeniyi(2007), in the revised edition of *National MathematicsCurriculumforBasicEducationProgramme*beginningfromBasic1to 9 stated the general objectives for the 9-Year Basic Mathematics Curriculumthus:'this revised edition of the National Mathematics CurriculumisfortheBasicEducationProgrammebeginningfromBasic 1 to 9. In this newcurriculum,there is no Primary Mathematics Curriculum for Junior Secondary curriculum. The two levels of education(Primaries1-6and JS1-3)havebeeninfusedintoBasic 1-9. Pupilsareexpectedtocontinuetheir educationfromBasiconetoBasicnine withoutinterruption'.

Thisrevisedcurriculumbecamenecessarybecausethe Universal BasicEducation (UBE) Bill 2004 mandated a nine- year compulsory education. Second, the National Economic,Empowerment and Development Scheme (NEEDS) and Millennium Development Goals (MDGs) necessitated also the needtorevisethiscurriculumwherenecessary.

The revised National Mathematics Curriculum for Basic Education in Nigeriais focusedongivingchildrentheopportunityto:

1) acquiremathematical literacy necessary to function in an informationage
2) cultivatetheunderstandingandapplicationof mathematicsskills and concepts necessary to thrive in the ever-changing technologicalworld 3) develop the essential element of problem solving, communication, reasoning and connection within their study of mathematics.
4) understandthemajorideasof mathematics,bearinginmindthatthe world has changed and is still changing since the first National MathematicsCurriculumwasdevelopedin1977. There is need to incorporate such changes in the areas of Information and CommunicationsTechnologies(ICT),PopulationandFamily Life Education, Environmental Degradation, Drug Abuse and HIV/AIDS.

These gaveriseto the needtomakethe curriculum more responsive to the survival and developmental needs of the Nigerian child. It should also be noted that this revised curriculumplacedemphasisonaffectivedomainandquantitativereasoning unlikethepreviouscurriculum. This is toboostpupils' chievementin cognitive and psychomotorcapabilities.The thematicapproachwasalsoadoptedin selectingthecontentand learningexperiences inthecurriculum.This isbecauseitis usefulin accommodating new contents/programmewithout necessarily disrupting the entire content or curriculum structure.

There are now six themes in this revised curriculum: Number and Numeration,BasicOperations,Measurement,AlgebraicProcess, GeometryandMensurationandEverydayStatistics.

This is a teaching curriculum. Thus, it provides maximal aid for the teacherbyprescribingtopics,objectivesor expectedlearningoutcomes stated in measurable terms, pupils' and teachers' activities and adequate evaluation guide. For this curriculum to be effective in achieving the purposeforwhichitismeant,the followingrecommendationsaremade:

1) it is strongly recommended that copies of these documents bemade available to every primary school teacher andmuchemphasisplacedonits useinordertoachievestatedobjectives
2) thereisneedtoorganizeworkshopsforteachers,supervisorsandinspectorsonhowtointerpretandusethecurriculum
3) Theminimumqualificationforteachersteachingin Basics1-9isNigeriaCertificateinEducation(NCE).Universitygraduatesoffirstdegreeandaboveinvariousdisciplinesteachingin Basics1-9 must have education qualificationfor the Basic EducationProgrammetosucceed.

Finally, itis ourhopethattherevisedNationalMathematicsCurriculumwill achieve the goals and objectives of the Universal Basic Education ProgrammeinNigeriaas containedintheNationalPolicyonEducation(2004) andtheUBE billof2004.

3.2 WhyDoWeTeach Mathematics?

Whatdowewanttoachieveinourmathematicslessons? Thisis agood questionwhichallteachersshouldaskthemselvesfromtimetotimein their daily practice. To look for a comprehensive answer, we inevitably need to address the question why mathematics is essential in our world as follows:

3.3 Mathematics as ProblemSolving

Although the definition of problem solving may differ from that ofNCTM's(1992), it, nevertheless, becomesthesignificantelementstobe emphasized in the teaching and learning of mathematics. Teachers areexpected to intentionally teach students on the heuristics of problem solving. Although teachers are free to choose the strategy suitable fortheir students, they are encouraged to follow those recommended byPolya(1974). Teachers are also encouraged to simulate mathematicalproblems based on their daily experiences. More specifically, teachers are expected to provide varied experiences through which students canworkindividually or in groups in tackling mathematical problems.Thecurriculum places heavy emphasis on the relationships betweenmathematicsandreallife problems.Problemsolvinginrealcontextsareconsidered essential in helping students appreciate mathematics. Inshort,problemsolving becomesthefocus inthecurriculum.

3.4Mathematics as Communication

The curriculum clearly states that one of the objectives in learning mathematics is to acquire the ability to communicateideas throughtheuse of mathematical s y m b o l s or ideas. An essential part of t h e curriculum is to help students attain the ability to comprehend mathematical statements encountered, for example, in the mass media.

Students are expected to be able to interpret the statistics used in various reports they encounter in the mass media. Inmathematics lessons, students are encouraged to work in groups oncertainprojectsorproblems.

3.5Mathematics as Reasoning

The main goal statement clearly states that the students need to develop the ability to think logically, systemically, creatively and critically. Although this is not clearly stated in the syllabus, teachers' guides and further elaboration of the syllabus specially encourage teachers to use approaches that can simulate mathematical thinking or reasoning. The use of statistics to critically examine information as part of the lessons, for example, can be said to be in line with the aim of promoting the above thinking abilities.

3.6 MathematicalConnections

There is a strong emphasis in making connections within mathematicsitself and across other subjects. In fact, the title of the curriculum suggests that making mathematical connections within itself or across other areas of study is strongly suggested. Making the connectionsbetweenmathematics studiedinclassandmaterialfrom everyday life orthe environment are explicitly stated in the documents accompanying the syllabus. Through the introduction o f certain facts concerning historical development in mathematics, the curriculum hopes that students should be able to see that mathematics has its origin in manycultures and is developed as responses to human needs that are both utilitarianandaesthetic.

SELF ASSESSMENT EXERCISE

Mathematics is often regarded as the "queen" of the sciences. Briefly explain why this could be true.

Organization of the 9-Year Basic Mathematics Curriculum Format

MARY ONE

ME: NUMBER AND UNMERATION

	Performance Objectives	Contents	Activities		Teaching and Learning materials	Evaluation Guide
			Teacher	Pupils		
Sample numbers	Pupils should be able to: 1. Sort and classify number of objects in a group or collection	i. Sort and classify objects leading to ideaof1-5	1. Brings objects such as: beans, bottle tops, buttons andnylonbags 2. Mixes the collections and asks pupils to sort them accordingtotypes	1.Bring various objects such as: beans, bottle tops, buttons and nylonbagsto class 2. Sort them according totypes	Courters: stones beans, bottle tops, buttons, leaves and nylonnbags etc	Pupilto: 1. Sort given number of objects from a collection.
	2. Identify number of objects in a group orcollection	ii.Identification of number of objects 1-5	1. Guides pupils to form groups: one for stones, two for bottle tops, three for beans, four for buttons and fiveforballs	Sort and classify themixed collection by forming groups for objects e.g. pick a stone, picktwobottletops etc	Counters: stones beans, bottle tops, buttons, leaves and nylonbags etc	2. Arrange given number of objects from a collection together.
	3. Count correctlyupto 5	iiiReading ofnumber 1-5	1. Asks pupils to show onebottle top, 2 bottle tops, uto5bottletops 2. Readsnumber1-5	Readthenumber1-5p		3. Read given numbersontheboard
	4. Write correctlynumber1-5	iv. writing of numbers1-5	1. writesnumbers1-5board 2. Leadspupilstowritethe numbers in order in their books	Write thenumber 1-5 inexercisebook		4. Writenumbers1-5on Theboard/exercise book
	5. Arrange numbers1-5 in order oftheirmagnitudes (quantities)	v. Ordering of number1-5	Arranges numbers in order of their magnitude using counters andotherobjects	Use counters to arrange objectsinmagnitude orin orderingform		5. Order given numbersin order of their magnitudesform
	6. Appreciate theneed for counting andordering		Leads pupils to appreciatenumbering in order of their magnitude	Appreciate the need forcounting and ordering ineverydayactivities		6. State why countingand ordering are important

4.0 CONCLUSION

The new basic mathematics curricular emphasizes thatmathematics should be taught in connection with its usefulness as an everyday activity

5.0 SUMMARY

Mathematicscouldbeseenas aproblemsolvingactivityinallareasof life;as awayof communication;as awayofreasoningindiverseareas of lifeactivities,andas awayofconnectionsofideas.

6.0 TUTOR-MARKEDASSIGNMENT

Describe the format of the new primary school curriculum in Nigeria andexplainthelinkages betweeneachcolumn.

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UNIT 3 COMPONENTS OF EFFECTIVE MATHEMATICSINSTRUCTION

CONTENTS

1.0 INTRODUCTION

Less is known about the components of effective mathematics instructionthanthecomponentsofeffectivereadinginstruction,because research in mathematics is less extensive than in reading. However,conclusions can still be drawn from some very good studies that doexist, as well as from typical grade level expectations in mathematics.As is trueforreading,thereis nosingle "best"programme forteachingmathematics. Rather, certain key abilities involved in learningmathematicsneedto beaddressedin instruction,withtheimportanceofdifferent abilities shifting somewhat across the elementary a n d secondarygrades.

2.0OBJECTIVES

Attheendofthisunit,you shouldbeableto:

- explainhowtodeveloppupils’abilityinconceptformations
- teachingthrougha concrete-to-representational-to-abstract sequenceofinstruction
- usingconcretemanipulativetoteachmathematics
- statetypesof manipulative.

3.0 MAINCONTENT

3.1 EffectiveNurserythroughPrimaryFourInstruction

At these levels, general education instruction in mathematics should include development of the following mathematics-related abilities:conceptsandreasoning(e.g., basicnumberconcepts,meaning ofoperationssuchas addition,geometricconcepts);automaticrecallof numberfacts (e.g.,memorisationofbasicadditionfactssuchas 3+4so that children know answers instantly instead of having to count); computational algorithms (the written procedure or series of steps forsolving more complex types of calculation, e.g., for two-digit additionwithregrouping,calculationstartsintheright-handcolumnandtens are"carried" from the ones to the tens column); functional mathematics (e.g., practical applications such as time and money); and verbalproblem-solving(e.g.,solvingwordproblems).

Becauseprogressinmathematicsbuilds heavilyuponpreviouslylearned skills, it is important for instruction to be clear, unambiguous, and systematic,withkeyprerequisiteskillstaughtinadvance. Forinstance, childrenshouldnotbeexpectedtodevelopautomaticrecallofaddition facts if they do not understand the basic concept of addition or themeaning of the addition sign. It is also essential for children to havesufficient practice to acquire new skills. For example, althoughmanipulative such as cubes or rods can be very helpful in developingbasic concepts, many children will not spontaneously progress fromaccurately solving facts with manipulative to automatic recall of facts.Instead, most children benefit from practice activities focusedspecifically o n helping them to memorisefacts. Similarly, learning computational a l g o r i t h m s such as those used in long division or two-digitmultiplicationoftenrequiresconsiderablepractice.

Scientificinvestigators interestedinlearningdisabilitieshaveidentified several patterns that may be found in youngsters with mathematics disabilities. Some of these children have difficulties that revolve primarilyaroundautomaticrecalloffacts,coupledwithgoodconceptual abilities in mathematics. This pattern characterisessome children withreading disabilities. Another common pattern involves difficulties withcomputational a l g o r i t h m s ; yet a third pattern involves visual-spatialdifficulties,suchas difficultyliningupcolumnsor withlearningspatialaspects of mathematics, such as geometry. Although effective generaleducationinstructioncanhelptopreventlow mathematicsachievement in many children, some youngsters with genuine mathematics disabilitieswillrequiremoreintensive, long-terminstructioninorderto make appreciable level of achievement.

3.2 TeachingPrimaryFiveandBeyond

In primary five andbeyond,general education instructioninmathematics focuses agreat deal on advanced concepts and reasoningreasoning (e.g., what avariable or a function is), learning of complexcomputationalalgorithms(e.g.,thoseinvolvedinaddingandsubtracting fractions and decimals), and more difficult kinds of verbal problem- solving (e.g., problems with multiple steps). By grade five,automatic recall of number facts is well-developed in most normallyachievingyoungsters. However, youngsters with mathematics disabilities oftencontinue to struggle with mathematics skills far below gradeexpectations, including not only automatic recall, but also manycomputational algorithms and mathematics concepts. A thoroughevaluation that assesses a range of important mathematics skills is essential, becausechildrencanhavedifferentstrengthsand weaknesses even within the domain of mathematics, and knowing the pattern ofstrengths and weaknesses is central to instructional planning. For instance, a child who has good conceptual abilities but whose difficulties centreon automatic recall and computation will need a different kind of instructional programmethan will one whose maindifficultiesareconceptualinnature.

As children advance into junior and secondary schools, trackingofstudentsintodifferentlevelsofmathematics (e.g., an acceleratedtrack, a grade-level track, and a remedial track) could be easier. Also, science courses begintodrawmoreheavilyonmathematicsskills, andstudents withmathematicsdisabilitiesmaybegintoexperiencemoredifficulties in science. Providing intensive remediation of basic mathematics skills to students who need it remains essential in these classes, not only tohelp students acquire the skills needed foreveryday life, but alsobecause mathematicsachievement serves as a gateway for highereducationandformanyoccupations.

3.3 Teaching through a Concrete-to-Representational-to-AbstractSequenceofInstruction

The purpose of teaching through a concrete-to-representational-to- abstract sequence of instruction is to ensure that students truly have athorough understanding ofthe mathematics concepts/skills they are learning. When students who have mathematics learning problems are allowed to first develop a concrete understanding of the mathematics concept/skill, then they are much more likely to perform that mathematicsskill and truly understand mathematical concepts at the abstractlevel.

What isit?

- Each mathematics concept/skill is first modeled with concrete materials (e.g. chips, unifixcubes, base ten blocks, beans and beansticks, patternblocks).
- Students are provided many opportunities to practice and demonstratemasteryusingconcretematerials
- The mathematicsconcept/skill is next modeled at the representational (semi-concrete) level which involves drawing pictures that represent the concrete objects previously used (e.g.tallies, dots,circles, stampsthatimprintpicturesforcounting)
- Students are provided many opportunities to practice and demonstratemasterybydrawing solutions
- Themathematicsconcept/skillisfinally modelledattheabstract level(usingonlynumbersandmathematicalsymbols)
- Students are provided many opportunities to practice and demonstratemasteryattheabstractlevelbeforemovingtoanew mathematicsconcept/skill.
- As a teacher moves through a concrete-to-representational-to- abstract sequence of instruction, the abstract numbers and/or symbols should be used in conjunction with the concrete materialsandrepresentationaldrawings(promotesassociationofabstract symbolswithconcrete&representationalunderstanding)

Whatarethecriticalelementsof thisstrategy?

- Useappropriateconcreteobjectstoteachparticularmathematics concept/skill (see Concrete Levelof Understanding/ Understanding Manipulatives-Examplesby mathematics concept area).Teachconcreteunderstandingfirst.
- Use appropriate drawing techniques or appropriate picture representationsofconcreteobjects(seeRepresentationalLevelof Understanding/Examples of drawing solutions by mathematics conceptarea).Teachrepresentationalunderstandingsecond.
- Use appropriate strategies for assisting students to move to the abstract level of understanding for a particular mathematics concept/skill (see Abstract Levelof Understanding/Potential barrierstoabstractunderstandingforstudentswhohavelearningproblems andhowtomanagethesebarriers).
- When teaching at each level of understanding, use explicit teaching methods (see the instruction strategy ExplicitTeacher Modeling).

HowdoIimplementthestrategy?

1. Wheninitiallyteachingamathematicsconcept/skill,describeand modelitusingconcreteobjects(concretelevelofunderstanding).

2. Provide students many practice opportunities using concrete objects.

3. When students demonstrate m a s t e r y o f skill by using concrete objects,describeand modelhowtoperformtheskillbydrawing or with pictures that represent concreteobjects (representational levelofunderstanding).

4. Provide many practice opportunities where students draw thesolutionsoruse picturestosolve problem

5. When students demonstrate m a s t e r y d r a w i n g solutions, describe and model how to perform the skill using only numbers and mathematicssymbols(abstractlevelofunderstanding).

6. Provide many opportunities for students to practice performing theskillusingonlynumbersandsymbols.

7. After studentshave masteredperformingtheskillattheabstract level of understanding, ensure students maintain their skill level byproviding periodicpractice opportunities forthe mathematics skills.

How does this Instructional Strategy Positively Impact Students

whohaveLearningProblems?

- Helpspassivelearnertomakemeaningfulconnections
- Teaches conceptual understanding by connecting concrete understandingtoabstractmathematicsprocess
- By linking learning experiences from concrete-to- representational-to- abstract levels of understanding, the teacher providesa graduatedframework for students to make meaningful connections.
- Blends conceptual and procedural understanding in structured way

SELF ASSESSMENTS EXERCISE

(1) Briefly describe how you would teach nursery level effectively.

(2) Describe in brief, Concrete –to- Representational –to- Abstract sequence of instructions.

3.4 UsingConcreteManipulativetoTeach Mathematics

What isit?

The concrete levelof understanding is the most basiclevel of mathematical understanding. It is also the most crucial level for developing conceptualunderstanding of mathematics concepts/skills. Concrete learning occurs when students have ample opportunities tomanipulate concreteobjects to solve problem. For students who havemathematicslearningproblems,explicitteachermodelingof theuseof specific concrete objects to solve specific mathematics problemsis needed.

Understandingmanipulative(concreteobjects)

To use mathematics manipulativeseffectively, it is important that youunderstand several basic characteristics of different types of mathematics manipulativesand how these specific characteristics impact students who have learning problems. As you read about thedifferent types of manipulatives, reflect on pictures of differentmanipulatives.

Generaltypesofmathematicsmanipulatives

Discrete- thosematerialsthatcanbecounted(e.g. cookies,children, countingblocks, toycars, etc.).

Continuous -materials that are not used for counting but are used for measurement(e.g.ruler, measuringcup, weightscale, trundlewheel).

Suggestions for using discrete and continuous materials with studentswhohavelearningproblems

Students who have learning problems need to haveabundant experiences using discrete materials before they will benefit from theuse of continuous materials. This is because discrete materials havedefining characteristicsthat students can easilydiscriminate throughsight andtouch. As students master an understandingofspecific readinessconceptsfor specificmeasurementconcepts/skillsthroughthe useofdiscretematerials(e.g.countingskills), thencontinuousmaterials canbeused.

Types of manipulativesused to teach the Base-10 System/place- value(Smith,1997):

Proportional - shows relationships by size (e.g. tencounting blocks groupedtogetheris tentimesthesizeofonecountingblock;abeanstick with ten beans glued to a popsicle stick is ten times bigger than one bean).

Non-linked proportional - single units are independent of each other, but can be "bundled together (e.g. Popsicle sticks can be "bundled together in groups of 'tens' with rubber bands; individual unifixcubes canbeattachedinrowsoftenunifixcubeseach).

Linked proportional - comes in single units as well as "already bundled" tensunits, hundreds units, and thousands units (e.g. baseten cubes/blocks;beansandbeansticks).

Examplesofmanipulative(concreteobjects)

Suggested manipulativesare listed according to mathematics concept/skillarea. Descriptionsofmanipulative are provided as appropriate.Abriefdescriptionofhoweachsetofmanipulativemaybeusedto teachthemathematicsconcept/skillis providedatthebottomofthe list for each mathematics concept area. This is not meant to bean exhaustive list, butthis list does include a variety of common manipulative. The list includes examples of "teacher-made" manipulativeaswellas "commercially-made"ones.Thesearediscussed under the followingheadings:

Counting/BasicAddition&Subtraction
PlaceValue
Multiplication/DivisionPositive
andNegativeIntegers Fractions Geometry
BeginningAlgebra

Counting/BasicAddition&SubtractionPictures

- Coloredchips
- Beans
- Unifixcubes
- Golftees
- Skittlesorother candypieces
- Packagingpopcorn

- Popsiclesticks/tonguedepressors

Description ofuse: studentscan usethese concrete materials tocount, to add, and to subtract. Students can count by pointing to objects andcounting aloud. Students can add by counting objects, putting them inone group and then counting the total. Students can subtract byremovingobjectsfromagroupandthencountinghowmanyare left.

PlaceValuePictures

- Base10cubes/blocks
- Beansandbeansticks
- Popsiclesticksandrubberbands forbundling
- Unifixcubes (individual cubes can be combined to represent "tens")
- Place value mat (a piece of tag board or other surface that has columnsrepresenting the "ones," "tens," and "hundreds" place values)

Description of use: studentsare first taught to represent 1-9 objects in the"ones"column.Theyarethentaughtto represent "10"bytradingin tensinglecountingobjectsforoneobjectthatcontainsthetencountingobjectsonit(e.g. ten separatebeansaretradedinforone"beanstick"-a popsicle stick with ten beans glued on one side. Students then beginrepresenting different values 1-99. At this point, students repeat thesametradingprocessfor"hundreds."

Multiplication/DivisionPictures

- Containersandcountingobjects(paperdessertplatesandbeans,paperor plastic cups and candy pieces, playing cardsand chips,cutouttag boardcirclesandgolftees, etc.). Containersrepresent the"groups"andcountingobjectsrepresentthenumberofobjects ineachgroup.(e.g.2x4=8:twocontainerswithfourcounting
objects on each container) Counting objects arranged in arrays (arrangedinrows andcolumns).Color-codethe"outside"verticalcolumn
andhorizontalrowhelpsemphasisethemultipliers

PositiveandNegativeIntegersPicture

- Counting objects,one set light coloredand one set dark colored (e.g. lightand dark colored beans; yellow and blue counting chips;circlescutoutoftagboardwithonesidecolored,etc.).

Descriptionof use:lightcoloredobjectsrepresentpositiveintegersanddark colored objects represent negative integers. When adding positiveand negative integers, the student matches pairs of dark and light colored objects. The color and number of objects remaining represent thesolution.

FractionsPictures

- Fractionpieces(circles, half-circles,quarter-circles,etc.)
- Fractionstrips(stripsoftagboardonefootinlengthandoneinch wide, dividedintowholes, ½'s, 1/3's,¼'s, etc.
- Fraction blocks or stacks. Blocks/cubes that represent fractionalparts by proportion (e.g. a"1/2" block is twice the height as a"1/4" block).

Descriptionof use:theteacher modelshowtocomparefractionalparts using onetypeofmanipulative.Studentsthencomparefractionalparts.

Asstudentsgainunderstandingoffractionalpartsandtheirrelationships with a variety of manipulatives, the teacher models and then students begintoadd,subtract,multiply, anddivideusingfractionpieces.

GeometryPictures

- Geoboards(squareplatformsthathaveraisednotchesorrodsthatare formed in an array). Rubber bands or stringcan be used to formvariousshapesaroundtheraisednotchesor rods.

Description of Use: concepts such as areaand perimeter can be demonstratedbycountingthenumberof notchorrod"units" insidethe shapeor aroundtheperimeteroftheshape.

BeginningAlgebraPictures

- Containers (representing the variable of "unknown") and countingobjects(representingintegers)-e.g.paperdessertplates and beans, small clear plastic beverage cups 7 counting chips, playingcardsandcandypieces, etc.

Description of use: The algebraic expression, "$4x$= 8," can be represented with four plates ("$4x$"). Eight beans can be distributed evenly amongthe four plates. The number of beans on one plate representthesolution("x"=2).

Suggestionsfor usingmanipulative(Burns:1996):

- talk with your students about how manipulativeshelp to learn mathematics
- setgroundrulesforusingmanipulative.
- developasystemfor storingmanipulative.
- allow time for your students to explore manipulative before beginninginstruction.
- encouragestudentstolearnnamesofthemanipulativetheyuse
- providestudentstimetodescribethemanipulativetheyuseorally orinwriting.Modelthisas appropriate
- introducemanipulativetoparents

Representational

Whatisit?

Examplesofdrawingsolutionsbymathematicsconceptlevel.

Whatisit?

At the representational level of understanding, students learn to problem-solvebydrawingpictures.Thepictures students drawrepresent the concrete objects students manipulated when problem-solving at theconcrete level.It is appropriate for studentsto begin drawing solutions toproblemsassoonastheydemonstratetheyhave masteredaparticularmathematics concept/skill at the concrete level. While not all students needtodrawsolutionstoproblems before movingfromaconcretelevel of understanding to an abstract level of understanding, studentswho havelearning problems in particular typically need practice solving problemsthroughdrawing.Whentheylearntodrawsolutions,students are provided an intermediate step where they begin transferring theirconcreteunderstandingtowardanabstractlevelofunderstanding.When studentslearntodrawsolutions,theygaintheabilitytosolveproblems independently. Through multiple independent problem-solving practiceopportunities, students gain confidence as they experience success.Multiple practice opportunities also assist students to begin to"internalise" the particular problem-solving process. Additionally, students' concrete understanding of the concept/skill is reinforced because of the similarity of their drawings to the manipulativestheyusedpreviouslyattheconcretelevel.

Drawing is not a "crutch" for students that they willuse forever. It simply provides students an effective way to practice problem solving independentlyuntiltheydevelopfluencyattheabstractlevel.
Examplesofdrawingsolutionsbymathematicsconceptlevel

The following drawing examples are categorisedby the type of drawings ("Lines, Tallies, and Circles," or "Circles/Boxes"). In eachcategory, thereareavarietyofexamples demonstratinghowtouse these drawingstosolvedifferenttypesofcomputationproblems.

Whatisit?

Potentialbarrierstoabstractunderstandingforstudentswhohave learningproblemsandhowtomanagethesebarriers

Whatisit?

Astudentwhoproblem-solvesattheabstractlevel,doessowithoutthe use ofconcrete objects or without drawing pictures. Understandingmathematicsconceptsandperformingmathematicsskillsattheabstract level requires students to do this with numbers and mathematics symbols only. Abstract understanding is often referred to as "doing mathematics in your head." Completing mathematics problems where mathematics problems are written and students solve these problems usingpaperandpencilis acommonexampleofabstractlevel problem solving. Potential barriers to abstract understanding for students whohavelearningproblemsandhowtomanagethesebarriers are:

1. studentswho are not successfulsolving problems at the abstract levelmaynotunderstandtheconceptbehindtheskill.

Suggestions:

- re-teach the concept/skill at the concrete level using appropriate

concreteobjects(seeConcreteLevelofUnderstanding).
- re-teach concept/skill at representational level andprovide opportunities for student to practice concept/skill by drawing solutions(seeRepresentationalLevelofUnderstanding).
- provideopportunitiesforstudentstouselanguagetoexplaintheirsolutions and how they got them (see instructional strategy StructuredLanguageExperiences).

2. Havedifficultywithbasicfacts/memoryproblems.

Suggestions:
- regularly provide students with a variety of practice activities focusing on basic facts. Facilitateindependent practice by encouraging students to draw solutions when needed (see the student practice strategies Instructional Games, Self-correcting Materials, StructuredCooperative Learning Groups, and StructuredPeerTutoring).
- conduct regular one-minute timings and chart student performance.Setgoalswithstudentsandfrequentlyreviewchart with students to emphasiseprogress. Focus on particular fact families that are most problematicfirst, and then slowly incorporate a variety of facts as the students demonstrate competence(seeEvaluationStrategyContinuousMonitoringand ChartingofStudentPerformance).
- teach students regular patterns that occur throughout addition, subtraction, multiplication, and division facts (e.g. "doubles" in multiplication,9'srule-add10and subtractone, etc.)
- provide student a calculator or table when they are solving multiple-stepproblems.

3. Repeatproceduralmistakes

Suggestions:
- providefewernumberofproblemsper page.
- provide fewer numbers of problems when assigning paper and pencilpractice/homework.
- provide ample space for students writing, cueing, and drawing. Provide problems that are already written on learning sheets rather than requiring students to copy problems from board or textbook.
- provide structure: turn lined paper sideways to create straight columns. Allowstudentstousedry-eraseboards/lapchalkboardsthat allowmistakestobewipedawaycleanly.Color cuesymbols; for multi-step problems, draw color-cued lines that signal studentswheretowriteandwhatoperationtouse;provideboxes that represent where numerals should be placed; provide visual directionalcues inasampleproblem;provideasampleproblem, completedstepbystepattopoflearningsheet
- provide strategy c u e cards that students can use to recall the correctprocedure for solvingproblem
- provide a variety of practice activities that require modes of

expressionotherthanonlywriting.

4.0 CONCLUSION

Students learning and mastery greatlydepend on the numberof opportunities a student has to respond. The more opportunities for successful practice that you provide (i.e.practicethat does not negativelyimpactstudents learningcharacteristics),themorelikelyitisthat your students will develop mastery of that skill especially when manipulativesareemployedinteaching.

5.0 SUMMARY

Teachingthroughaconcrete-to-representational-to-abstractsequenceof instruction involves the use of manipulativesboth concrete and representational. Concrete o b j e c t s should be used when teaching the following topics in primary schools**:** Counting/Basic Addition and Subtraction; Place Value; Multiplication/Division; P o s i t i v e and Negative Integers;Fractions;GeometryandBeginningAlgebra.

6.0 TUTOR-MARKEDASSIGNMENT

i. Ineachcasegivefiveconcreteobjectsthatcanbeusedtoteach:
   a. Counting/BasicAdditionandSubtraction
   b. PlaceValue/ Multiplication/Division
   c. PositiveandNegativeIntegers
   d. Fractions
   e. Geometry
   f. BeginningAlgebra.

ii. Brieflydescribehowyouwillusegeoboardtoteachareaofa rectangle.

iii. Listdifferenttypesof manipulativeswithatleasttwoexamples each.

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UNIT 4 MATHEMATICS INSTRUCTION FOR STUDENTSWITHLEARNINGDIFFICULTIES

CONTENTS

1.0 INTRODUCTION

Itisimportant for teachers tounderstandthe characteristics ofstudents with learning difficulties and be able to adopt instruction to their peculiar needs.

2.0 OBJECTIVES

Attheendofthisunit,you shouldbeableto:

- statefour approaches for teaching students with learning difficultiesinmathematics
- distinguish between the learning instructions for teaching studentswithlearningdifficulties.

3.0 MAINCONTENT

3.1 Effective Mathematics Instruction forStudents with Learning Difficulties inMathematicsFour

Approaches thatImproveResults

Students have a variety of disabilities—most notably, learning difficulties.Butotherdisabilitiesas wellmayoccur suchas mildmental retardation, AD/HD, behavioral disorders, and cognitive disabilities. Meta-analyses have found strong evidence of instructional approaches thatappeartohelpstudentswithdisabilities improvetheirmathematics achievement. According to these studies, four methods of instructionshowthemostpromise.Theseare:

- systematicandexplicitinstruction
- self-instruction
- peer-tutoring
- visualrepresentation

Of course, to make use of this information, an educator would need to know much more about each approach. So let us take a closer look at them.

3.2 ExplicitandSystematicInstruction

Explicit instruction, often called direct instruction, refers to an instructional practice that carefully constructsinteractions between students and their teacher. Teachers clearly state a teaching objectiveand follow a defined instructional sequence. They assess how much students already know on the subject and tailor subsequent instruction, based upon that initial evaluation of student skills. Students move through the curriculum, both individually andin groups, repeatedly practicingskillsatapacedeterminedbytheteacher'sunderstandingof student needs and progress (Swanson, 2001). Explicit instruction hasbeenfoundtobeespecially successfulwhenachildhas problems withaspecificorisolated skill(Kroesbergen&VanLuit,2003).

The Center for Applied Special Technology (CAST) offers a helpfulsnapshot of an explicit instructional episode (Hall, 2002), shown inFigure 4.1 below. Consistent communication betweenteachers andstudents creates the foundation for the instructional process.Instructional episodesinvolve pacing a lesson appropriately, allowing adequate*processing* and *feedback* time,encouraging f*requent student responses*,and*listening*and*monitoring*throughoutalesson.

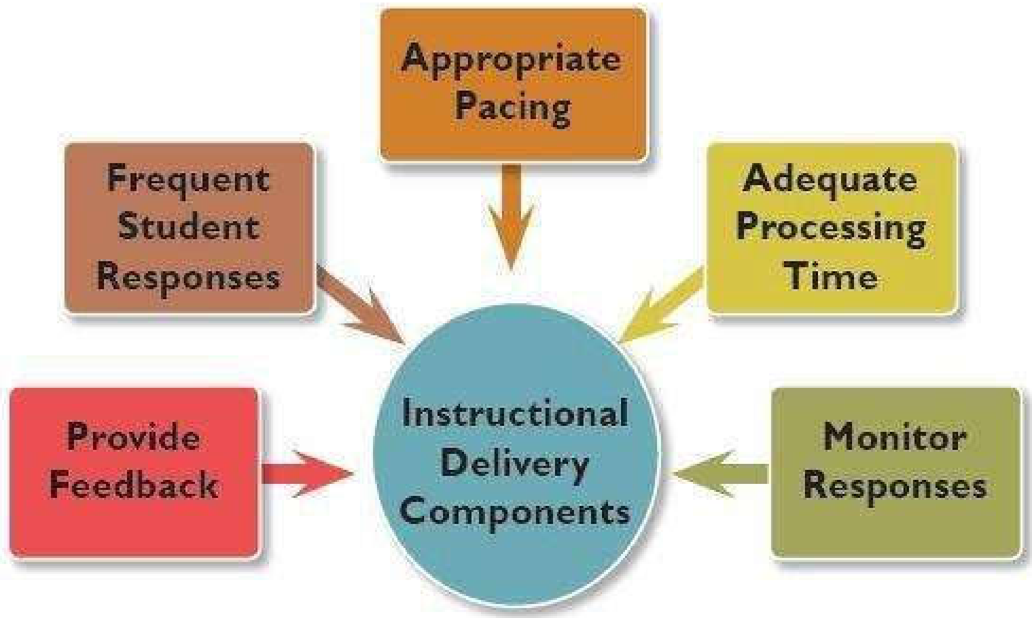

Figure 1. Standard instructional delivery components essential to all explicit instructional episodes (Hall, 2002).

Systematic instruction focuses on teaching students *how to learn* by giving them the tools and techniques that efficient learners use to understand and learn new material or skills. Systematic instruction, sometimes called "strategy instruction" refers to the strategies Students learn that help them integrate new information with what is already knowninawaythatmakessenseandbeableto recalltheinformationor skill later, even in a different situation or place. Typically, teachersmodel strategy use for students, including thinking aloud through the problem-solving process, so students can see when and how to use aparticular strategy and what they can gain by doing so. Systematicinstructionisparticularlyhelpfulin strengtheningessentialskillssuchas organisationandattention,andoftenincludes:

- memory devicesto help students remember the strategy (e.g., a first-letter mnemonic created by forming a word from the beginninglettersofotherwords)
- strategy stepsstated in everyday language and beginning with actionverbs(e.g.,readtheproblemcarefully)
- strategy stepsstated in the order in which they are to be used (e.g., studentsarecuedtoreadthewordproblemcarefullybefore tryingtosolvetheproblem)
- strategystepsthatprompt studentstousecognitiveabilities(e.g., the critical steps needed in solving a problem) (Lenz, Ellis, &Scanlon,1996, as citedinMaccini&Gagnon,n.d.).

All students can benefit from a systematic approach toinstruction, not just those with disabilities. That is why many of the textbooks being published today include overt systematic approaches to instruction in their explanations and learning activities. That is also why NICHCY's first *Evidence for Education* was devoted to the power of strategyinstruction. The research into systematic and explicit instruction isclear—the approaches taken together positively impactstudents'learning(Swanson,inpress). TheNationalMathematicsAdvisory Panel Report(2008)foundthatexplicitinstructionwasprimarilyeffectivefor computation(i.e.,basicmathematics operations),butnotas effectivefor higherorderproblemsolving.Thatbeingunderstood,meta-analysesand research reviews by Swanson (1999; 2001) and Swanson and Hoskyn(1998)assertthatbreakingdowninstructionintosteps, workinginsmallgroups, questioning students directly, and promoting ongoing practiceand feedback seem to have greater impact when combined with systematic "strategies." What does a combined systematic and explicit instructionalapproachlooklikeinpractice?TammyCihylik,alearningsupport teacher at Harry S. Truman Elementary Schoolin Allentown, Pennsylvania,describes a first-grade lessonthatuses money to explore mathematicalconcepts:

[Students] use manipulatives, she explains, "looking at the penny, identifying the penny." Cihylikprompts the students with explicit questions: "what does the penny look like? How much is it worth?" Then she provides the answers herself, with statements like, "the penny is brown, andis worthonecent."Cihylikencouragesstudents torepeatthedescriptivephrasesafter her,andthenleads theminapplyingthatbasicunderstandingina systematic fashion.After countingoutfivepennies and demonstrating their worth of five cents, she instructs the students to count out six pennies and report their worth. She repeats this activity each day, and incorporates other coins and questions as studentsmastertheideaofvalue.

Within this example, the relationship between explicit and systematic instruction becomes clear. The teacher is leading the instructional process through continually checkingin, demonstration, and scaffolding/extending ideas as students build understanding. She uses specific strategies involving prompts that remind students the value ofthe coins, simply stated action verbs, and metacognitive cues that askstudents to monitor their money. Montague (2007) suggests that "theinstructionalmethodunderlyingcognitivestrategy instructionis explicitinstruction."

3.3 Self-Instruction

Self-instruction refersto a variety of self-regulation strategies that studentscanusetomanagethemselvesaslearnersanddirecttheirown behavior, including their attention (Graham, Harris, &Reid, 1992). Learning is essentially broken down into elements that contribute tosuccess:

- settinggoals
- keepingontask
- checkingyourworkas yougo
- rememberingtouseaspecificstrategy
- monitoringyourownprogress
- beingalerttoconfusionordistractionandtakingcorrectiveaction
- checking your answer to makesure it makes sense and that the mathematicscalculationswerecorrectlydone.

Whenstudents discuss the nature of learning in this way, they develop both a detailed picture of themselves as learners (known as metacognitive awareness) and the self-regulation skillsthat good learners use to manage and take charge of the learning process. Some examplesofself-instructionstatementsareshownonthenextpage.

To teach students to "talk to themselves" while learning new information, solving a mathematics problem, or completing a task, teachers should firstmodel self-instructionaloud. Theytakeataskand thinkaloudwhileworkingthroughit, craftingamonologuethatovertly includes the mental behaviors associated with effective learning: goal-setting,self-monitoring,self-questioning,and self-checking. Montague (2004) suggests that both correct and incorrect problem-solvingbehaviorsbemodelled.

Modellingof correct behaviors helps students understand how goodproblem solvers use the processes and strategies appropriately.Modellingof incorrect behaviors allows students to learn how to useself-regulation strategies to monitor their performance and locate andcorrecterrors.Self-regulationstrategiesare learnedandpracticedintheactual context of problem solving. When students learn the modelling routine, they then can exchange places with the teacher and become modelsfortheir peers.

The self-statements that students use to talk themselves through the problem-solvingprocessareactuallypromptingstudentstousearange ofstrategies andtorecognisethatcertainstrategiesneedtobedeployed atcertaintimes(e.g.,self-evaluationwhenyouaredone,tocheckyour work). Because learning is a very personal experience it is important thatteachersandstudentsworktogethertogenerateself-statementsthat are not only appropriate to the mathematics tasks at hand but also toindividual students. Instruction also needs to include frequentopportunitiestopracticetheiruse, withfeedback(Grahametal., 1992) untilstudentshaveinternalisedtheprocess.

Self Assessment Exercise

What is self-instruction? How does it compare with your experience in Distance Learning System?

3.4 Peer-Tutoring

Peer- tutoring is a term that is been used to describe a wide array oftutoringarrangements,but most ofthe research on its success refers to students working in pairs to help one another learn or practice an academic task. Peer - tutoring works best when students of differentabilitylevelsworktogether(Kunsch,Jitendra,&Sood, 2007). Duringapeer - tutoringassignment,itis commonfortheteachertohave studentsswitch roles partway through, so the tutor becomes the tutee. Sinceexplainingaconcepttoanotherpersonhelpsextendone's ownlearning, this practicegivesboth studentstheopportunitytobetter understandthematerialbeing studied.

Research has also shown that a variety of peer-tutoring programs are effective in teaching mathematics, including ClasswidePeer -Tutoring (CWPT), Peer-Assisted Learning Strategies (PALS), and Reciprocal Peer -Tutoring (RPT) (Barley *et al.,* 2002). Successful peer-tutoring approachesmayinvolvetheuseofdifferentmaterials,rewardsystems,and reinforcementprocedures,butattheir coretheysharethe following characteristics(Barley*etal.,*2002):

- theteachertrainsthestudentstoactbothastutorsandtutees,so they are prepared totutor, andreceive tutoring from their peers. Before engaging in a peer-tutoring program, students need to understand how the peer- tutoring process works and what is expectedofthemineachrole.
- peer-tutoringprograms benefit from using highly structured activities. Structured activitiesmay include teacher-prepared materials and lessons (as in ClasswidePeer- Tutoring) or structured teaching routines that students follow when it is their turntobetheteacher (as in ReciprocalPeer-Tutoring).
- materialsused for the lesson (e.g., flashcards, worksheets, manipulatives, and assessment materials) should be provided to thestudents.Studentsengaginginpeer tutoringrequirethesamematerials to teach each other as a teacher would use for thelesson.
- continualmonitoringandfeedbackfromtheteacher helpstudents engagedinpeer tutoringstayfocusedonthelessonandimprove their tutoringandlearningskills.

Finally,thereismountingresearchevidencetosuggestthat, whilelow- achieving students may receive moderate benefits from peer tutoring, effects forstudentsspecificallyidentifiedwithlearningdifficultiesmay beless noticeableunlesscareis takentopairthesestudentswithamore proficient peer who can model and guide learning objectives (Kunsch, Jitendra,&Sood,2007).

3.5 VisualRepresentations

Mathematics instruction is a complex process that attempts to makeabstract concepts tangible, difficult ideas understandable and multifaceted problems solvable. Visual representations bring research- basedoptions,tools, andalternativestobearinmeetingtheinstructional challengeofmathematicseducation(Gersten*etal.*,2008).

Visual representations, broadly defined, can include manipulatives, pictures,numberlines,andgraphs offunctionsand relationships. “representationapproaches to solving mathematical problems includepictorial (e.g., diagramming); concrete (e.g., manipulatives); verbal(linguistic training); and mapping instruction (schema-based)” (Xin&Jitendra,1999,p. 211).Researchhasexploredtheways inwhichvisual representationscanbeusedin solvingstoryproblems (Walker&Poteet, 1989); learning basic mathematics skills such as addition, subtraction,multiplication, and division (Manalo, Bunnell, &Stillman, 2000); andmasteringfractions(Butler, Miller, Crehan, Babbitt,&Pierce, 2003)andalgebra(Witzel,Mercer,& Miller, 2003).

Concrete-Representational-Abstract (CRA) techniques are probably themostcommon example of mathematics instruction incorporating visualrepresentations.TheCRAtechniqueactuallyrefers toasimpleconceptthathas proventobeaveryeffectivemethodofteachingmathematics tostudents with disabilities (Butler *et al.,* 2003; Morin &Miller, 1998). CRAis athree-partinstructionalstrategyinwhichtheteacherfirstuses*concrete* materials (such as colored chips, base-ten blocks,geometricfigures, pattern blocks, or unifixcubes) to model themathematicalconcepttobelearned,thendemonstratestheconceptin *representational* terms (such as drawing pictures), and finally in *abstract* or *symbolic* terms(suchasnumbers, notation,ormathematicalsymbols).

Duringafractionlessonusing CRAtechniques, for example,theteacher might first show thestudents plastic pie pieces, andexplain that,when the circle is split into 4pieces,each of those pieces is ¼ of the whole,and when a circle is split into 8 pieces, each piece is ⅛ ofthe whole.After seeing the teacher demonstrates fraction concepts using concretemanipulatives, students would then be given plastic circles split into equal pieces and asked what portion of the whole one section of that circlewouldbe.Byholdingtheobjects intheirhandsandworkingwith them concretely, students are actually building a *mental* imageof the realitybeingexploredphysically.

After introducing the concept of fractions with concrete manipulatives, theteacherwouldmodeltheconceptin*representational* terms, eitherbydrawingpicturesorbygivingstudentsaworksheetof unfilled-incircles splitintodifferentfractionsandaskingstudentstoshade insegmentstoshowthefractionofthecircletheteachernames.

In the final stage of the CRA technique, the teacher demonstrates how fractionsarewrittenusingabstracttermssuchas numbersandsymbols (e.g., ¼ or ½). The teacher would explain what the numerator anddenominator are and allow students to practice writing differentfractionsontheir own.

As the Access Center (2004) points out, CRA works well with individual students, in small groups, andwith an entire class. It is alsoappropriate at both the elementary and secondary levels. The National Council of Teachers of Mathematics (NCTM) recommends that, when using CRA, teachers should make sure that students understand whathas beentaughtateachstepbeforemovinginstructiontothenext stage(Berkas&Pattison,2007).In somecases,studentsmayneedto continue usingmanipulativesintherepresentationalandabstractstagesas away ofdemonstratingunderstanding.

4.0 CONCLUSION

We have briefly examined four approaches to teaching mathematics to students with disabilities which research has shown to be effective. Each is worthy ofstudy in its own right and the sources of additional information provided will help teachers, administrators, and families bringtheseresearch-basedpracticesintothemathematicsclassroom.

When it is time to determine how you can best teach mathematics to your students, select an instructional intervention that supports the educational goals of those students based on age, needs, and abilities. Research findings can and do help identify effective and promising practices,butitis essentialtoconsider how well-matchedanyresearchactually istoyour localsituationandwhetherornotaspecificpractice will be useful or appropriate for a particular classroom or child. Interventions are likely to be most effective when they are applied to similar content, insimilarsettings,andwiththeagegroupsintendedfor them. That is why it is important to look closely at the components of any research study todetermine whether the overall findings provide appropriate guidance for your specific students, subjects, and grades— applestoapples, sotospeak.

5.0 SUMMARY

Systematicandexplicitinstructionis adetailedinstructionalapproachinwhich teachersguidestudentsthroughadefinedinstructionalsequence.Within systematic and explicit instruction, students learn to regularlyapply strategies that effective learners use as a fundamental partof masteringconcepts.

Self-instruction is a way by which students learn to manage their own learningwithspecificpromptingorsolution-orientedquestions.

Peer tutoring, is an approach that involves pairing students together to learnorpracticeanacademictask.

Visual representation uses manipulatives, pictures, number lines, and graphsoffunctionsandrelationshipstoteachmathematicalconcepts.

6.0 TUTOR-MARKEDASSIGNMENT

Distinguishbetweenthefollowingstypesofinstructions;:systematicand explicitinstruction;self-instruction;peer tutoringandvisualrepresentation instructions.

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