

MODULE 2: Cognitive Development and Mathematics Learning

UNIT 1: Gagne's Hierarchy of Concept and Meaning and Mathematics Learning

UNIT 2: Piaget Theory of Intellectual Development and Mathematics

UNIT 3: Writing Objectives Using Bloom's Taxonomy

UNIT 4: Innovations in Teaching of Mathematics

UNIT 1: Gagne's Hierarchy of Concept and Meaning and Mathematics Learning**CONTENTS**

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1.0 Introduction

Cognitive development is a field of study in neuroscience and psychology focusing on a child's development in terms of information processing, conceptual resources, perceptual skill, language

learning, and other aspects of brain development and cognitive psychology compared to an adult's point of view. A large portion of research has gone into understanding how a child imagines the world. A major controversy in cognitive development has been "nature versus nurture" or nativism versus empiricism. However, it is now recognized by most experts that this is a false dichotomy: there is overwhelming evidence from biological and behavioral sciences that from the earliest points in development, gene activity interacts with the events and experiences in the environment. Another issue is how culture and social experience relate to developmental changes in thinking. Another question is phylogeny convergence or homology with non-human animals. Most aspects of learning and cognition are similar in humans and non-human animals. These issues propagate to nearly every aspect of cognitive development.

2.0 Objectives

At the end of the unit you should be able to:

- Identify Gagne's Hierarchy of Concept and Meaning
- Explain How the Present-day Mathematics Teaching Violates Gagne's Principle
- Discuss the implication of Gagne's Hierarchy to the teaching and learning of mathematics

3.0 Main content

3.1 Gagne's Hierarchy of Concept and Meaning

Robert Gagne in his book 'On the Conditions of Learning', has given a taxonomy of learning types (Gagne, 1970 Chap. 4). that he has arranged hierarchically.

1. *Signal learning.* This is a type of associative learning that has been initially studied by Pavlov who has called it conditioned reflex. A subject that responds in a certain way (R) to a stimulus S1 is given two stimuli (S1 and S2) simultaneously. After a sufficient number of repetitions she learns to give the response (R) to S2 even in the absence of S1. Much of the learning that we do without giving conscious thought is of this type. Much of the initial learning of a young child in childhood is signal learning.
2. *Stimulus-response learning.* This is another type of associative learning that has been called trial and error learning by Thorndike. Skinner has used the term operant learning for it. It involves some goal or objective that the subject attempts to achieve. The process is essentially a successive approximation process. The initial efforts are almost random. The subject modifies his approach in every attempt. Each successful attempt is remembered while failed attempts are forgotten. The success rate improves with more attempts. A good example is a child learning to walk. Initially he falls down often. But with more attempts he is able to master the skill.
3. *Chaining.* Chaining is the process of establishing a sequential connection of a set of stimulus-response pairs for the purpose of attaining a particular goal. For example, the opening of a lock involves a number of simpler steps connected in a sequence (locate the key-hole - insert the key - turn the key clockwise - watch for lever unlocking - take off the lock). Successful chaining requires prior learning of each component response. Algorithms are generally such chaining sequences.
4. *Verbal Association.* Human beings have the ability to encode and express knowledge through sound patterns. Verbal association here refers to the most elementary kind of verbal behaviour - learning of verbal associations (object « name) and verbal sequences (chains of verbal associations).
5. *Multiple Discrimination Learning.* discrimination is the ability to distinguish between two or more stimulus objects or events. There are two different kinds of capabilities involved. The first is where the learner is able to make different responses to different members of a collection of

stimulus events and objects. The second type involves the capability of the learner to respond in a single way to a collection of stimuli belonging to a single set. (This involves recognition of the defining rule for the set and responding accordingly.)

6. *Concept learning.* Concept learning involves discrimination and classification of objects. We will distinguish between two types of concept learning: concrete and abstract. Concrete concepts are those that are formed through direct observation. For example, consider the edge of a table, the edge of a razor blade and the edge of a cliff. It is possible to formulate a rule that defines an edge. But the concept of edge is formed more easily through direct observation of several examples. A learner can respond to a set of stimulus objects in two ways—one by distinguishing among them and the other by putting them into a class and responding to any instance of that class in the same way. Both these types are examples of concept learning. The significance of concept learning is that it frees the learner from the control by specific stimuli.

7. *Principle (or rule) learning.* Some concepts are not concrete. They are based on rules that involve other concepts. So they have to be learnt through definition. Definitions are statements that express rules for classifying, i.e. rules that are applicable to any instance of a particular class. Definitions are used for objects as well as for relations. A salient feature of principle learning is that the learner cannot acquire the concept through memorizing its statements verbatim unless she knows the referential meaning of the component concepts. For example, $ax^2+bx+c=0$ is meaningless unless you understand what a , b , c , and x represented.

8. *Problems solving.* Problems solving, here, refers to something more than classroom mathematical drills. Also referred to as heuristics (Polya, 1957), the process of problems solving is one in which the learner discovers a combination of previously learnt rules that can be applied to achieve a solution for a novel situation. The following sequence of events is typically involved in problem solving.

- (a) presentation of the problem,
- (b) definition of the problem,
- (c) formulation of hypothesis,
- (d) verification of hypothesis.

The learning outcome of problems solving is a higher order rule that becomes a part of the student's repertory. According to Gagne, cognition and concept formation is a multi-layered phenomenon, each layer consisting of a particular learning type. Signal learning, Stimulus-response learning, Chaining, Verbal Association and Multiple Discrimination Learning are all pre-requisites for the formation of concepts and the ability to solve problems. The process of concept formation involves all these eight processes.

A very important point here is that if the learning has not been sufficiently accomplished at any level, then there is perceptible deterioration at all higher levels (Gagne and Wigand, 1970).

3.2 Rote Learning in the Context of Gagne's Hierarchy

Let us examine the different hierarchy levels of Gagne and see where the traditional methods of teaching fit. Signal learning, Stimulus-response learning, Chaining, Verbal Association and Multiple Discrimination Learning constitute the basic forms of learning. They are the basic building tools that enable the mind to acquire a working set-up for concept formation. It is this area where rote learning is most effective and insufficient learning at this level impairs the student's abilities for higher learning.

Signal learning refers to learning through unconditional association. When small children memorize alphabets and digits symbols, they are unconditionally associating the symbols with their form. Since the child does not as yet possess any related pre-formed associations, this is the only learning alternative available at this stage. Rote learning is the most effective learning tool at this stage because it directly does what is required. Stimulus-response learning or Operant Conditioning is a process based on successive approximation. Once the basic nodal associations have been formed in the mind, a successive approximation process or shaping takes place on the basis of positive and negative reinforcements. In the traditional elementary education, this step is accomplished through a lot of rote exercises.

The next step, chaining, is the process of combining a set of individual S-R's in sequence. In fact, the concept of *Sutra* developed in ancient India (Namita, 1996), is a formalisation of this step. *Sutra* has been identified with algorithm by Vernekar. The term 'algorithm' refers to a step-by-step method for solving any problem (Rajaraman, 1980, p.3). According to Vernekar (1994), the basic idea of the algorithmic method is that the various steps in an operation are arranged like beads in a thread (*sutra*). Thus *sutra* as well as algorithm refers to the same process as chaining or forming mental links.

The next step is verbal association. Most of the beginners' verbal associations are definitions and fact-snippets to be memorized. Here again, the rote methods are applicable. Although memorizing the vocabulary is a very boring job, once a student acquires good vocabulary through whatever means, its role in understanding verbal and written material cannot be denied.

SELF ASSESSMENT EXERCISE

Under which of Rober Gagne's learning types does early childhood learning fall? Give a brief illustration

3.3 How the Present-day Mathematics Teaching Violates Gagne's Principle

Present day curriculum stresses the role and necessity of concept formation in education (National Curriculum Framework 2000, 2005). This cognitive approach appears to be quite reasonable. A cognitive approach can be very useful in this context (Redish, 1994). At present, the heuristic constructivist approach is being implemented in the modern schools for teaching mathematics as well as other sciences subjects. A majority of students who go to higher classes are found to be extremely poor in concepts

(Agnihotri et al., 1994). Arons (1997) has pointed out several deep conceptual flaws in the thinking of average Physics students. Why do conceptual flaws occur? Assuming Gagne's model, the following learning types heavily rely on previously learned materials. (1) Chaining, (2) Multiple discrimination learning, (3) Concept learning, (4) Principle learning, and (5) Problem solving. And as we have seen, the kind of learning material that these learning types are based on is most effectively done through memorization.

In mathematics, one who has memorized the multiplication tables and rigorously practiced basic mathematical operations through oral methods is much more confident in higher mathematics because he has less stumbling blocks to overcome.

Modern school education has gradually done away with basic mathematical drills. So the prerequisites for formation of higher concepts as pointed out by Gagne are not being fulfilled. Mathematical knowledge is cumulative in nature. So with a weak foundation the majority of students are bound to display an overall weakness in their concepts. This, according to my view, is the main reason why many of today's students are weak in concepts.

4.0 Conclusion

Gagne's information processing model as shown above, that the rote and algorithmic methods could be used in traditional schools for effective building a strong base for formation of higher concepts. We should develop teaching methodology for mathematics and other subjects that incorporates rote learning in an effective way so that knowledge is better conveyed and represented in the mind of students. The rote learning of basic mathematical facts and word-meaning in primary schools will in particular be a very useful preparation for higher concepts. For better results a balance between heuristic approach and algorithmic approach will have to be established. We should also develop effective uses of sutra in mathematics teaching.

5.0 Summary

In this unit Gagne's Hierarchy of Concept and Meaning was identified. How the present-day mathematics teaching violates Gagne's principle as well as the implications to the teaching and learning of mathematics were discussed.

6.0 Tutor-Marked Assignment

- 1 List Gagne's Hierarchy of Concept and Meaning
- 2 What are the implications of Gagne's Hierarchy of Concept and Meaning to the teaching and learning of mathematics.

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UNIT 2: Piaget Theory of Intellectual Development and Mathematics**CONTENTS**

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1.0 Introduction

Jean Piaget (1896-1980) was a biologist who originally studied molluscs (publishing twenty scientific papers on them by the time he was 21) but moved into the study of the development of children's understanding, through observing them and talking and listening to them while they worked on exercises he set. His view of how children's minds work and develop has been enormously influential, particularly in educational theory. His particular insight was the role of maturation (simply growing up) in children's increasing capacity to understand their world: they cannot undertake certain tasks until they are psychologically mature enough to do so. His research has spawned a great deal more, much of which has undermined the detail of his own, but like many other original investigators, his importance comes from his overall vision. He proposed that children's thinking does not develop entirely smoothly: instead, there are certain points at which it "takes off" and moves into completely new areas and capabilities. He saw these transitions as taking place at about 18 months, 7 years and 11 or 12 years. This has been taken to mean that before these ages children are not capable (no matter how bright) of understanding things in certain ways, and has been used as the basis for scheduling the school curriculum. Whether or not should be the case is a different matter.

2.0 Objectives

At the end of the unit you should be able to:

- Identify Piaget's stages of intellectual development
- Distinguish between assimilation and accommodation
- Discuss the implication of Piaget's stages of intellectual development to the teaching and learning of mathematics

3.0 Main content

3.1 The Nature of Intelligence: Operative and Figurative Intelligence

Piaget believed that reality is a dynamic system of continuous change, and as such is defined in reference to the two conditions that define dynamic systems. Specifically, he argued that reality involves transformations and states. Transformations refer to all manners of changes that a thing or person can undergo. States refer to the conditions or the appearances in which things or persons can be found between transformations. For example, there might be changes in shape or form (for instance, liquids are reshaped as they are transferred from one vessel to another, humans change in their characteristics as they grow older), in size (e.g., a series of coins on a table might be placed close to each other or far apart) in placement or location in space and time (e.g., various objects or persons might be found at one place at one time and at a different place at another time). Thus, Piaget argued, that if human intelligence is to be adaptive, it must have functions to represent both the transformational and the static aspects of reality. He proposed that operative intelligence is responsible for the representation and manipulation of the dynamic or transformational aspects of reality and that figurative intelligence is responsible for the representation of the static aspects of reality.

Operative intelligence is the active aspect of intelligence. It involves all actions, overt or covert, undertaken in order to follow, recover, or anticipate the transformations of the objects or persons of interest. Figurative intelligence is the more or less static aspect of intelligence, involving all means of representation used to retain in mind the states (i.e., successive forms, shapes, or locations) that intervene between transformations. That is, it involves perception, imitation, mental imagery, drawing, and language. Therefore, the figurative aspects of intelligence derive their meaning from the operative aspects of intelligence, because states cannot exist independently of the transformations that interconnect them. Piaget believed that the figurative or the representational aspects of intelligence are subservient to its operative and dynamic aspects, and therefore, that understanding essentially derives from the operative aspect of intelligence.

At any time, operative intelligence frames how the world is understood and it changes if understanding is not successful. Piaget believed that this process of understanding and change involves two basic functions: Assimilation and accommodation.

3.2 Assimilation and Accommodation

Through studying the field of education Piaget focused on accommodation and assimilation. Assimilation, one of two processes coined by Jean Piaget, describes how humans perceive and adapt to new information. It is the process of taking one's environment and new information and fitting it into pre-existing cognitive schemas. Assimilation occurs when humans are faced with new or unfamiliar information and refer to previously learned information in order to make sense of it. Accommodation, unlike assimilation is the process of taking one's environment and new information, and altering one's pre-existing schemas in order to fit in the new information.

Through a series of stages, Piaget explains the ways in which characteristics are constructed that lead to specific types of thinking; this chart is called Cognitive Development. To Piaget, assimilation is

integrating external elements into structures of lives in environments
 or those we could have through experience. It is through assimilation
 that accommodation is derived. Accommodation is imperative because it shows people will
 continue to interpret new concepts, schemas, frameworks, etc. Assimilation is different from
 accommodation because of how it relates to the inner organism due to
 the environment. Piaget believes that the human brain has been programmed through evolution to
 bring equilibrium, and to move upwards in a process to equilibrate what is not. The equilibrium is
 what Piaget believes ultimately influences structures because of the internal and external processes
 through assimilation and accommodation.

Piaget's understanding is that these two functions cannot exist without the other. To assimilate an
 object into an existing mental schema, one first needs to take into account or accommodate to the
 particularities of this object to a certain extent; for instance, to recognize (assimilate) an apple as an
 apple one needs first to focus (accommodate) on the contour of this object. To do this one needs to
 roughly recognize the size of the object. Development increases the balance or equilibration between
 these two functions. When in balance with each other, assimilation and accommodation generate men-
 tal schemas of the operative intelligence. When one function dominates over the other, they
 generate representations which belong to figurative intelligence.

Following from this conception Piaget theorized that intelligence is active and constructive. It is
 active in the literal sense of the term as it depends on the actions (overt or covert, assimilatory or
 accommodatory), which the thinker executes in order to build and rebuild his model of the world. It
 is also constructive because actions, particularly mental actions, are coordinated into more inclusive
 and cohesive systems, thus they are raised to more stable and effective levels of functioning.

3.3 Piaget's Stages of Intellectual Development

Sensorimotor stage

The **sensorimotor stage** is the first of the four stages in cognitive development which "extends from
 birth to the acquisition of language". In this stage, infants construct an understanding of the world by
 coordinating experiences (such as seeing and hearing) with physical, motoric actions. Infants gain
 knowledge of the world from the physical actions they perform on it. An infant progresses from
 reflexive, instinctual action at birth to the beginning of symbolic thought toward the end of the stage.
 Piaget divided the sensorimotor stage into six sub-stages: 0–2 years, Infants just have senses—
 vision, hearing, and motor skills, such as grasping, sucking, and stepping. ---
 from Psychology Study Guide by Bernstein, Penner, Clarke-Stewart, Roy

Sub-Stage	Age	Description
1 <i>Simple Reflexes</i>	Birth-6 weeks	"Coordination of sensation and action through reflexive behaviors". Three primary reflexes are described by Piaget: sucking of objects in the mouth, following moving or interesting objects with the eyes, and closing of the hand when an object makes contact with the palm (palm grasp). Over the first six weeks of life, these reflexes begin to become voluntary actions; for example, the palm reflex becomes intentional grasping.
2 <i>First habits and primary circular reactions phase</i>	6 weeks-4 months	"Coordination of sensation and two types of schemes: habits (reflex) and primary circular reactions (reproduction of an event that initially occurred by chance). Main focus is still on the infant's body." As an example of this type of reaction, an infant might repeat the motion of passing their hand before their face. Also at this phase, passive reactions, caused by classical or operant conditioning, can begin.
3 <i>Secondary circular reactions phase</i>	4-8 months	Development of habits. "Infants become more object-oriented, moving beyond self-preoccupation; repeat actions that bring interesting or pleasurable results." This stage is associated primarily with the development of coordination between vision and prehension. Three new abilities occur at this stage: intentional grasping for a desired object, secondary circular reactions, and differentiations between ends and means. At this stage, infants will intentionally grasp their hand in the direction of a desired object, often to the amusement of friends and family. Secondary circular reactions, or the repetition of an action involving an external object begin; for example, moving a switch to turn on a light repeatedly. The differentiation between means and ends also occurs. This

		child's growth as it signifies the dawn of logic.
4 <i>Coordination of secondary circular reactions stages</i>	8–12 months	"Coordination of vision and touch--hand-eye coordination; coordination of schemes and intentionality." This stage is associated primarily with the development of logic and the coordination between means and ends. This is an extremely important stage of development, holding what Piaget calls the "first proper intelligence." Also, this stage marks the beginning of goal orientation, the deliberate planning of steps to meet an
5 <i>Tertiary circular reactions, novelty, and curiosity</i>	12–18 months	"Infants become intrigued by the many properties of objects and by the many things they can make happen to objects; they experiment with new behavior." This stage is associated primarily with the discovery of new means to meet goals. Piaget describes the child at this juncture as the "young scientist," conducting pseudo-experiments to discover new methods of meeting challenges.
6 <i>Internalization of Schemes</i>	18–24 months	"Infants develop the ability to use primitive symbols and form enduring mental representations." This stage is associated primarily with the beginning of insight, or true creativity. This marks

By the end of the sensorimotor period, objects are both separate from the self and permanent.

Object permanence is

the understanding that objects continue to exist even when they cannot be seen, heard, or touched. Acquiring this sense of object permanence is one of the infant's most important accomplishments, according to Piaget.

Preoperational stage

The preoperative stage is the second of four stages of cognitive development. Cognitive Development Approaches. By observing sequences of play, Jean Piaget was able to demonstrate that towards the end of the second year, a qualitatively new kind of psychological functioning occurs.

(Pre)Operatory Thought is any procedure for mentally acting on objects. The hallmark of the preoperational stage is sparse and logically inadequate mental operations. During this stage, the child learns to use and to represent objects by images, words, and drawings. The child is able to form stable concepts as well as mental reasoning and magical beliefs. The child however is still not able to perform operations; tasks that the child can do mentally rather than physically. Thinking is still egocentric. The child has difficulty taking the viewpoint of others. Two substages can be formed from preoperative thought.

• **The Symbolic Function Substage**

Occurs between about the ages of 2 and 7. During 2-4 years old, kids cannot yet manipulate and transform information in logical ways, but they now can think in images and symbols. The child is able to formulate designs of objects that are not present.

Other examples of mental abilities are language and pretend play. Although there is an advancement in progress, there are still limitations such as egocentrism and animism. Egocentrism occurs when a child is unable to distinguish between their own perspective and that of another person's. Children tend to pick their own view of what they see rather than the actual view shown to others. An example is an experiment performed by Piaget and Barbel Inhelder. Three views of a mountain are shown and the child is asked what a traveling doll would see at the various angles; the child picks their own view compared to the actual view of the doll. Animism is the belief that inanimate objects are capable of actions and have lifelike qualities. An example is a child believing that the sidewalk was mad and made them fall down.

• **The Intuitive Thought Substage**

Occurs between about the ages of 4 and 7. Children tend to become very curious and ask many questions; begin the use of primitive reasoning. There is an emergence in the interest of reasoning and wanting to know why things are the way they are. Piaget called it the intuitive substage because children realize they have a vast amount of knowledge but they are unaware of how they know it.

'Centration' and 'conservation' are both involved in preoperative thought.

Centration is the act of focusing all attention on one characteristic compared to the others. Centration is noticed in conservation; the awareness that altering a substance's appearance does not change its basic properties. Children at this stage are unaware of conservation. Example, In Piaget's most famous task, a child is presented with two identical beakers containing the same amount of liquid. The child usually notes that the beakers have the same amount of liquid. When one of the beakers is poured into a taller and thinner container, children who are typically younger than 7 or 8 years old say that the two beakers now contain a different amount of liquid. The child simply focuses on the height and width of the container compared to the general concept.

Concrete operational stage

The **concrete operational stage** is the third of four stages of cognitive development in Piaget's theory. This stage, which follows the preoperational stage, occurs between the ages of 7 and 11 years and is characterized by the appropriate use of logic. Important processes during this stage are:

Seriation—the ability to sort objects in an order according to size, shape, or any other characteristic. For example, if given different-shaded objects they may make a color gradient.

Transitivity—The ability to recognize logical relationships among elements in a serial order, and perform 'transitive inferences' (for example, If A is taller than B, and B is taller than C, then A must be taller than C).

Classification—the ability to name and identify sets of objects according to appearance, size or other characteristic, including the idea that one set of objects can include another.

Decentering—where the child takes into account multiple aspects of a problem to solve it. For example, the child will no longer perceive an exceptionally wide but short cup to contain less than a normally wide, taller cup.

Reversibility—the child understands that numbers or objects can be changed, then returned to their original state. For this reason, a child will be able to rapidly determine that if $4+4$ equals t , $t-4$ will equal 4, the original quantity.

Conservation—understanding that quantity, length or number of items is unrelated to the arrangement or appearance of the object or items.

Elimination of Egocentrism—the ability to view things from another's perspective (even if they think incorrectly). For instance, show a child a comic in which Jane puts a doll under a box, leaves the room, and then Melissa moves the doll to a drawer, and Jane comes back. A child in the concrete operations stage will say that Jane will still think it's under the box even though the child knows it is in the drawer.

Children in this stage can, however, only solve problems that apply to actual (concrete) objects or events, and not abstract concepts or hypothetical tasks.

Formal operational stage

The formal operational period is the fourth and final of the periods of cognitive development in Piaget's theory. This stage, which follows the Concrete Operational stage, commences at around 11 years of age (puberty) and continues into adulthood. In this stage, individuals move beyond concrete experiences and begin to think abstractly, reason logically and draw conclusions from the information available, as well as apply all these processes to hypothetical situations. The abstract quality of the adolescent's thought at the formal operational level is evident in the adolescent's verbal problem solving ability. The logical quality of the adolescent's thought is when children are more likely to solve problems in a trial-and-error fashion. Adolescents begin to think more as a scientist thinks, devising plans to solve problems and systematically testing solutions. They use hypothetical-deductive reasoning, which means that they develop hypotheses or best guesses, and systematically deduce, or conclude, which is the best path to follow in solving the problem. During this stage the adolescent is able to understand such things as love, "shades of gray", logical proofs and values. During this stage the young person begins to entertain possibilities for the future and is fascinated with what they can be. Adolescents are changing cognitively also by the way that they think about social matters. Adolescent Egocentrism governs the way that adolescents think about social matters and is the heightened self-consciousness in them as they are which is reflected in their sense of personal uniqueness and invincibility. Adolescent egocentrism can be dissected into two types of social thinking, imaginary audience that involves attention getting behaviour, and personal fable which involves an adolescent's sense of personal uniqueness and invincibility.

The Stages and Causation

Piaget sees children's conception of causation as a march from "primitive" conceptions of cause to those of a more scientific, rigorous, and mechanical nature. These primitive concepts are characterized as magical, with a decidedly non-natural or non-mechanical tone. Piaget attributes this to his most basic assumption: that babies are phenomenists. That is, their knowledge "consists of assimilating things to schemas" from their own actions such that they appear, from the child's point of view, "to have qualities which in fact stem from the organism." Consequently, these "subjective conceptions," so prevalent during Piaget's first stage of development, are dashed upon discovering deeper empirical truths. Piaget gives the example of a child believing the moon and stars follow him on a night walk; upon learning that such is the case for his friends, he must separate himself from the object, resulting in a theory that the moon is immobile, or moves independently of other agents. This second stage, from around three to eight years of age, is characterized by a mix of this type of magical, animistic, or "non-natural" conceptions of causation and mechanical or "naturalistic" causation. This conjunction of natural and non-natural causal explanations supposedly stems from experience itself, though Piaget does not make much of an attempt to describe the nature of the differences in conception; in his interviews with children, he asked specifically about natural phenomena: what makes clouds move? What makes the stars move? Why do rivers flow? Thenature of fall the answers given, Piaget says, are such that these objects must perform their actions to "fulfill their obligation towards men." He calls this "moral explanation

SELF ASSESSMENT EXERCISE

Distinguish between assimilation and accommodation

3.4 Challenges to Piagetian stage theory

Piagetians' accounts of development have been challenged on several grounds. First, as Piaget himself noted, development does not always progress in the smooth manner his theory seems to predict. 'Decalage', or unpredicted gaps in the developmental progression, suggest that the stage model is at best a useful approximation. More broadly, Piaget's theory is 'domain general', predicting that cognitive maturation occurs concurrently across different domains of knowledge (such as mathematics, logic, understanding of physics, of language, etc.). During the 1980s and 1990s, cognitive developmentalists were influenced by "neo-nativist" and evolutionary psychology ideas. These ideas de-emphasized domain general theories and emphasized domain specificity or modularity of mind. Modularity implies that different cognitive faculties may be largely independent of one another and thus develop according to quite different time-tables. In this vein, some cognitive developmentalists argued that rather than being domain general learners, children come equipped with domain specific theories, sometimes referred to as 'core knowledge', which allow them to break into learning within that domain. For example, even young infants appear to be sensitive to some predictable regularities in the movement and interaction of objects (e.g. that one object cannot pass through another), or in human behavior (e.g. that a hand repeatedly reaching for an object has that object, not just a particular path of motion), as is the building block out of which more elaborate knowledge is constructed. More recent work has strongly challenged some of the basic presumptions of the 'core knowledge' school, and revised ideas of domain generality—but from a newer dynamic systems approach, not from a revised Piagetian perspective. Dynamic systems approaches harkent to modern neuroscientific research that was not available to Piaget when he was constructing his theory. One important finding is that domain-specific knowledge is constructed as children develop and integrate knowledge. This suggests more of a "smooth integration" of learning and development than either Piaget, or his neo-nativist critics, had envisioned. Additionally, some psychologists, such as Vygotsky and Jerome Bruner, thought differently from Piaget, suggesting that language was more.

3.5 Post Piagetian and Neo-Piagetian stages

In the recent years, several scholars attempted to ameliorate the problems of Piaget's theory by developing new theories and models that can accommodate evidence that violates Piagetian predictions and postulates. These models are summarized below.

The neo-Piagetian theories of cognitive development, advanced by Case, Demetriou, Halford, Fischer, and Pascual-Leone, attempted to integrate Piaget's theory with cognitive and differential theories of cognitive organization and development. Their aim was to better account for the cognitive factors of development and for intra-individual and inter-individual differences in cognitive development. They suggested that development along Piaget's stages is due to increasing working memory capacity and processing efficiency. Moreover, Demetriou's theory ascribes an important role to hypo-cognitive processes of self-recording, self-monitoring, and self-regulation and it recognizes the operation of several relatively autonomous domains of thought (Demetriou, 1998; Demetriou, Mouyi, Spanoudis, 2010).

- Postformal stages have been proposed. Kurt Fischer suggested two, Michael Commons presents evidence for four postformal stages: the systematic, metasytematic, paradigmatic and cross paradigmatic. (Commons & Richards, 2003; Oliver, 2004).
- A "sentential" stage has been proposed, said to occur before the early preoperational stage. Proposed by Fischer, Biggs and Biggs, Commons, and Richards.
- Searching for a micro-physiological basis for human mental capacity, Traill (1978, proposed that there may be "pre-sensorimotor" stages developed in the womb and/or transmitted genetically.

Postulated physical mechanisms underlying "schemes" and stages

Piaget himself (1967) considered the possibility of RNA molecules as likely embodiments of his still-abstract "schemes" (which he promoted as units of action)—though he did not come to any firm conclusion. At that time, due to works such as that of Holger Hydén, RNA concentrations had indeed been shown to correlate with learning, so the idea was quite plausible.

However, by the time of Piaget's death in 1980, this notion had lost favour. One main problem was over the protein which (it was assumed) such RNA would necessarily produce, and that did not fit in with observation. It then turned out, surprisingly, that only about 3% of RNA does code for protein (Mattick, 2001, 2003, 2004). Hence most of the remaining 97% (the "ncRNA") could now theoretically be available to serve as Piagetian schemes (or other regulatory roles now under investigation). The issue has not yet been resolved experimentally, but its theoretical aspects have been reviewed; (Traill 2005/2008).

4.0 Conclusion

Piaget believed that all children try to strike a balance between assimilation and accommodation, which is achieved through a mechanism Piaget called equilibration. As children progress through the stages of cognitive development, it is important to maintain a balance between applying previous knowledge (assimilation) and changing behaviour to account for new knowledge (accommodation). Equilibration helps explain how children are able to move from one stage of thought into the next.

5.0 Summary

Piaget's theory, however vital in understanding child psychology, did not go without scrutiny. A main figure whose ideas contradicted Piaget's ideas was the Russian psychologist Lev Vygotsky. Vygotsky stressed the importance of a child's cultural background as an effect to the stages of development.

Because different cultures stress different social interactions, this challenged Piaget's theory that the hierarchy of learning development had to develop in succession. Vygotsky introduced the term Zone of proximal development as an overall task a child would have to develop that would be too difficult to develop alone.

6.0 Tutor-Marked Assignment

- 1 Identify Piaget's stages of intellectual development
- 2 Describe in details all the stages of Piaget's stages of intellectual development.
- 3 What are the implications of Piaget's stages of intellectual development to the teaching and learning of mathematics.

7.0 References/Further Readings

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UNIT 3: Writing Objectives Using Bloom's Taxonomy**CONTENTS**

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Writing Objectives Using Bloom’s Taxonomy
 - 3.2 Bloom’s Ranking of Thinking Skills
 - 3.3 Examples of objectives written for each level of Bloom’s Taxonomy
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
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1.0 Introduction

Behavioural objectives are means of conceiving instructional strategy in a form that requires a specification of what tasks the students are expected to be able to perform, under what conditions and how such tasks will be evaluated. The process of learning is an individual experience for each student.

According to behaviourist school of psychology, learning takes place whenever an individual's behaviour is modified, that is when he thinks or acts differently; or when he has acquired new knowledge or a new skill and so forth. Thus the concept of behavioural objectives as a significant educational strategy is similar to the concept of operational definition of terms developed in science some years ago "to eliminate hypothetical concepts by defining a concept in terms of the steps or operations whereby the physical reality of the concept could be observed or measured" (Dressel, 1977).

2.0 Objectives

At the end of the unit you should be able to:

- state Bloom’s Taxonomy of education objectives
- identify the attributes of each level of the taxonomy
- write behavioural objectives

3.0 Main content

Writing Objectives Using Bloom's Taxonomy

Various researchers have summarized how to use Bloom’s Taxonomy. Following are four interpretations that you can use as guides in helping to write objectives using Bloom’s Taxonomy. Bloom’s Taxonomy divides the way people learn into three domains. One of these is the cognitive domain, which emphasizes intellectual outcomes. This domain is further divided into categories or levels. The keywords used and the type of questions asked may aid in the establishment and encouragement of critical thinking, especially in the higher levels.

Level	Level Attributes	Keywords	Questions
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<p>1 Knowledge</p>	<p>Exhibits previously learned material by recalling facts, terms, basic concepts and answers.</p>	<p>who, what, why, when, omit, where, which, choose, find, how, define, label, show, spell, list, match, name, relate, tell, recall, select</p>	<p>What is...? How is...? Where is...? When did --- happen? How did - happen? How would you explain...? Why did...? How would you describe...? When did...? Can you recall...? How would you show...? Who were the main...? Can you list three...? How would you classify the type of...? How would you compare...? Contrast...? Will you state or interpret in your own words...? How would you rephrase the meaning...? What factors or ideas show...?</p>
<p>2 Comprehension</p>	<p>Demonstrating understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions and stating main idea.</p>	<p>compare, contrast demonstrate, interpret explain, extend, illustrate, infer, outline, relate rephrase, translate summarize, show, classify</p>	<p>Which statements support...? Can you explain what is happening... what is meant...? What can you say about...? Which is the best answer...? How would you summarize...?</p>
<p>3 Application</p>	<p>Solving problems by applying acquired knowledge, facts, techniques and rules in a different way.</p>	<p>apply, build, choose construct, develop interview, make use of organize, experiment with plan, select, solve, utilize model, identify</p>	<p>How would you use...? What examples can you find to...? How would you solve using what you have learned...? How would you organize to show...? How would you show your understanding of...? What approach would you use to...? How would you apply what you learned to develop...? What other way would you plan to...? What would result if...? Can you make use of the facts to...? What elements would you choose to change...? What facts would you select to show...? What questions would you ask in an interview with what...?</p>

4: Analysis	Examining and breaking information into parts identifying motives or causes; making inferences and finding evidence to support generalizations.	analyze, categorize, classify, compare, contrast, discover, dissect, divide, examine, inspect, simplify, survey, take part in, test for, distinguish, list, distinction, theme, relationships, function, motive, inference, assumption, conclusion	What are the parts or features of...? How is related to...? Why do you think...? What is the theme...? What motive is there...? Can you list the parts...? What inference can you make...? What conclusions can you draw...? How would you classify...? How would you categorize...? What evidence can you find...? What is the relationship between...? Can you make a distinction between...? What is the function of...? What ideas justify...?
5 Synthesis	Compiling information together in a different way by combining elements in a new pattern or proposing alternative solutions.	build, choose, combine, compile, compose, construct, create, design, develop, estimate, formulate, imagine, invent, make up, originate, plan, predict, propose, solve, solution, suppose, discuss, modify, change, original, improve, adapt, minimize, maximize, delete, theorize, elaborate, test, improve, happen, change	What changes would you make to solve...? How would you improve...? What would happen if...? Can you elaborate on the reason...? Can you propose an alternative...? Can you invent...? How would you adapt to create a different...? How could you change (modify) the plot (plan)...? What could be done to minimize (maximize)...? What way would you design...? What could be combined to improve (change)...? Suppose you could what would you do...? How would you test...? Can you formulate a theory for...? Can you predict the outcome if...? How would you estimate the results for...? What facts can you compile...? Can you construct a model that would change...? Can you think of an original way for the...?
6 Evaluation	Presenting and defending opinions by making judgments about information, validity of ideas, quality of work based, on a set of	award, choose, conclude, criticize, decide, defend, determine, dispute, evaluate, judge, justify, measure, compare, mark, rate, recommend, rule on, s	Do you agree with the actions...? with the outcomes...? What is your opinion of...? How would you prove...? disprove...? Can you assess the value or importance of...? Would it be better if...? Why did they (the character) choose...? What would you recommend...? How would you rate the...? What would

	criteria.	elect, agree, interpret, explain, appraise, prioritize, opinion, support, importance, criteria, prove, disprove, assess, influence, perceive, value, estimate, influence, deduct	you cite to defend the actions...? How would you evaluate...? How could you determine...? What choice would you have made...? What would you select...? How would you prioritize...? What judgment would you make about...? Based on what you know, how would you explain...? What information would you use to support the view...? How would you justify ...? What data was used to make the conclusion...? Why was it better than...? How would you prioritize the facts...? How would you compare the ideas ...? People ...?
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3.2 Bloom’s Ranking of Thinking Skills

Knowledge	Comprehension	Application	Analysis	Synthesis	Evaluation
List, name, identify, show, define, recognize, recall, state, visualize	Summarize, explain, Interpret, describe, compare, paraphrase, differentiate, demonstrate, classify	Solve, illustrate, calculate, use, interpret, relate, manipulate, apply modify	Analyse, organize, deduce, contrast, compare, distinguish, discuss, plan, devise	Design, hypothesis, support, schematize, write, justify	Evaluate, choose, estimate, judge, report, defend, criticize

According to Benjamin Bloom, and his colleagues, there are six levels of cognition:

1. Knowledge: rote memorization, recognition, or recall of facts
2. Comprehension: understanding what the facts mean
3. Application: correct use of the facts, rules, or ideas
4. Analysis: breaking down information into component parts
5. Synthesis: combination of facts, ideas, or information to make a new whole
6. Evaluation: judging or forming an opinion about the information or situation

Ideally, each of these levels should be covered in each course and, thus, at least one objective should be written for each level. Depending on the nature of the course, a few of these levels may need to be given more emphasis than the others.

3.3 Examples of objectives written for each level of Bloom's Taxonomy and activities and assessment tools based on those objectives.

Common key verbs used in drafting objectives are also listed for each level.

Level	Level Attributes	Keywords	Example Objective	Example Activity	Example Assessment
1 Knowledge	Rote memorization, recognition, recall of facts.	List, recite, define, name, or match, recall, label, recognize	“By the end of this course, student will be able to recite Newton’s laws of motion.”	Have students group up and perform simple experiments to the class showing how one of the laws of motion works.	Use the following question on an exam or home work. “Recite Newton’s three laws of motion.”
2 Comprehension	Understanding what the facts mean.	Describe, explain, paraphrase, restate, give original examples of, summarize, interpret, discuss	“By the end of this course, the student will be able to explain Newton’s three laws of motion in his/her own words.”	Group students in to pairs and have each pair think of words that describe motion. After a few minutes, ask pairs to volunteer some of their descriptions and write these words on the board.	Assign the student to write a simple essay that explains what Newton’s laws of motion means in his/her words.
3 Application	Correct use of the facts, rules, or ideas.	Calculate, predict, apply solve, illustrate, use, demonstrate, determine, model	By the end of this course, the student will be able to calculate the kinetic energy of projectile.	After presenting the kinetic energy equation in class, have the students pair up for just a few minutes and practice using it so that they feel comfortable with it before been assessed.	On a test, define a projectile and ask the students to “Calculate the kinetic energy of the projectile.”

4 Analysis	Breaking down information in to component parts.	Classify, outline, breakdown, categorize, analyze, diagram, illustrate	“By the end of this course, the student will be able to differentiate between potential and kinetic energy.”	Present the student with different situations involving energy and ask the students to categorize the energy as either kinetic or potential then have them explain in detail why they categorized it the way they did, thus breaking down what exactly makes up kinetic and potential energy	Give the student an assignment that asks them outline the basic principles of kinetic and potential energy. Ask them to point out the differences between the two as well as how they are related.
5 Synthesis	Combining parts to make a new whole	Design, formulate, build, invent, create, compose, generate, derive, modify, develop	By the end of this section of the course, the student will be able to “Design an original homework problem dealing with the principle of conservation of energy.”	Tie each lecture or discussion to the previous lecture or discussion before it, thus helping the student assemble all the discrete classroom sessions in to a unified topic or theory.	Give the student a project in which they must design an original homework problem dealing with the principle of conservation of energy.
6 Synthesis	Judging the value or worth of information or ideas.	Choose, support, relate, determine, defend, judge, grade, compare, contrast, argue, justify, convince, select, evaluate	“By the end of the course, the student will be able to determine whether using conservation of energy or conservation of momentum would be more appropriate following a	Have different groups of students solve the same problem using different methods, then have each groups present pros and cons of the method they choose.	On a test, describe a dynamic system and ask the students which method they will use to solve the problem and why.

			dynamic problem.”		
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4.0 Conclusion

Behavioural objectives have been defined as desired outcome of learning which is expressed in terms of observable and/or measurable behaviour or performance. In contrast to an educational aim which only stipulates changes that cannot be observed or measured, behavioural objectives spell out what the learners should be able to do as a consequence of the learning experiences associated with the objectives. It has been found that Bloom's taxonomy on cognitive domain lends itself to a number of adaptations suitable for formulating behavioural objectives in science instruction.

5.0 Summary

The six levels of cognition as proposed by Bloom are:

1. Knowledge: rote memorization, recognition, or recall of facts
2. Comprehension: understanding what the facts mean
3. Application: correct use of the facts, rules, or ideas
4. Analysis: breaking down information into component parts
5. Synthesis: combination of facts, ideas, or information to make a new whole
6. Evaluation: judging or forming an opinion about the information or situation

6.0 Tutor-Marked Assignment

1. List Bloom's taxonomy of educational objectives
2. Pick a mathematics topic and write behavioural objectives for each of the Bloom's levels of cognition.

7.0 References/Further Readings

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<http://teaching.uncc.edu>

UNIT 4: INNOVATIONS IN TEACHING OF MATHEMATICS**CONTENTS**

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main content
 - 3.1 Need for Innovations in Teaching Mathematics
 - 3.2 Innovations in Teaching Mathematics
 - 3.3 Guidelines for a Teacher in Incorporating Innovations in Teaching Mathematics
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
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1.0 Introduction

It is a common saying that a business man who does today's work with yesterday's tools should not expect to remain in business tomorrow. Life is dynamics, so is education and teaching techniques. There are lots of innovations in the teaching of mathematics. This unit shall explore these innovations.

2.0 Objectives

At the end of the unit you should be able to:

List and discuss at least four innovative methods of teaching mathematics

3.0 Main content**3.1 Need for Innovations in Teaching Mathematics**

Though Mathematics being so important a subject and occupying a central position since the Ancient period, still it has not been the interest of many students. The gaps are found between aspiration and achievement. Mathematics is highly abstract. It is concerned with ideas rather than objects; with the manipulation of symbols rather than the manipulation of object. It is a closely-knit structure in which ideas are interrelated. Mathematical concepts are hierarchical and interconnected, much like a house of cards. Unless lower-level concepts are mastered, higher-level concepts cannot be understood. Students who discover some of the structures of mathematics, are often impressed by its beauty. They note the lack of contradiction, and they see how a new technique can be derived from one that has already been learned.

Teaching of mathematics is not only concerned with the computational know-how of the subject but is also concerned with the selection of the mathematical content and communication leading to its

understanding and application. So while teaching mathematics one should use the teaching methods, strategies and pedagogic resources that are much more fruitful in gaining adequate responses from the students than we have ever had in the past. The teaching and learning of mathematics is a complex activity and many factors determine the success of this activity. The nature and quality of instructional material, the presentation of content, the pedagogic skills of the teacher, the learning environment, the motivation of the students are all important and must be kept in view in any effort to ensure quality in teaching-learning of mathematics.

Innovations and innovative practices in teaching mathematics, is discussed under teaching methods, strategies and pedagogic resources. The process of innovation is generally described as consisting of three essential steps, starting with the conception of an idea, which is then proposed and is finally adopted. Though many ideas have been conceived to bring about change in the teaching of mathematics, it is yet to be proposed and adopted. So, the innovations discussed may not be new in terms of the idea but is new in terms of practice.

Looking to the aims of teaching mathematics it can be seen that more focus is laid to the higher level of objectives underlying the mathematics subject, like critical thinking, analytical thinking, logical reasoning, decision-making, problem-solving. Such objectives are difficult to be achieved only through verbal and mechanical methods that are usually used in the class of mathematics. The verbal methods of instruction give all importance to speech and texts, to the book and to the teacher. From an historical point of view this method was majorly used until the end of the nineteenth century.

In one of these verbal methods teachers are simply satisfied with giving the mathematical rules to pupils and having them memorize it. They justify this method by saying pupils would not understand explanations. Their task is to transmit to their pupils the knowledge which has accumulated over the centuries, to stuff their memory while asking them to work exercises, e.g. the rule of signs and formulas in algebra; students memorize this and remember it! Another verbal method involves explanation. Teachers who use this method assume that the mental structure of the child is same as the adult's. But a developmental stage according to Piaget is a period of years or months during which certain developments take place. Teachers think teaching must imply logic, and logic being linked to language, or at least to verbal thought, verbal teaching is supposed to be sufficient to constitute this logic. This method leads to series of explanations and students at the initial steps of logical explanation trying to understand and grasp but slowly the gap is created between the explanations transmitted by teacher and received by students which lead to the poor understanding on part of students and they develop a fear of the subject: Math phobia. The Education Commission (1964-66) points out that "In the teaching of Mathematics emphasis should be more on the understanding of basic principles than on the mechanical teaching of mathematical computations". Commenting on the prevailing situation in schools, it is observed that in the average school today instruction still confirms to a mechanical routine, continues to be dominated by the old besetting evil of verbalism and therefore remains dull and uninspiring.

SELF ASSESSMENT EXERCISE

Write a brief note on the need for innovations in the teaching of mathematics

3.2 Innovations in Teaching Mathematics

Innovations in teaching of mathematics can be diversified in terms of Methods, Pedagogic Resources and Mastery Learning Strategy used in teaching-learning process.

1. Mastery Learning Strategy

Teaching Strategy is a generalized plan for a lesson and includes a specific structure to be followed. B.S. Bloom has developed Mastery Learning Strategy. It is a new instructional strategy that is used for developing mastery learning and objectives of curriculum can be realized. It consists of different steps: division of content into units, formulation of objectives related to each unit, teaching and instruction are organized for realizing objectives of each unit, administering unit tests to evaluate the mastery level and diagnose the learning difficulties, remedial instructions are given to remove the difficulties and attain mastery level by every student. This strategy plays an important role for learning of basics and fundamentals e.g. operations in different number systems – Natural numbers, Integers, Rational numbers, Real numbers.

2. Methods

Method is a style of the presentation of content in classroom. The following are the innovative methods that can be used to make teaching-learning process of Mathematics effective.

Inducto-Deductive Method

It is a combination of inductive and deductive method. Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples. In classroom usually the instructions directly start with the abstract concepts and are being taught in a way that does not bring understanding on the part of a majority of the students. Formulas, theorems, examples, results are derived, proved and used. But teacher needs to start with specific examples and concrete things and then move to generalizations and abstract things.

The teacher again needs to show how generalization can be derived and it holds true through specific examples. This method will help students for better understanding, students don't have to cram the things and will have long lasting effect.

Example: Pythagoras Theorem - In a right-angled triangle $\triangle ABC$ right angled at B , $AB^2 + BC^2 = AC^2$ (Considering right angle triangles of different measurement leading to generalization and then establishing it through the theoretical proof).

Analytico-Synthetic Method

It is a combination of Analytic and Synthetic method. Analytic is breaking down and moving from unknown to known and Synthetic is putting together known bits of information and moving from known to unknown. These methods are basically used in proving the results and solving sums. In textbooks mostly synthetic method is used, to prove something unknown we start with a certain

known thing, but that leaves doubt in mind of students why we have started with that step and using this particular known thing. So teacher has to use combination in order to explain and relate each step logically.

Example: If $ba = dc$ then prove that $d(a - ab) = b(c - 2ad)$.

Synthetic Method Analytic Method

$$ba = dc \quad \square \quad ba - 2a = dc - 2a \quad (\text{Why??})^* \quad d(a - 2ab) = b(c - 2ad)$$

*the doubt raised in students mind is being solved with the help of analytic method

$$d(a - 2ab) = b(c - 2ad) \quad ba \quad 2ab = dc \quad 2ad \quad ba - 2a = dc - 2a \quad ba = dc$$

$$\square \quad -\square \quad -\square \quad \square \quad \square$$

Problem-Solving Method

This method aims at presenting the knowledge to be learnt in the form of a problem. It begins with a problematic situation and consists of continuous meaningful well-integrated activity. Choose a problem that uses the knowledge that students already have, i.e. you as a teacher should be able to give them the problem and engage them without spending time in going over the things that you think they should know. After students have struggled with the problem to get solution, have them share their solutions. This method will help them in developing divergent thinking.

Example: Put a problem of finding the amount of water in a given container instead of deriving the formula of volume (cylinder filled with water).

Play-Way Method

This method consists of the activities that include a sort of fun or play and give joy to the students. Students don't realize that they are learning but in a way they are gaining knowledge through participating in different activities.

This method helps to develop interest in mathematics, motivates students to learn more and reduces the abstract nature of the subject to some extent.

Example: Mathematical games and puzzles.

Laboratory Method

Laboratory method is based on the principles of "learning by doing" and "learning by observation" and proceeding from concrete to abstract. Students do not just listen to the information given but do something practically also.

Principles have to be discovered, generalized and established by the students in this method. Students learn through hands-on experience. This method leads the student to discover mathematical facts. After discovering something by his own efforts, the student starts taking pride in his achievement, it gives him happiness, mental satisfaction and encourages him towards further achievement.

Example: Making and observing models, paper folding, paper cutting, construction work in geometry.

3. Pedagogic Resources

Pedagogic resources are the resources that a teacher may integrate in a method for the transaction of a particular content and draw upon to advance the students' learning.

Teaching Aids

Teaching aids are the materials used for effective teaching and enhancing the learning of students. It can be anything ready-made or made by the teacher or made by students. Different teaching aids should be used in teaching mathematics like Charts, Manipulatives, Programmed Learning Material (PLM), computers and television.

Charts—It can be used to display formulae, symbols, mathematical and geometrical figures. Charts can be used for making students familiar to the symbols and for memorization of basic formulae. Even it can be used to bring to the students two-dimensional geometry and the graphical representation in a better way.

Manipulatives—They are objects or materials that involve mathematics concepts, appealing to several senses, which can be touched and moved around by the students (not demonstrations of materials by the teacher). Each student needs material to manipulate independently. With students actively involved in manipulating materials, interest in mathematics will be aroused. Canny (1984) has shown that mathematics instruction and students' mathematics understanding will be more effective if manipulative materials are used. Models can be used to make things concrete like three dimension figures in geometry.

Programmed Learning Material (PLM)—It is a self-learning material in which learner can proceed at his own pace. It has the characteristic of all sequential steps, learner's response, self-pacing, immediate feedback, reinforcement and self-evaluation. It is helpful in acquisition of concepts like fractions, numbers systems, etc. and can be used as a remedy for slow learners for a specific content.

Computers and Television—Computer can be used for multimedia presentation for the concepts that require visualization and imagination. Computer can also be used for providing Computer Assisted Instruction (CAI), it is similar to PLM, i.e. it is a computerized PLM. Television can be used to show some good mathematics education show.

Activities

Activities here include all such work where in students play an active role, have to interact with different resources and generate knowledge. It includes Quiz competition, Projects, Roleplay, Seminars, Discussion, Mathematics club, Assignment, Field trips, etc.

Name of the Activity Examples/Situations where Activity can be used

Quiz Competition

Logic, Properties of Numbers, Mathematical Rules and Results

Projects Contribution by Different Mathematicians

Role Play

Arithmetical concepts like Profit & Loss, Simple & Compound Interest

Seminars

Shortcut through Vedic Mathematics,

Application of Mathematics in other Disciplines

Discussion

Properties of 'Zero', Difference between Rational and Irrational Numbers, Relating Different Concepts in Mathematics

Mathematics Clubs

Application of the concept studied, Preparing Models, Paper Folding (Origami) Assignment Self-Study, Extension of Knowledge

Field Trips

Experiencing the Functional use of Mathematics in Bank, Insurance Company

In any curriculum, content and presentation of content are the two most important and inseparable components. It is difficult to say anything definitely about which method and pedagogic resource is going to be most effective for presentation of a particular type of content. Selection of method and pedagogic resource depends on many factors like type of content, objectives to be achieved, level of the students, entry behaviour, availability of resources. Also acceptance of innovative methods and positive attitude of teacher towards it, is an important factor for the selection of method and pedagogic resource. The things included under innovations are existing in books, also there are researches which show that some innovations are carried out in the classroom and has shown the positive effect on teaching learning process but their practical usage and implementation in classroom is not seen to the expected level.

3.3 Guidelines for a Teacher in Incorporating Innovations in Teaching Mathematics

For effective transaction of the curriculum and achievement of curricular objectives appropriate method and pedagogic resources should be used in providing learning experiences to the students.

A number of factors need to be considered while making use of a particular method and pedagogic resource: learners' capabilities, availability of resources, entry behavior, school environment, objectives to be achieved, the nature of content and the teacher's own preparation and mastery.

Decide on and plan in advance the innovative idea that the teacher would be incorporating to transact a particular concept so that loss of instructional time is prevented or minimized.

The immediate environment of the learner both natural and human should be used when and where possible for making learning concrete and meaningful.

Involve the students in the process of learning by taking them beyond the process of listening to that of thinking, reasoning and doing.

In order to promote self-study skills use of library and resource center needs to be encouraged.

Receiving regular feedback for teaching and learning should be an inbuilt component of teaching-learning process. Continuous and comprehensive evaluation has to be ensured as it plays an important role for the required modification in teaching-learning process. Mathematics-teachers' organizations at different levels should be formed where sharing of ideas and experiences, developing resources in a collaborative manner and the mechanism that enable teachers to carry out innovations is being discussed. Mathematics-teachers' organizations can be instrumental in establishing a climate of confidence in carrying out innovations and a positive attitude to new approaches in teaching mathematics.

Properly instruct and guide the students for carrying out different activities and precautionary measures should be taken so that students are not misguided. Study mathematical journals and modern books of professional interest. Any facilities of in-service training should be availed off for improving teaching of mathematics.

The teacher can always ask himself two questions: 1. 'Is there some new way in which I can present this material in order to make it more meaningful and more interesting?' 2. 'What activities, demonstrations, teaching aids, etc. would enrich the classroom presentation and direct attention of students to the important elements?' Once the teacher discovers innovative ways to arouse interest and enthusiasm in the class, he will be able to use these ideas again the following year, since those will be new and fascinating to a different class. But teachers should keep in mind that as time passes, the world undergoes a change, the environments surrounding students' changes and their needs also changes, so one has to continuously go on modifying and discovering new ways of teaching which proves him a better teacher.

4.0 Conclusion

It is important for teachers to have a repertoire of knowledge of several innovative ways of presenting mathematics to their students.

5.0 Summary

In this unit some innovative methods were discussed namely, inductive-deductive, analytic-synthetic, problem-solving, play-way, laboratory methods. Some teaching resources were also discussed such as: charts, manipulatives, programmed learning material (PLM), computers and television. Some guidelines for a teacher in incorporating innovations in teaching mathematics were provided.

6.0 Tutor-Marked Assignment

- (a) Why is innovative teaching necessary in mathematics classroom?
- (b) Discuss as much as possible four innovative ways of teaching mathematics.

7.0 References / Further Readings

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