MODULE 4 ASSESSMENT IN MATHEMATICS EDUCATION AND BASIC MATHEMATICSPROPERTIES

UNIT 1 CLASSROOMASSESSMENT TECHNIQUE

CONTENTS

1.0 INTRODUCTION

Assessment is important because of the decisions you will make about children when teaching and caring for them. You will be calleduponevery day to make decisions before, during, and after yourteaching. Whereas some of these decisions will seem small and inconsequential,otherswillbe“highstakes,”influencingthelifecourseof children. All of your assessment decisions taken as a whole will direct and alterchildren’s learning outcomes. Assessment can enhance your teachingand students’ learning and if you use assessment procedures appropriately, youwillhelpallchildrenlearnwell.

2.0 OBJECTIVES

Attheendofthisunit,you shouldbeableto:

- statethecharacteristicsofclassroomassessment
- mention the assumptions on which classroom assessment is baseduponanddistinguishbetweenthem.

3.0 MAINCONTENT

3.1 ClassroomAssessment

In the 1990's, educational reformers are seeking answers to two fundamental questions: (1) how well are students learning? (2)howeffectively are teachers teaching? Classroom research and classroomassessmentresponddirectlytoconcernsaboutbetterlearningandmore effective teaching. Classroom research was developed to encourage teacherstobecomemoresystematicand sensitiveobserversoflearning as it takes place every day in their classrooms. Teachers have an exceptional opportunity to use their classrooms as laboratories for thestudy of learning and through such study to develop a better understanding of the learning process and the impact of their teachingupon it. Classroom assessment, a major component of classroomresearch, involvesstudentandteachersinthecontinuous monitoring ofstudents' learning. It provides teacher with feedback about their effectiveness, and it gives students a measure of their progress as learners. Most important, because classroom assessments are created, administered, and analysedby teachers themselves on questions of teaching and learning that are important to them, the likelihood thatinstructorswillapplythe resultsoftheassessmenttotheirownteachingisgreatlyenhanced.

Through close observation of students in the process of learning, thecollection of frequent feedback onstudents' learning, and the design ofmodest classroom experiments, teachers can learn much about howstudents learnand,morespecifically,how students respondtoparticular teaching approaches. Classroom assessment helps individual teachers obtainusefulfeedbackonwhat, howmuch, andhowwelltheirstudents are learning. Teachers can then use this information to refocus theirteaching to help students make their learning more efficient and moreeffective.

Teachers,whousuallyassumethattheirstudentsarelearningwhatthey aretryingtoteachthem,areregularlyfacedwithdisappointingevidenceto the contrary when they mark tests and at the end of theterm. Toooften, students have not learned as much or as well as was expected. Therearegaps, sometimesconsiderableones, betweenwhatwas taught and what has been learned. By the time teachers notice these gaps inknowledge or understanding, it is frequently too late to remedy theproblems.Toavoidsuchunhappy surprises, teachers andstudentsneedbetterways to monitor learning throughout the term. Specifically, teachers need a continuous flow of accurate information on students’ learning.

For example, ifateacher'sgoalis tohelp studentslearnpoints"A"through "Z" duringthecourse,thenthatteacherneedsfirsttoknowwhetherallstudents are really starting at point "A" and, as the teaching proceeds, whethertheyhavereachedintermediatepoints "B," "G," "L," "R," "W," and so on. To ensure high-quality learning, it is not enough to teststudents whenthesyllabus has arrivedatpoints "M" and"Z." Classroomassessment is particularly useful for checking how well students are learning at those initial and intermediate points, and for providing informationforimprovementwhenlearningisless thansatisfactory.

Throughpractice inclassroom assessment, teachers become better able tounderstandandpromotelearning,andincreasetheir abilitytohelpthe students themselves become more effective, self-assessing and self-directed learners. Simply put, the central purpose of classroom assessment is to empower both teachers and their students to improvethequalityoflearningintheclassroom.

Classroomassessmentis anapproachdesignedtohelpteachersfindout what students are learning in the classroom and how well they are learningit.Thisapproachhasthefollowingcharacteristics:

- **Learner-Centred**
  Classroom assessment focuses the primary attention of teachers and studentsonobservingandimprovinglearning,ratherthanonobservingand improving teaching.Classroomassessmentcanprovideinformationtoguideteachers and studentsinmakingadjustmentstoimprovelearning.

- **Teacher-Directed**
  Classroom assessment respects the autonomy, academic freedom, and professional judgementof teachers. The individual teacher decides what toassess, how to assess, and how to respond to the information gained throughtheassessment. Also,theteacheris notobliged tosharetheresultofclassroom assessmentwithanyoneoutsidetheclassroom.

- **MutuallyBeneficial**
  Because it is focused on learning, classroom assessment requires the active participation of students. By cooperating in assessment, students reinforce their grasp of the course content and strengthen their own skills at self- assessment. Their motivation is increased when they realisethat teachers areinterestedandinvestedintheirsuccess aslearners.Teachers alsosharpentheirteachingfocus bycontinuallyaskingthemselves three questions:"whatarethe essential skills and knowledge I am trying to teach?" "How can I find outwhetherstudentsarelearningthem?""HowcanI helpstudents learn better?"

  As teachers work closely with students to answer these questions, they improvetheir teachingskillsandgainnewinsights.

- **Formative**
  The purpose of classroom assessment is to improve the quality of studentlearning, not to provide evidence for evaluating or grading students. Theassessmentsarealmostnevergradedandarealmostalways anonymous.

Context-Specific
Classroom assessments have to respond to the particular needs and characteristics of the teachers, students, and disciplines to which they areapplied. Whatworkswellinoneclasswillnotnecessaryworkin another?

Ongoing
Classroom assessment is an ongoing process, best thought of as the creating and maintenance of a classroom "feedback loop." By using anumber of simple classroom

assessment techniques that are quick andeasy to use, teachers get feedback from students on their learning.Teachers thencompletetheloopbyprovidingstudents withfeedbackon theresultsoftheassessment and suggestionsforimprovinglearning.To check on the usefulness of their suggestions, teachers use classroomassessment again, continuing the "feedback loop." As the approachbecomes integrated into everyday classroom activities, thecommunications loop connectingteachersand students-andteachingand learning-becomes moreefficient andmoreeffective.

SELF ASSESSMENT EXERCISE

Write a brief note on the characteristics of classroom assessment.

3.2 ClassroomAssessmentandGoodTeaching Practice

Classroom assessment is an attempt to build on existing good practice by making feedback on students' learning more systematic, more flexible, and more effective. Teachers already ask questions, react to students'questions, monitorbodylanguageandfacialexpressions,readhomework andtests, andsoon. Classroomassessmentprovidesawayto integrate assessment systematically andseamlessly into the traditional classroomteachingandlearningprocess

As they are teaching, teachers monitor and react to student questions, comments, bodylanguage,andfacialexpressionsinanalmostautomatic fashion. This "automatic" information gathering and impression formation is a subconscious and implicit process. Teachers depend heavily on their impressions of student learning and make important judgments based on them, butthey rarely make thoseinformal assessments explicit or check them against the students' own impressions or ability to perform. In the course of teaching, teachers assume a great deal about their students' learning, but most of theirassumptionsremainuntested.

Even when teachers routinely gatherpotentially useful information on students’learningthroughquestions,quizzes, homework,andexams, it is oftencollectedtoolate-atleastfromthestudents'perspective-toaffect their learning. In practice, it is very difficult to "de-program" students whoareusedtothinkingofanythingtheyhavebeentestedandgraded on as being "over and done with." Consequently, the most effective timestoassessandprovide feedbackarebeforestartinganewtopicor the midterm and final examinations. Classroom assessment aims at providingthatearly feedback.Classroomassessmentis basedon seven assumptions:

1. thequalityof students’ learning is directly, although not exclusively, related to the quality of teaching. Therefore, one of the most promising waystoimprovelearning isto improve teaching

2. toimprove their effectiveness, teachers need first to make their goals and objectives explicit and then to get specific, comprehensible feedback on the extent to which they are achievingthosegoalsandobjectives

3. toimprove their learning, students need to receive appropriate andfocusedfeedbackearlyandoften;theyalsoneedtolearnhow toassesstheir

ownlearning

4. thetype of assessment most likely to improve teaching and learning is that conducted by teachers to answer questions they themselveshaveformulatedinresponsetoissuesorproblems intheir ownteaching

5. systematicinquiryandintellectualchallengearepowerfulsources of motivation, growth, and renewal for teachers, and classroom assessmentcanprovidesuchchallenge

6. classroomassessmentdoesnotrequirespecialisedtraining;itcan becarriedoutbydedicatedteachersfromalldisciplines

7. bycollaborating with colleaguesand actively involving students in classroom assessment efforts, faculty (and students) enhance learningandpersonalsatisfaction.

Tobeginclassroomassessment,itis recommendedthatonlyoneortwo of the simplest classroom assessment techniques are tried in only one class.Inthisway, verylittleplanningorpreparationtimeandenergyof the teacher and students is risked. In most cases, trying out a simple classroomassessmenttechniquewillrequireonlyfivetotenminutes of class timeandless thananhouroftimeoutofclass.Aftertryingoneor two quick assessments, the decisionasto whether thisapproach is worth further investments of time and energy can be made. This process of startingsmallinvolvesthree steps.

FiveSuggestionsforaSuccessfulStart:

1. ifa classroom assessment techniques does not appeal to your intuitionandprofessionaljudgmentasateacher, donotuseit

2. donot make classroom assessment into a self-inflicted chore orburden

3. do not ask your students to use any classroom assessmenttechniqueyouhavenotpreviouslytriedonyourself

4. allowfor moretimethanyouthinkyouwillneedtocarryoutand respondtotheassessment

5. makesureto"closetheloop." Letstudentsknowwhatyoulearnfromtheir feedback and how you and they can use thatinformationtoimprovelearning.

4.0 CONCLUSION

The aim of assessment is to improve students' performance and not merely to audit it. Assessment should be learner-centered and focused on students'achievementinrelationtothegoalsofacourse.Ratherthanbeing separate from learning, assessment plays a central role in the instructionalprocess.

5.0 SUMMARY

Assessment helps teachers develop more complex relationships with their students by providing concrete pieces of work for students and teachers to discuss, as well as opportunities for formal and informal conversations about the work. Similarly, students work closely with eachother providingandreceivingfeedbackontheir work.

Assessmenthelpsstudentsanswer thequestions"AmIgettingit?"and"HowamIdoing?" Earlyandfrequentfeedbackfromtheteacher,peers,and mentors will also provide students with the practice and the knowledge to better assess themselves and find answers to these questions.

Assessment can help make content connections clear. Teachers promptstudents to make connections between their work and other subjectmatter.

Assessment also sheds light on which methods of instruction are most effective.Throughassessment,ateachergainstherequisiteinformationfor choosing and utilisingthose teaching strategies that best help alearnerprogresstowardsthegoalsofacourse.

6.0 TUTOR-MARKEDASSIGNMENT

i. Whyisclassroomassessmentimportant?
ii. Discuss the assumptions on which classroom assessment is based.

7.0 REFERENCE/FURTHERREADING

Thomas, A. A.&Cross, P. K. (n.d).*From ClassroomAssessmentTechniques*:*AHandbookforCollegeTeachers*.

UNIT 2 PURPOSEANDTOOLSOFASSESSMENT

CONTENTS

1.0 INTRODUCTION

The popular conception of assessment is restricted to evaluating individual student's performance by tests designed to determine, at the endofaunitoftimeorinstruction,whatthestudenthasalreadylearned.But assessment should also be used during the learning process to enable teachers to monitor students' understanding and to modify curriculum and instruction, as well as to assess the effectiveness ofschool programs. Assessment of an individual student's performanceshouldbeacontinuousprocessthatinvolvesmanytypesof assessmentactivity. Students should play active roles in assessment so that eachassessmentexperienceis alsoaneducationalexperience.

2.0 OBJECTIVES

Attheendoftheunit,you shouldbeableto:

- statethepurposeofassessment
- distinguishbetweenthecontent principle,thelearningprincipleand theequityprincipleastheyrelatetomathematicsteachingand learning.

3.0 MAINCONTENT

3.1 PurposeofAssessment

It is important to establish at the outset that the major purpose of assessmentis topromotelearning.Theassessmentis notthegoal, buta means to achieve a goal. Three fundamental educational principles which form the foundation of all assessment that supports effectiveeducation(measuringwhatcounts)are:

- **TheContentPrinciple:**thissuggeststhatassessment should reflectthemathematicsthatismostimportantfor studentsto learn.
- **TheLearningPrinciple**:thissuggeststhatassessment should enhancemathematics learningandsupportgoodinstructional practice.
- **TheEquityPrinciple**:thissuggeststhatassessment should supporteverystudent'sopportunitytolearnimportant mathematics.

New Jersey'smathematicsstandardstatesthatexperienceswillbesuch thatallstudents:

(i) areengagedinassessmentactivitiesthatfunctionprimarilyto improvelearning
(ii) areengagedinassessmentactivities baseduponrich, challengingproblemsfrommathematicsandotherdisciplines
(iii) areengagedinassessmentactivitiesthataddressthecontentof thecurriculum

The content principle, the learning principle, and the equity principle wereincorporatedintothe firstthreeofthesixassessment standardsintheNCTM*AssessmentStandardsforSchoolMathematics*(2000).

(i) Assessment shouldreflectthemathematicsthatall studentsneed toknowandbeabletodo.
(ii) Assessmentshouldenhancemathematicslearning.

Assessments should be learning opportunities as well as opportunitiesfor students to demonstrate what they know and can do. Althoughassessmentis doneforavarietyofreasons,itsmaingoalis to improvestudents' learning and inform teachers as they make instructionaldecisions. As such, it should be a routine part of ongoing classroomactivityratherthananinterruption.

(i) Assessment shouldpromoteequity

Assessment should be a means of fostering growth toward high expectationsratherthanafilterusedtodenystudentstheopportunityto learn important mathematics. In an equitable assessment, each studenthasanopportunitytodemonstratehisorher mathematicalpower; this can only be accomplished by providing multiple approaches to assessment,adaptationsfor bilingualandspecialeducation students, and other adaptations for students with special needs. Assessment is equitable when students have access to the same accommodations andmodificationsthattheyreceiveininstruction.

(ii) Assessment shouldbeanopenprocess

Threeaspectsofassessmentareinvolved here.First,information aboutthe assessment process should be available to those affected by it, thestudents.Second,teachersshouldbeactiveparticipantsinallphasesof theassessmentprocess.Finally, theassessmentprocessshouldbe opentoscrutinyandmodification.

(iii) Assessment shouldpromotevalidinferencesaboutmathematics learning

Avalidinferenceis basedonevidencethatis adequateandrelevant.The amount and type of evidence that is needed depends upon the consequences of the inference. For example, a teacher may judge students' progress in understanding place value through informal interviewsandusethisinformationtoplanfutureclassroomactivities.

(iv) Assessment shouldbeacoherentprocess

Threetypes ofcoherenceareinvolvedinassessment.First, thephases of assessment must fit together. Second, the assessment must match thepurpose forwhichitis beingconducted.

Finally, the assessment must be aligned with the curriculum and with instruction.

These principles should be kept in mind as changes in assessmentstrategies are contemplated, developed, tested, and implemented. Theyshouldbekeptinmindbyclassroomteachersandall othersinvolvedinassessment — for example, local education authority committees selectingastandardisednorm-referencedtest, local educationinspectors or school headmasters analysingdata from a collection of students' portfolios, and mathematics curriculum planners reviewing proposedtestitems forthestatewidetests.

Self Assessment Exercise

At what point in time is ideal to assess your pupils' performance and why?

4.0 CONCLUSION

Assessment plays a crucial role in the education process. It determinesmuch of the work students undertake (possibly all in the case of themoststrategicstudent),andaffectstheir approachtolearning. Itcanbeargued that assessment is an indication of which aspects of the courseare valued most highly. The assessment of outcomes provides thefeedbacknecessarytomakesoundeducational decision.

5.0 SUMMARY

Assessmentissupportedbythecontentprinciple**,** thelearningprinciple andequityprinciple. Assessment shouldreflectthemathematicswhichall students need to know and be able to do, should enhance mathematicslearning,shouldbeanopenprocess, shouldpromotevalid inferencesaboutmathematicslearningandshouldbeacoherentprocess.

6.0 TUTOR-MARKEDASSIGNMENT

i. Identifythree fundamentaleducationalprincipleswhichformthe foundationofallassessmentthat supportseffectiveeducation.
ii. Listareasinmathematicsthatassessment should support.

7.0 REFERENCES/FURTHERREADINGS

OxfordCentreforStaffandLearningDevelopment.(2002).*Purposes andPrinciplesofAssessment*.www.brookes.ac.uk/services/ocsd/

Tewksbury,B.J &Macdonald,R.H(2005).'AssessingStudent Learning:On-lineCourseDesignTutorial'.http://serc.carleton.edu/NAGTWorkshops/coursedesign/tutorial/T OC.html

UNIT 3 BASICNUMBERPROPERTIES: ASSOCIATIVE,COMMUTATIVEAND DISTRIBUTIVE

CONTENTS

1.0 INTRODUCTION

There are three basic properties of numbers, and your textbook will probably have just a little section on these properties, somewhere near thebeginningofthecourse,andthenyouwillprobablyneverseethem again (until the beginning of the *next* course). My impression is thatcovering these properties is a holdover from the "New mathematics" fiasco of the 1960s. While the topic will become relevant in matrixalgebra and calculus (and become amazingly important in advancedmathematics,acouple yearsafter calculus),theyreallydonot matterawholelotnow.

Why not? Because every mathematics system you have ever workedwith has obeyed these properties! You have never dealt with a systemwhere $a\times b$didnotinfactequal$b\times a$, forinstance,or where$(a\times b)\times c$did notequal $a\times(b\times c)$.Thatiswhythepropertiesprobablyseemsomewhat pointless to you. Do not worry a b o u t their "relevance" for now; just makesureyoucankeepthepropertiesstraight soyoucanpassthenext test.Thelessonbelowexplains howIkepttrackoftheproperties.

2.0 OBJECTIVES

Attheendofthisunit,you shouldbeableto:

- statethedistributivepropertyofnumbers
- applythedistributivepropertytosolvearithmeticand simple
- algebraicproblems
  statetheassociativepropertyofnumbers

- applytheassociativepropertytosolvearithmeticandsimple algebraicproblems
- statethecommutativepropertyof numbers
- applythecommutativepropertytosolvearithmeticandsimple algebraicproblems.

3.0 MAINCONTENT

3.1 DistributiveProperty

The distributive property is easy to remember, if you recallthat "multiplication *distributes*over addition". Formally, they write thispropertyas "$a(b+c)=ab+ac$".Innumbers,this means,that$2(3+4)=2\times3 + 2\times4$. Any time they refer in a problem to using the distributive property, they want you to take something through theparentheses*(or factor something out)*; any time a computation depends on multiplying throughaparentheses*(orfactoringsomethingout)*, theywantyoutosaythatthecomputation usedthedistributiveproperty.

Whyis thefollowingtrue?$2(x+y)=2x+2y$

Since they distributed through the parentheses, this is true **by theDistributiveProperty.**

UsetheDistributivePropertytorearrange:$4x–8$

Thedistributivepropertyeither takessomethingthroughaparenthesisorelse factors something out. Since there are not any parentheses to go into, you must need to factor out of. Then the answer is "**By the DistributiveProperty,$4x–8=4(x–2)$".**

"But wait!"yousay "thedistributive property says multiplicationdistributesover*addition,* not *subtraction!* Youcan either view the contents of the parentheses as the subtraction of a positive number ("$x–2$") or else as the additionof a negative number ("$x+(–2)$"). In the latter case, it is easy to see thatthe distributive property applies, because you are still adding; you are just adding a negative.

The other two properties come in two versions each: one for addition and the other for multiplication. (Note that the distributive property referstobothadditionandmultiplicationtoo, buttobothwithinjustonerule).

3.2 AssociativeProperty

The word "associative" comes from "associate" or "group". TheAssociativePropertyis therulethatreferstogrouping. For addition,the ruleis "$a+(b+c)=(a+b)+c$";innumbers,this means2+(3+4)= (2+3)+4.For multiplication,theruleis"$a(bc)=(ab)c$";innumbers,this means 2(3×4) = (2×3)4. Any time they refer to the associativeproperty, they want you to regroup things; any time a computation depends on things being regrouped, they want you to say that the computationusestheassociativeproperty.

Rearrange,usingtheAssociativeProperty:2(3*x*)

They want you to regroup things, not simplify things. In other words, they do not want you to say "6*x*". They want to see the following regrouping:**(2×3)*x*.**

Simplify2(3*x*),andjustifyyoursteps.

Inthiscase, they *do*wantyoutosimplify, butyouhavetotellwhyitis okayto do just exactly what you have *always* done. Here is how this works:

2(3*x*)	**original(given)statement**
(2×3)*x*	**bytheAssociativeProperty**
6*x*	**simplification(2×3= 6)**

Whyis ittruethat2(3*x*)= (2×3)*x*?

Since all they did was regroup things, this is true by the AssociativeProperty.

3.3 CommutativeProperty

Theword "commutative" comesfrom"commute" or"movearound",so thecommutativepropertyistheonethatreferstomovingstuffaround. For addition,theruleis "$a+b=b+a$";innumbers,thismeans2+3=3+2. For multiplication,theruleis "$ab=ba$";innumbers,thismeans2×3=3×2. Anytimetheyrefertothecommutativeproperty,theywantyou to move stuff around; any timeacomputation depends on moving stuff around, they want you to say that the computation uses the commutativeproperty.

Use the Commutative Property to restate "3×4×*x*" in at least two ways.

They want you to move stuff around, not simplify. In other words, the answeris not"12*x*";theansweris anytwoofthefollowing:

4× 3× *x*, 4×*x*× 3, 3× *x*× 4,*x*× 3× 4, and*x*× 4× 3

Whyis ittruethat3(4x)= (4x)(3)?

Sincealltheydidwas movestuffaround(theydidnotregroup),thisis truebytheCommutativeProperty.

3.4 WorkedExamples

Simplify3a–5b+ 7a. Justifyyoursteps.

Iam goingtodotheexactsamealgebraIhavealwaysdone,butnowI havetogive thenameof thepropertythatsaysitis okayformetotake eachstep.Theanswerlooks likethis:

3*a*–5*b*+ 7*a*	**Original(given)statement**
3*a*+ 7*a*–5*b*	**CommutativeProperty**
(3*a*+ 7*a*)–5*b*	**AssociativeProperty**
a*(3+ 7) –5*b	**DistributiveProperty**
a*(10) –5*b	**Simplification(3+7=10)**
10*a*–5*b*	**CommutativeProperty**

The only fiddly part was moving the "– 5b" from the middle of the expression (in the first line of the table above) to the end of the expression(inthesecondline).Ifyouneedhelpkeepingyour negativesstraight, convert the "–5b" to "+ (–5b)". Just do not lose that minussign!

Simplify23+ 5x+ 7y–x–y–27. Justifyyoursteps.

23+ 5*x*+ 7*y*–*x*–*y*–27	**Original(given)statement**
23–27+ 5*x*–*x*+ 7*y*–*y*	**CommutativeProperty**
(23–27)+(5*x*–*x*)+(7*y*–*y*)	**AssociativeProperty**
(–4)+ (5*x*–*x*)+ (7*y*–*y*)	**Simplification(23–27=–4)**
(–4)+ *x*(5–1)+*y*(7–1)	**DistributiveProperty Simplification**
–4+ *x*(4) + *y*(6)	**CommutativeProperty**
–4+ 4*x*+ 6*y*	

Simplify 3(x + 2)–4x. Justify your steps.

3(x+ 2) –4x	**original(given)statement**
3x+ 3×2–4x	**DistributiveProperty**
3x+ 6–4x	**simplification(3×2= 6)**
3x–4x+ 6	**CommutativeProperty**
(3x–4x)+ 6	**AssociativeProperty**
x(3–4)+ 6	**DistributiveProperty**
x(–1)+ 6	**simplification(3–4= –1)**
–x+ 6	**CommutativeProperty**

4.0 CONCLUSION

This unit has treated: the distributive property ofnumbers; how to applythe distributive property to solve arithmeticand simple algebraic problems, and how to apply the commutative property to solve arithmeticandsimplealgebraicproblems.

5.0 SUMMARY

In this unit, you have learnt how to state the distributive property of numbers, howtoapplythedistributivepropertytosolvearithmeticand simplealgebraic problems, and how to apply associative propertytosolvearithmeticandsimplealgebraicproblems,etc.

6.0 TUTOR-MARKEDASSIGNMENT

i. Whyisittruethat3(4+x)=3(x+4)? Whyis3(4x)=(3×4)x?
ii.
iii. Whyis12–3x=3(4–x)?

7.0 REFERENCES/FURTHERREADINGS

Basic Number Properties: Associative, Commutative, and Distributive. http://www.purplemathematics.com/modules/numbprop.htm

Stapel, E.(2011). ‘Basic Number Properties: Associative, Commutative, and Distributive.Purple Mathematics’.Availablefromhttp://www.purplemathematics.com/modules/numbprop.htm.

UNIT 4 OTHER NUMBER PROPERTIES: IDENTITIES, INVERSES, SYMMETRY

CONTENTS

1.0 INTRODUCTION

We need to know that "the identity" is whatever does not change your number atall,and"theinverse" is whateverturns your numberintothe identity. For addition, "the identity" is zero, because adding zero toanything does not change anything. The "inverse" is theadditiveinverse:itis thesamenumber, butwiththeoppositesign. For instance,supposeyournumberis –6,andyouareadding.Theidentityis zero,and theinverseis6,because–6+6=0.

For multiplication, "the identity" is one, because multiplying by onedoes not change anything. The "inverse" is the multiplicative inverse:the same number,but on the opposite side ofthe fraction line.Forinstance,supposeyournumberis –6, andyouaremultiplying.The identity is one, and the inverse is–1/6, because(–6)(–1/6)=1.

Youalsoknow(ifyouhavedoneanyequationsolving)thatyoucando anything you want to an equation,as long asyoudo the same thingtobothsides. This is the"propertyofequality".

2.0 OBJECTIVES

Attheendofthisunit,you shouldbeableto:

- apply trichotomylaw, transitive (moving across) property, the reflexiveproperty, thesymmetricproperty,theadditiveidentityand themultiplicativeinversetosolvemathematicalproblem.

The basic fact that you need for solving many equations, especially quadratics, is that, if $p\times q=0$, then must have either $p = 0$ or else$q = 0$. Theonlywayyoucanmultiplytwothingsandendupwithzeroisif one (or both) of those two things was zero to start with. This is the "zero-product property".And there are some properties thatyouuseto solve word problems, especially where substitution is required. Anything equals itself: this is the "reflexive" (reflecting onto itself)property.Also, itdoesnotmatterwhichordertheequalityisin;if $x=y$,then$y=x$:this is the"symmetric"(theymatch)property. Youcan "cutoutthemiddleman",sotospeak;if$x=y$and$y=z$,thenyoucansay that$x=z$:thisis the"transitive"(movingacross)property.Twonumbers areeither equaltoeachotherorunequal;this is the"trichotomy"law(socalled becausetherearethreecasesfortwogivennumbers,$a<b$, $a=b$, or$a>b$). Andyoucanplugin forvariables,soif$x$=3,then4x=12, because$x$=4(3):this is the"substitution"property.

Here are some examples.Note: textbooks vary somewhat in the names theygivetheseproperties;youwillneedtorefertotheexamples inyour booktoknowtheexactformatyou shoulduse.

3.0 MAINCONTENT

3.1 DeterminewhichPropertywas Used

1×7= 7

They multiplied, and they did not change anything: **the multiplicative identity.**

–7*y*= –7*y*

'his is obvious:anything equals itself. They used **the reflexive ›roperty.**

f 10= *y*, then*y*= 10.

Whensolvinganequation,Imightrearrangethingsso Iendupwiththe variableontheleft.ButIonlyswitchedsides;Idid notactuallychange anything:**thesymmetricproperty.**

x*+ 0= *x

Theyadded, andtheydidnotchangeanything:**theadditiveidentity. If 2(*a***

+ *b*)= 3*c*, and*a*+ *b*= 9, then2(9)= 3*c*.

You might be confused here between the transitive property andthe substitution property. If you look closely, what they did was substitute "9"for"$a+b$",sotheyused**thesubstitutionproperty.**

2= *x*, so2+5=*x*+ 5

Theydidthebackwardsofsolvinganequation, butthepointis thattheywere working with an equation. They changed the equation by adding equalthingstobothsides: **theadditivepropertyof equality.**

If x+ 2= 10,then$x$+ 2+(–2) equalswhat,andwhy?

They solvedtheequation by gettingrid ofthe2fromboth sides.Since theyaddedthesamethingtobothsides, theygot**x=8bytheadditive propertyof equality.**

(x–3)(x+ 4) = 0, so$x$= 3or$x$= –4.

They set the quadratic equal to zero, factored, and then solved each factor:**thezero-productproperty.**

4x= 8, so$x$= 2

They solved the equation by dividing both sides by 4, or, which is thesame thing, multiplying both sides by (1/4). In other words, theychanged the equation by doing thesame multiplying to both sides:**the multiplicativepropertyof equality.**

If $x$is not equalto$y$andnot lessthan$y$, whatmust betrueof$x$,andwhy?

By the trichotomylaw, there are only three possible relationships between$x$and$y$,andtheyhaveeliminatedtwoofthem. Then **$x>y$, by thetrichotomylaw.**

$x+(-x)=0$

Theyadded, andtheyendedupwithzero:**theadditiveinverse.**

$$\left(3/_3\right)\left(2/_5\right)+\left(5/_5\right)\left(4/_3\right)=6/_{15}+20/_{15}$$

Theyconvertedtoacommondenominatorbymultiplyingbothfractions byausefulformof1;rememberthat$3/_3$and$5/_5$arejust1!Sotheyused **themultiplicativeidentity.**

If 5x= 0, what is x, andwhy?

You can do this in either of two ways: multiply both sides by$1/_5$**(the multiplicative property of equality)** and then get**x = 0**, oryou couldsaythat, since5doesn'tequalzero,then$x$mustequalzero**(bythezero- product property)**.

$$\left(2/_3\right)\left(3/_2\right)=1$$

They multiplied, and they ended up with one: **the multiplicative inverse.**

If 3x+ 2=$y$and$y$= 8,then3x+ 2= 8.

You might be confused here between the transitive property and the substitutionproperty. Whatthey did here was "cut outthe middleman"by removing the "*y*" in the middle, so they used **the transitive property.**

If –*x*= 14, whatdoes*x*equal,andwhy?

To solve this, you would multiply both sides by a negative one, to canceloffthe minussign. So:

***x*= –14,bythemultiplicativepropertyof equality.**

If *x*= 3and*y*= –4,thenwhatdoes*xy*equal,andwhy?

Bysubstitution(plugginginforthevariables), youget(3) (–4).Inother words:

xy=–12, bythesubstitutionproperty**.**

Can*x*<*x*?Whyorwhynot?

Bythereflexiveproperty, *x*=*x*. Bythetrichotomylaw,if*a*=*b*then*a* cannotbelessthan *b*.Sotheansweris "**no, bythereflexiveproperty andthetrichotomylaw**"

4.0 CONCLUSION

Applying the trichotomylaw, transitive (moving across) property, the reflexiveproperty,thesymmetricproperty,theadditiveidentityandthe multiplicative inverse to solve mathematics problem are the key basics thatleadtotheunderstandingof mathematicsandteachersshould stressthelearningofthesebasicprinciples.

5.0 SUMMARY

Identities, inverses, and symmetry are the basic concepts needed to prove some basic properties of mathematics such as: the multiplicativeidentity; the reflexive properties; the symmetric property; the identitypropertyandthesubstitutionproperty.

6.0 TUTOR-MARKEDASSIGNMENT

i. List and define the basic concepts needed to propertiesof provesome mathematics.

7.0 REFERENCE/FURTHERREADINGS

Stapel,E. (2011). ‘Other Number Properties: Identities, Inverses, Symmetry.’.http://www.purplemathematics.com/ modules/numbprop2.htm.